THE TUTORIAL PHYSICS

VOLUME III.

A TEXT-BOOK OF LIGHT

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THE TUTORIAL PHYSICS

VOLUME III.

A TEXT-BOOK OF LIGHT

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PREFACE TO THE FOURTH EDITION.

THIS book now contains a very full treatment of the elements of Geometrical Optics; but those portions of the subject which require advanced mathematics are not included, and no mention is made of Physical Optics, with the exception of the necessary references to the wave-theory.

The practical side of the subject has received very careful attention. Light is prolific of simple experiments, and these are essential for the verification and explanation of the mathematical theories. Most of the experiments given in the book require only very simple apparatus, and many practical illustrations are of course derived from the common phenomena of daily life

A large number of diagrams has been given, for in this subject the figures are especially important. The necessity for making clear and correct drawings for the solution of problems should be strongly impressed on the student.

The book is supplied with a good collection of problems, for practice in calculation is another very important element in the study of this subject. Students should remember that it is an absurdity to give a numerical result to four of five significant figures, if the original data are probably correct only to three, and a worse absurdity to give the numerical result as a recurring decimal.

Many students experience much difficulty in applying

the convention of signs; but it is hoped that the rules and explanations here given will be found very helpful on this point.

The chapter on Dispersion is longer than is usual in books of this size, but this should be sufficiently justified by the importance and interest of this branch of the subject. Spectrum analysis has already proved of the greatest use in chemistry and astronomical physics, and most of the future discoveries in Light will probably be based upon the ideas contained in this chapter.

NOTE ON THE FIFTH EDITION.

The only change in this edition is the insertion of a chapter on Polarisation, of Intermediate University standard, written by Mr. A. W. Humphreys, B.Sc. (Lond.).

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CHAPTER I.

INTRODUCTORY.

1. **Light** is the external physical cause of the sensation called sight.

2. **The Cause of Light.** Light is generally believed to have its origin in vibration of the luminous body, just as sound originates in vibration of the sonorous body, and to be transmitted to the eye as sound is to the ear by means of undulations in the intervening medium. There are, however, these important differences between the two phenomena. First, that whereas the sonorous body vibrates as a whole, it is only the *molecules* of the luminous body that are in vibration. Secondly, that while the frequency of sonorous vibrations is only a few hundreds or thousands per second, the frequency of luminous vibrations amounts to *hundreds of billions* (about 400 to 800 billions) per second.

Bodies which of themselves produce light are said to be *self-luminous*. Many bodies are able to reflect light from self-luminous objects, and thus become luminous themselves. Thus the sun is self-luminous, but the moon is rendered luminous only by reflecting the sun's light.

3. **The Transmission of Light. The Luminiferous Ether.** A further point of difference between light and sound is found in the medium by which they are propagated. A

properly isolated bell sounding in a good vacuum is quite inaudible, but it is no less visible than before removal of the air. And we know that light from the stars reaches our eyes after traversing millions of millions of miles of "empty" space. As our senses give us no evidence of anything in these vacuum spaces, physicists have been forced to fill them with an *imaginary substance* which they call the *luminiferous ether*.* It is quite unlike any form of matter with which we are acquainted. It is probably devoid of weight, and is perfectly elastic, and so far would appear to most nearly resemble an exceedingly rarefied gas; but the undulations it transmits are known to be transverse, and such therefore as no gas, by reason of its being devoid of cohesion, is competent to transmit. The kind of matter to which it appears to bear the closest resemblance is an extremely attenuated jelly.

This ether is supposed not only to occupy all space, but to interpenetrate all matter, and to lie between the molecules of even the densest solids, as air lies between the leaves and branches of a tree.

It may now be possible for us to picture to ourselves the vibration of the molecules of the luminous body setting up undulations in the ether which travel with inconceivable speed† in all directions. Some of these undulations falling on the eyes, there set up changes in the optic nerve, which, when transmitted to the brain, produce the sensation of light. But whatever may be the nature of light, there are some fundamental properties, established by experiment, which may be studied quite independently of any hypothesis on this point.

In the pages that follow we propose to study in this way a few of the more important of these fundamental properties; but, as the undulatory theory is now so completely established, reference will be made to it whenever it seems advisable.

* This must not be confounded with the very tangible liquid called ether by chemists.

† About 186,000 miles per second, a speed that would carry it about 7½ times round the earth in a second.

4. Light is invisible. Remembering what light is—simply undulations in an invisible medium—this statement ought to cause little surprise. When we apparently see a beam of sunlight entering through a small window into an otherwise dark room, what we really see is not the light itself but a number of floating particles in the air illuminated by the beam. Many of these are so large as to be easily visible separately as dancing motes. If a lighted Bunsen burner be brought below the beam so as to burn up or volatilize these particles, the luminous track will be interrupted by what appears to be black smoke rising from the flames. But the Bunsen flame is perfectly smokeless, and the black spaces are full of dust-free air, consequently there is nothing in those parts to reflect the light, and it remains invisible.

5. A Medium is the name given to any substance through which light passes. A *transparent medium* is one which transmits the light which enters it more or less completely. Possibly no medium except the ether allows all the light that enters it to pass through; a portion of the light is reflected or absorbed by the medium, and that which emerges is consequently less bright. However, a medium which transmits the greater portion of the light entering it, is called *transparent*, *e.g.*, water, glass, mica. Media which permit little or none of the light which falls on them to pass through are *opaque*, *e.g.*, wood, iron, lampblack. Media which transmit light to some extent but do not enable one to see clearly through them are called *translucent*, *e.g.*, wax, ground glass, china.

Bodies which in their ordinary state appear perfectly opaque, really transmit a very considerable amount of light when obtained in sufficiently thin laminæ.

If a gold-leaf, supported between two pieces of glass, be held to the light, it will appear semi-transparent and bright green. A stone may be ground down sufficiently thin to become transparent. A piece of cardboard, which under ordinary conditions appears perfectly opaque, transmits much light when held close before an electric arc; and if

the hand be similarly held it is possible to see the bones through the semi-transparent flesh. On the other hand, water, though very transparent, absorbs so much light as to make the sea-bottom below a few hundred fathoms perfectly dark. Therefore the terms *transparent*, *translucent*, and *opaque* refer to a difference in degree more than in kind, and for this reason it is perhaps more correct to speak of transparent, translucent, and opaque *bodies* than to apply these terms to *substances*.

A medium is *homogeneous* when it is uniform throughout in composition, structure, and properties. A medium which is not uniform is called *heterogeneous*.

6. Ray—Beam—Pencil. These terms are of such frequent use in connection with light that it becomes necessary to

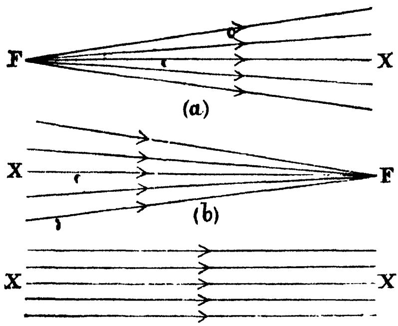


Fig. 1.

define them. A *ray* is strictly only a mathematical conception, a line which may be taken as lying in a direction at right angles to the wave fronts.

In other media than air, glass and liquids, the rays are not necessarily at right angles to the wave fronts. The direction of the ray at a point may be defined as the straight line joining the centre of a small spherical obstacle, situated at that point, to the centre of the shadow produced by it on a screen, an infinitely small distance beyond it in the direction in which the light is travelling.

A *beam* of light is a collection of adjacent rays, and may be *divergent*, *convergent*, or *parallel*—that is, the component rays may diverge from, or converge to, a point, or run parallel (Fig. 1, *a*, *b*, *c*). A convergent or divergent beam has the form of a cone of small, but finite angle, while a parallel beam is a cylinder of small cross section. The *axis* of a beam, F X (Fig. 1), is the central ray passing along the geometrical axis of the figure of the beam, and the point, F, from or to which the rays of a beam diverge or converge, is called its *focus*. The focus of a parallel beam is at infinity. *Pencils* are very narrow beams, and like beams may be parallel, divergent, or convergent. When a pencil of light comes from a point on a very distant source such as the sun, moon, or a star, it is considered to be parallel, although, strictly speaking, it is very slightly divergent.

CHAPTER II.

RECTILINEAR PROPAGATION OF LIGHT.

7. Light travels in straight lines through the same homogeneous medium. Many familiar phenomena point to the fact that light travels through the same homogeneous medium in straight lines. If two screens be each pierced with a small hole and then held one in front of the other, in such a position (Fig. 2) that the two holes and a candle flame are in the same straight line, a ray of light can pass from the candle through the holes to an eye placed in the same straight line behind the screens; but, if either of the screens be but slightly displaced in its own plane, the candle becomes invisible.

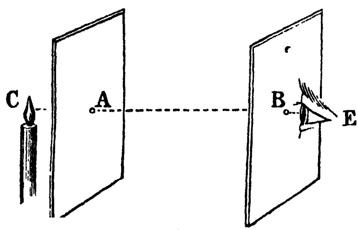


Fig. 2.

Similarly, if a scale be laid on the bottom of a vessel, and looked at over the edge of the vessel, in the way indicated in Fig. 3, it will be found that the line E A S is a straight line. In both these experiments the same medium (air) extends between the eye and the object seen; but if, in the latter example, water be poured into the vessel, it will be found that a point, S′, on the scale can be seen, and that E A O S′ is not a straight line. Hence, when light passes from one medium to another, it is in general bent out of its direct rectilinear path; and, from what has been said, it is evident that the bending must take place at the surface of separation of the two media. When, however, a ray of light travels

through a non-homogeneous medium, it may suffer gradual and continuous change of direction, if the change in the properties of the medium along its path are also gradual and continuous; it is only on passing from one homogeneous medium to another that sudden change of direction takes place. The magnitude and direction of this change depend on conditions which we shall consider more fully in later chapters.

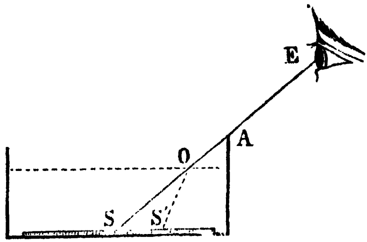


Fig. 3.

It should be noticed in connection with this point that the eye takes no cognizance of change of direction in a ray of light; every object is seen in the direction taken by the axis of the pencil of light which enters the eye. For example, if rays of light, starting from O (Fig. 4), be bent, as indicated in the figure, then O appears to the eye to be at O′. The point O′ is the *virtual focus* of the pencil entering the eye, called *virtual* because its rays do not really diverge from O′, but only appear to do so.

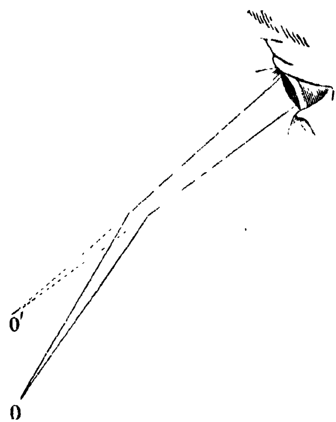


Fig. 4.

8. The pinhole camera. If a sheet of cardboard, pierced, at its centre, with a large pinhole, be placed between a candle and a thin paper screen, shaded from external light, a more or less distinct representation of the candle flame will be seen, in an inverted position, on the screen (Fig. 5). If the cardboard form the front, and

the screen the back of a closed box, the representation can be seen very distinctly from behind, through the paper (or ground glass) screen. The explanation of this is simple. Let A B (Fig. 6) represent the candle flame or other brightly illuminated object, O the hole in the cardboard, and S S the screen. From every point on A B rays are given off in all directions, and consequently from every point of A B a small pencil of rays passes through O, and forms a small circular or elliptical spot on the screen. The result of this is that we have on the screen an assemblage of nearly circular spots, which, owing to the crossing of the rays at O, define an *inverted* representation of the object A B. If these spots are large

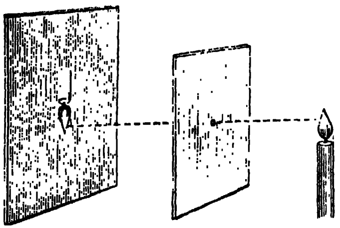


Fig. 5.

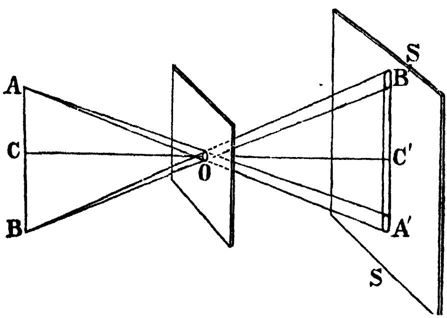


Fig. 6.

they overlap one another, and the representation is blurred and indistinct; hence, in order to obtain a well-defined picture of A B on the screen, the aperture at O must be very small; for the size of the spot on the screen depends, for given positions of A B and S S, upon the size of aperture.

9. Shadows. The formation of shadows is a direct consequence of the rectilinear propagation of light. If an opaque body B (Fig. 7) be placed so as to intercept a portion of the light emitted by a luminous point L, the cone of light incident on the surface of the body is stopped, and the space beyond, enclosed by the geometrical continuation of this cone, is screened from the rays diverging from L.

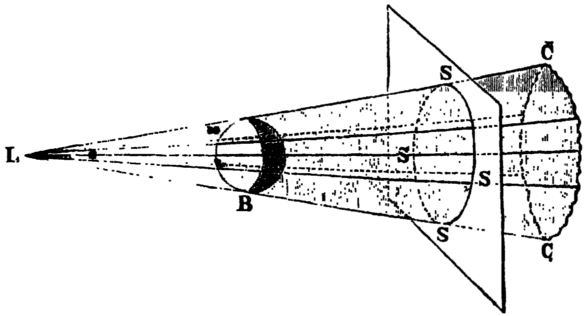


Fig. 7.

The cone here considered ,L C C, is called the *shadow cone*, and its trace ,S S S S, on any surface intersecting it beyond B, outlines the *shadow* cast on this surface. When, however, the source of light is not a luminous point, but a luminous

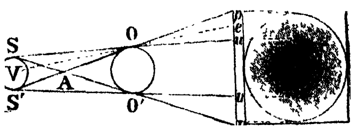


Fig. 8, *a*.

body, the case is somewhat more complicated. Let S S′ (Fig. 8, *a*) represent a spherical source of light, and O O′ an opaque sphere placed near it. Consider the single cone S S′ *u u*, which encloses S S′ and O O′; it is evident from the figure that *no* light from S S′ falls within the portion of this cone lying beyond O O′; and for this reason it has been called the cone of *total shadow*, or the cone of the *umbra*;

and the portion of it just referred to as being completely screened from the light is called the umbra. Consider again the double cone S S′ A pp, enclosing S S′, and O O′, and having its apex at A. This is the cone of *partial shadow*, or the cone of the *penumbra*, and the portion of it beyond O O′, and surrounding the umbra, is known as the *penumbra*. From any point in the cone, not within the total shadow, a portion of the source of light can be seen, and for this reason the shadow is only partial. The depth of shadow at any point depends on the extent of the source invisible from that point; to an eye placed at e all below $e\,V$ is invisible, while all above is visible; hence, at points near the outer boundary of the penumbral cone, the shadow is very light, but gradually deepens as we approach the outer boundary of the umbral cone.

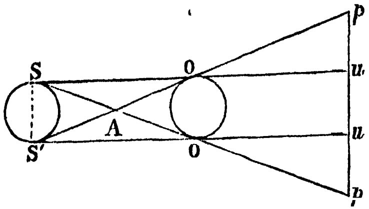


Fig. 8, *b*.

The penumbral cone always has the form of a cone diverging from a point (A) lying between S S′ and O O′, but the form of the umbral cone depends on the relative size of the source of light and the opaque body; when the latter is the greater the cone diverges from a point behind the former (Fig. 8, *a*), when equal it takes the form of a cylinder (Fig. 8, *b*), and when smaller the cone converges to a point beyond the opaque body (Fig. 8, *c*). If the shadow of the opaque body be cast upon a suitably placed screen (Fig. 8, *a*), it will be found to consist of a central region of total shadow, the umbra, surrounded by a zone of partial shadow, the penumbra. The former is of uniform depth all over, but the latter passes gradually from the

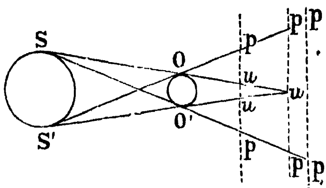


Fig. 8, *c*.

total shadow of the umbra to a complete absence of shadow at its outer boundary, and consequently neither its outer nor its inner edge is sharply defined. The relative size of the umbra and penumbra in any particular case depends upon the conditions illustrated in Figs. 8, and upon the position of the screen. The reader will find it instructive to draw diagrams for a number of different cases.

10. Eclipses. These are of two kinds, lunar and solar. If at full moon the centres of the sun, earth, and moon are very nearly in a straight line, the earth, acting as a screen, will stop the sun's rays before they reach the moon, and the moon will therefore be either wholly or partially darkened. This phenomenon is called a **Lunar Eclipse**, and may be either total or partial. On the other hand, if the three centres are nearly in a straight line when the moon is new, the moon, by coming between the earth and sun, will cut off the whole or a portion of the sun's rays from certain parts of the earth's surface. In such parts the earth will be darkened, and the sun will appear either wholly or partially hidden. This phenomenon is a **Solar Eclipse**, and may be either total, annular, or partial. It is called annular when the moon is too near the sun to hide it completely, but leaves the rim of the sun's disc visible, like a ring of light round its own dark body.

CHAPTER III.

PHOTOMETRY.

11. Sources of Light. Nearly all light is produced either by burning or by incandescence. The earliest domestic sources of light consisted either of wicks immersed in oil (lamps), or of wicks immersed in solid fat (candles). Both these sources are still in use, and have been greatly improved in recent years. Coal-gas was first artificially prepared (by Murdoch) in 1792, and was then only burnt at naked burners. Argand greatly improved the light of both oil and gas lamps by putting a glass chimney around the flame to increase the draught. Bunsen increased the temperature of the flame by mixing air with the gas before burning. This, however, made the flame non-luminous; but if now a body, especially a refractory oxide, is placed in the flame, it is heated to incandescence and gives out a brilliant light. The outcome of this discovery is the Welsbach lamp. The mantle, which is a gauze-like structure, is made up of 99 per cent. thoria and 1 per cent. ceria; the ceria possessing great light-emitting properties. Acetylene gas, owing to its strongly luminous flame and ease of preparation, is also largely used for lighting.

With the advent of electricity, the electric arc and the electric incandescent lamps * were introduced. The former was invented by Davy in 1801, and is admirable for street illumination and optical lantern work, its light being very white. A newer form of the arc lamp uses carbons, impregnated with metallic salts; the arc itself is the source of light in this case, but the light is strongly coloured. The ordinary incandescent electric lamp is provided with carbon filaments; in more recent forms the filament is

* See *Higher Textbook of Magnetism and Electricity*, Art. 163.

made of osmium or tantalum. The Nernst lamp consists of a filament of certain oxides of rare metals heated to incandescence by an electric current. It is not a vacuum lamp, but has the disadvantage of being rather fragile.

At present there is severe competition between the Welsbach light and the incandescent electric lamp for domestic use, and between compressed gas lamps and arc lamps for street illumination. A still newer lamp is the Cooper-Hewitt mercury vapour lamp. It is an electric vacuum lamp, the light—a brilliant green—being given out by an arc of mercury.

Light as a measurable quantity. Light, like radiant heat, is undoubtedly a form of energy, and, as such, is capable of measurement. The quantity of light in any space at any instant is measured by the corresponding amount of energy, available for purposes of illumination, which is in that space at the instant considered, and the physical intensity of the light is measured by the energy transmitted through that space in unit time. For light of a given colour, the intensity conditions the brightness as perceived by the eye (or by a sensitive plate in a photographic camera). The physical intensity is also strictly proportional to the heating effect, produced in a black surface exposed to the light.

There are thus three methods of measuring the intensity of light—the *ocular* or *photometric*, the *photographic*, and the *calorimetric*. In a photographic camera the light does work in producing chemical changes in the salts on the sensitive film. In the calorimetric method a thermopile,* or some similar instrument, is exposed to the radiation, and the energy received is transformed into heat energy, which produces an electrical effect measurable by a delicate galvanometer. The defect of both the photographic and the calorimetric methods is that they measure other forms of energy physically similar to luminous energy in every respect, except that of being detected by the eye.

* See *Higher Textbook of Magnetism and Electricity*, § 218, and *Higher Textbook of Heat*, § 97 *et seq.*

In this chapter we shall consider the photometric method only; the other methods will be more fully dealt with in Chapter IX.

12. Intensity of light emitted in any direction. Let q denote the quantity of light energy emitted per second by a luminous point or small element of a luminous surface within the limits of a cone of a small solid angle,* ω. Then the limit of the ratio $\frac{q}{\omega}$, when ω, and consequently q, are indefinitely diminished, is the intensity of the emission of light in the direction of the axis of the cone.

13. Intensity of illumination of a surface at a point. Let q denote the quantity of light energy incident per second on a small element of a surface of area s, then the limit of the ratio $\frac{q}{s}$, when s, and therefore q, are indefinitely diminished gives the intensity of illumination of the surface at the point at which s vanishes. If $\frac{q}{s}$ is constant for all points, then the surface is said to be *uniformly illuminated*; and if Q denote the quantity of light energy incident per second upon a portion of the surface of area S, then $\frac{Q}{S}$ determines the intensity of this uniform illumination.

14. Law of inverse squares. The intensity of light diminishes as we recede from the source of light, and increases when we come nearer. With light, as with sound and with radiant heat and other influences which spread from a centre, the intensity at any point in free space depends on the distance of that point from the source in a manner which is expressed in *The Law of Inverse Squares*. The law is that the intensity is inversely proportional to

* Page 29.

the square of the distance, provided the medium is perfectly transparent. That is to say, if the intensity be a certain amount at a distance of 1 ft., then at a distance of 2 ft. the intensity is lessened in proportion to the square of 2 to the square of 1. The square of 2 being 4, the intensity of 2 ft. is one-fourth; at 3 ft. one-ninth of the intensity at 1 ft., and so on.

To illustrate the effect of distance upon radiant energies, the wire pyramid represented in Fig. 9 may be used. The outside lines represent the boundary rays of a cone of light emanating from L. A, B, C are cross sections of the cone at a distance of 1, 2, and 3 ft. respectively. B is clearly

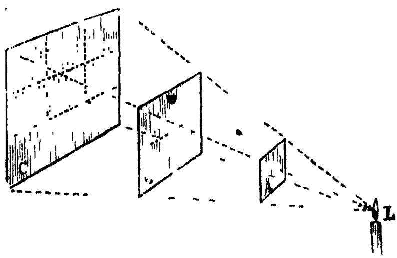


Fig. 9.

four times and C nine times as large as A. Thus the same cone of light which is spread over A, is at B distributed over an area four times as great. The intensity at B is therefore one-fourth of that at A. At C the rays are thinned out so as to illuminate nine times the area they did at A. The intensity at C is one-ninth of that at A.

If a square frame made of wire be held midway between a small source of light and a screen, the shadow of the frame encloses just four times the area which the frame itself does. Hence the light which passes through the frame is scattered over four times the area at double the distance. The intensity at the screen is one-fourth of the intensity at the frame.

To prove the general case let P (Fig. 10) represent a luminous point, and let us conceive this point to be surrounded by a sphere $S_1 S_1 S_1$ of radius, R_1, having its centre at P. Then the inner surface of this sphere will be uniformly illuminated, and if no light be absorbed by the medium the intensity of illumination, I_1, is given by—

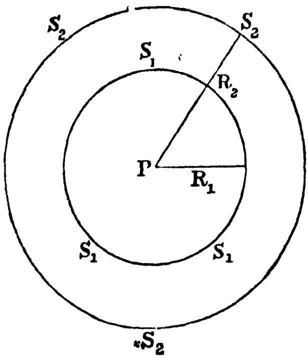


Fig. 10.

$$I_1 = \frac{Q}{4\pi R_1^2} \text{ *}$$

where Q denotes the total quantity of light energy emitted by P per second. Similarly, if we consider the sphere $S_2 S_2 S_2$, the intensity of illumination of its inner surface is given by—

$$I_2 = \frac{Q}{4\pi R_2^2}.$$

Hence we have—

$$\frac{I_1}{I_2} = \frac{Q}{4\pi R_1^2} \Big/ \frac{Q}{4\pi R_2^2} = \frac{R_2^2}{R_1^2}.$$

That is, *the intensity of illumination, of any uniformly illuminated surface, is inversely proportional to the square of its distance from the source of light*; or, in more general terms, *the intensity of illumination, at any point of a surface, is inversely proportional to the square of the distance of that point from the source of light.*†

For a direct experimental proof of this law see Exp. 1.

It follows from the above that if I_1 denote the intensity

* The area of the surface of a sphere of radius $R = 4\pi R^2$.

† In this and the following articles the dimensions of the source of light are considered to be so small, compared with the other distances involved, that it may be treated as a luminous point. If the source of light have a large surface, the illumination for points close up or near to the surface is practically independent of the distance.

of illumination of a surface at unit distance from the source of light, and perpendicular to the rays, then the intensity of illumination I_0 on a similarly placed surface at a distance, R, is given by—

$$I_0 = \frac{I_1}{R^2}, \text{ for, } \frac{I_0}{I_1} = \frac{1}{R^2}; \text{ that is, } I_0 = \frac{I_1}{R^2}.$$

15. The intensity of illumination of a surface varies with the angle of incidence of the light. The angle of incidence is the angle made by the axis of the incident pencil of light with the normal to the surface. Let P A B (Fig. 11) denote a pencil of light emanating from P, and incident on the surface A B at an angle represented by P O N. If Q denote the quantity of light energy emitted per second by P in the pencil P A B, the intensity of illumination of $AB = \frac{Q}{\text{area of } AB} = \frac{Q}{A} = I$. Now imagine the surface A B to be rotated round O into a position A′ B′, where it is at right angles to P O.

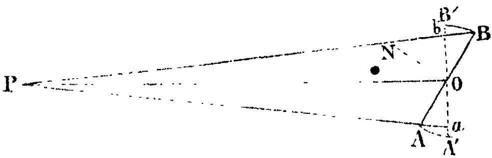


Fig. 11.

The same quantity of light now falls on the smaller area, *ab*, and the intensity of illumination of this area $= \frac{Q}{\text{area of } ab} = \frac{Q}{A_0} = I_0$.

It can, however, be shown (Art. 19) that $A_0 = A \cos BOB' = A \cos PON$, assuming that the area A B is very small, and the angle B b O thus very nearly a right angle.

Therefore we have—

$$\frac{I}{I_0} = \frac{Q}{A} \Big/ \frac{Q}{A_0} = \frac{A_0}{A} = \frac{A \cos PON}{A} = \cos PON.$$

That is, $I = I_0 \cos PON$.

This result may be expressed thus:—If I_0 denote the intensity of illumination of a given surface when the light is incident normally, then, for incidence at an angle θ, the intensity of illumination is given by $I = I_0 \cos\theta$; that is, *the intensity of illumination varies directly as the cosine of the angle of incidence.*

This partly explains why the strength of the sun's heat and light is greater on fields and hillsides which lie directly facing the sun, and less upon places which slope away from the sun, and so lie obliquely to the rays.

Now since $I_0 = I_1/R^2$ and $I = I_0 \cos\theta$ we have $I = \frac{I_1 \cos\theta}{R^2}$, which gives the intensity of illumination of a surface placed at a distance R from the source of light, the angle of incidence being θ, and the source being of such power that the intensity of illumination of a surface placed at unit distance from the source, and perpendicular to the rays, is I_1.

16. The illuminating power of any source of light. The intensity of any source of light is proportional to its illuminating power. *This quantity is measured by the intensity of illumination of unit area of a surface placed at unit distance from the given source, the light being incident normally on this surface.* If L_1 denote the illuminating power of a given source of light (A), then $L_1 = I_1$, and the intensity of illumination produced by this source on a surface, at a distance R_1 when the angle of incidence of the light is θ_1, is given by—

$$I = \frac{L_1 \cos\theta_1}{R_1^2}. \quad \text{(Arts. 14, 15.)}$$

Similarly, if another source of light (B) be so placed as to produce the *same* intensity of illumination (I) for an angle of incidence of the light θ_2, then—

$$I = \frac{L_2 \cos\theta_2}{R_2^2}$$

where L_2 denotes the illuminating power of B, and R_2 its

distance from the illuminated surface. Now, equating these two expressions for I, we have—

$$\frac{L_1 \cos \theta_1}{R_1^2} = \frac{L_2 \cos \theta_2}{R_2^2}.$$

This is the general formula. Usually in the comparison of sources of light it is arranged that $\theta_1 = \theta_2$.

In this case—

$$\frac{L_1}{R_1^2} = \frac{L_2}{R_2^2} \text{ or } \frac{L_1}{L_2} = \frac{R_1^2}{R_2^2}.$$

This shows that *the illuminating powers of different sources of light are* **directly** * *proportional to the squares of the distances they must be placed from a given surface, in order to produce on it the same intensity of illumination.* It should be noticed that the above is true only when the angle of incidence, θ, is the same in each case; hence it is evident, that in an experimental comparison of illuminating powers care must be taken to satisfy this condition.

Note.—It is important to distinguish between the quantities *intensity of light*, *illuminating power*, and *intensity of illumination*. Intensity of light is a quantity involving energy (Arts. **11**, **12**); illuminating power of a source of light is proportional to the intensity of the light, and is measured as stated in Art. **16**; intensity of illumination refers to the surface illuminated and not, like the other two quantities, to the source of light (Arts. **13-15**).

17. Photometers. Photometry is the experimental comparison of the illuminating powers of different sources of light, and the different forms of apparatus by which this comparison is effected are called *photometers*. The practical unit employed in photometry is the light emitted by a standard sperm candle, six to the pound, burning 120 grains per hour; and hence the illuminating power of any source of light is generally expressed as being equivalent to that of a certain number of standard candles. It has been found that the eye is unable to estimate the ratio of the intensity of illumination due to different sources of light, but that it is a better judge of the equality of the

* This statement must be carefully distinguished from that made at the end of Art. 14.

illumination of two adjacent surfaces. For this reason nearly* all methods of photometry depend on the equalisation of the illuminations of two adjacent white screens, and the details of the construction of photometers are devised to facilitate this adjustment. All photometrical experiments must be made in a dark room.

Foucault's photometer. This photometer (Fig. 12) consists of a semi-transparent screen, A B, of thin paper, ground glass, or thin white porcelain, fixed vertically in front of, and at right angles to, a partition, C D, which is movable by means of a screw in the direction of its length. The two sources of light to be compared, L_1 and L_2, are placed on opposite sides of this partition in such positions

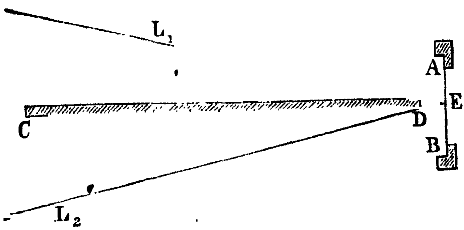


Fig. 12.

that the angle $L_1 E L_2$ is bisected by C D. By this arrangement one portion of the screen is illuminated by one source and the other portion by the other; and, by adjusting the position of C D until these separately illuminated portions become contiguous, their illuminations may be more accurately compared.

When both portions appear equally bright the comparison is complete, and we have, if L_1 and L_2 represent the final positions of the sources of light—

$$\frac{\text{Illuminating power of } L_1}{\text{Illuminating power of } L_2} = \frac{(E\,L_1)^2}{(E\,L_2)^2}.$$

* The dimensions of the screen are small compared with the distances $E\,L_1$ and $E\,L_2$

Rumford's photometer. In this photometer the illuminating powers of two sources of light are compared by adjusting to equality the intensities of the two shadows of a vertical rod cast on a screen by the two given sources.

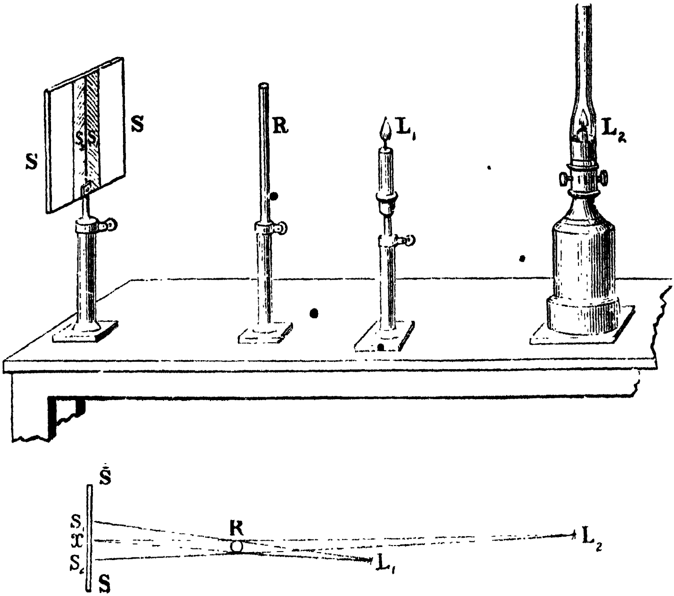


Fig. 13.

The lights (L_1, L_2), rod (R), and screen (*S S*) are arranged, as shown in Fig. 13, so that the two shadows (S_1x, S_2x), whose edges should be well defined, appear close together, and of equal intensity. In this way, since each shadow is illuminated by the source to which the other is due, equality of intensity of the shadows cast by L_1 and L_2 means equality of illumination due to L_2 and L_1; and hence we have, as in the previous case—

$$\frac{L_1}{L_2} = \frac{(L_1 x)^2}{(L_2 x)^2}.$$

Bunsen's photometer. Bunsen has devised a very simple form of photometer. If a sheet of paper having a spot of grease on it be held up to the light, it will be seen that the spot of grease is semi-transparent, and looks brighter than the rest of the paper when viewed from the side remote from the light, but darker when seen from the other side. The reason of this is evident; more light passes through the region of the grease spot than through the rest of the paper, and hence when seen from the side remote from the light it looks brighter than the rest of the screen, through which little or no light passes; when looked at on the other side, however, the spot looks comparatively dark, because a large proportion of the light incident upon it passes through, and is therefore not spent in illuminating its surface. It will now be understood that if a suitable paper screen, having a grease spot at its centre, be placed betweeen two lights, A and B, and its position adjusted until the spot cannot be seen on either side, except by close inspection, then the screen must be equally illuminated on both sides. For, if the grease spot is not readily distinguishable from the adjacent surface of the screen, the amount of light coming from unit area of both must be the same. Let Q denote the quantity of light incident from A on unit area of the surface of the screen, and q the quantity that passes through, per unit area, in the region of the spot. Then $Q-q$ denotes the quantity of light spent in illuminating the surface of unit area on the grease spot; whereas the whole quantity,* Q, is spent in illuminating the surface of unit area of the screen in the neighbourhood of this spot. Hence, on the side of A, the spot will appear dark unless a quantity of light, q, is transmitted through it from B to make up for the quantity that has passed through from A. It thus appears that a necessary condition for the disappearance of the spot is that the quantities of light passing through it in opposite directions must be equal. But, if the surface of

* To simplify matters the screen is considered to be opaque, except at the grease spot, and its surface to be such that there is no regular reflection. The general case is easily treated, and leads, in a similar way, to the same result.

the spot is the same on both sides, the quantity of light that passes through will be, for each side, the same fraction of the light incident on that side; and consequently, if the

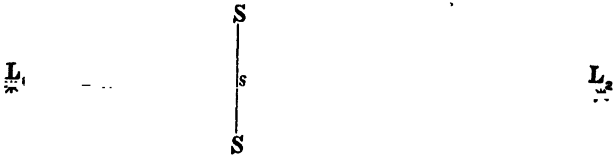


Fig. 14.

quantities of light passing through the unit area of the grease spot are equal, then the quantities of light incident on unit area of opposite sides of the screen are also equal*—that is, the two sides of the screen are equally illuminated. If L_1, L_2, and S S (Fig. 14) represent the relative positions of the lights and the screen when finally adjusted, we have—

$$\frac{L_1}{L_2} = \frac{(L_1 s)^2}{(L_2 s)^2}.$$

In practically carrying out the necessary measurements, at least four different adjustments should be made: (1) Adjust for disappearance of the spot when seen from the side of screen facing L_1. (2) Turn the screen round through 180°, and again adjust for disappearance of spot from the same

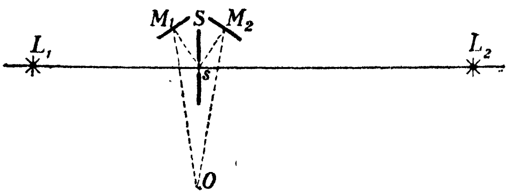


Fig. 15.

surface now facing L_2. (3) Repeat (1) and (2) with the other surface of the screen.

* That is, if $q = \frac{Q}{n}$, then, when q and n are the same for both sides of the screen, Q must also be the same for both sides.

The screen is usually mounted in a light frame, so that it can be easily turned round in its stand, or as a whole.

The stand also often carries two mirrors M_1 M_2, arranged as in Fig. 15, so that an observer at O can view both sides of the grease spot at the same time and nearly under the same conditions.

The **Lummer-Brodhun photometer** is a commercial application of the grease-spot photometer.

Joly's photometer. This consists of two equal blocks of paraffin wax, W_1, W_2 (Fig. 16), about a quarter-inch thick separated by a smooth thin sheet of tinfoil T, and mounted on an open wooden slider B B. The light coming from either source enters the wax and is scattered both before and after

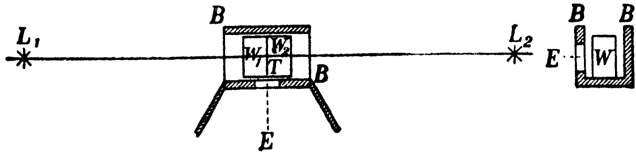


Fig. 16.

reflection from the surface of the tin. The tin is opaque, so that W_1 is illuminated solely by L_1 and W_2 by L_2. In taking a measurement the compound block is moved up and down the line joining the luminous points until a position is found in which W_1 and W_2 appear equally bright to an observer looking at them through a hole in B B. The position of T is then read and we have—

$$\frac{L_1}{L_2} = \left(\frac{L_1T}{L_2T}\right)^2.$$

Flicker photometers. In these instruments a white surface is alternately illuminated by the lights under comparison. When the lights are so placed that the intensities of illumination they produce on this surface are unequal, a "flickering" effect is visible to an eye viewing this surface. The lights are adjusted till this flickering vanishes, and the ordinary law is then applied.

Exp. 1. To prove the Inverse Square Law. Since until the law is proved we have not the means of comparing different sources of light, we must in this experiment use sources which are multiples of some chosen unit source of light. Set up a Bunsen or Joly photometer and compare the illuminating power of a single candle with that of four similar candles arranged together as one source of light. It will be found, on adjusting for equality of illumination, that the distance of the four-candle source from the screen is twice that of the single candle. Repeat for different distances and different numbers of candles Show for each observation that—

$$\frac{L_1}{L_2} = \frac{R_1^2}{R_2^2},$$

from which it follows that for the same source, and a screen at different distances,

$$I \propto \frac{1}{R^2}.$$

The position of the screen is determined more quickly and accurately if it be made to oscillate between two positions, in one of which one side is too bright and in the other too dark. While making it oscillate the extent of the oscillation should be gradually decreased to zero. In this way the position of equality will be found with great exactness.

18. Standards of Light. The standard candle is defined in Art. **17**. Its employment was quite sufficient in the early stages of photometry, but it is too inaccurate for present-day work, different candles often varying in illuminating power by as much as 20 per cent. Other standards have therefore been devised. One standard taken is a portion of a certain sized flame in an Argand burner; another is a flame of amyl acetate burning in a special lamp, but the most accurate is the **Pentane lamp,** which consists of a flame of pentane vapour mixed with a certain definite proportion of air and burnt at a ring burner made of steatite. Its power is equal to that of **ten standard candles.** An electric glow lamp with a certain constant potential difference between its terminals and carrying a certain constant current has also been proposed as a standard, but the light-giving powers of such lamps diminish with age, and hence as standards they are worthless.

Another proposed standard is that of unit area of platinum, heated to the temperature of its melting point, but many practical difficulties forbid its use in commercial life at present.

The following table gives the candle-power of some common and other sources of light:—

Ordinary gas jet . . .	10-18
Welsbach burner and mantle .	45
Common Argand burner . .	11-17
Electric glow lamp . . .	16 (usual house size)
Electric arc lamp . . .	1,000-2,000
Lamp on the Eddystone . .	80,000
" " Belle Isle (Finisterre)	30,000,000

Celestial sources of Light. The sun, moon, and stars being so far away, their illuminating powers are not expressed in candle-power. Instead of this the illumination they produce on a surface is expressed in foot-candles, the foot-candle being the illumination produced on a screen by a standard candle placed one foot in front of it. Sunlight is approximately equal to 600,000 foot-candles; the light due to the full moon to one foot-candle. The sun is therefore about 600,000 times as bright as the full moon. The sun sends us 17,000,000,000 times more light than Sirius, the brightest of the stars, and it has been calculated that the total effect of starlight is about one-hundredth of that of the full moon.

The **Efficiency** of a lamp is the energy required to light the lamp for a second divided by the candle-power. The approximate efficiencies of the following sources expressed in watts per candle-power are:—Ordinary glow lamp, 3 to 4; Electric arc lamp, 1; Nernst lamp, 2; Osram lamp, $1\frac{3}{4}$; Cooper-Hewitt lamp, $\frac{1}{2}$.

The light given out by an ordinary candle contains only 2 per cent. of the total energy consumed. The ratios for electric incandescent and arc lights are 7 and 15 per cent. respectively. The light given out by the sun carries 35 per cent. of the total output of energy, while that given out by the Cuban firefly carries 99 per cent., so that, in comparison with Nature, man's appliances are very feeble.

In all photometric measurements the lights to be compared should be of the same colour.* If this is not the case it will be found impossible to adjust accurately for equal illumination, owing to the difference in colour of the illuminated surfaces. In general, different sources of light emit differently coloured rays in different proportions (Art. 100), so that an accurate comparison of intensity can only be made by means of an instrument which forms the light from each source into a spectrum (Art. 101), and admits of a comparison of corresponding parts of the spectra so formed.

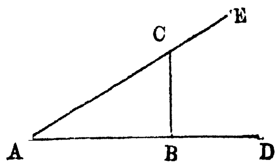
* In working with flicker photometers it has been found that the difference in colours of the two lights scarcely affects the results, the results obtained by the direct use of the flicker photometer agreeing very closely with those obtained by dealing with each part of the spectra in turn. This result has not been fully accepted.

CALCULATIONS.

19. **The calculations** connected with the subject-matter of the preceding chapters are simple applications of the elements of geometry or algebra to the principles there explained, and need no further illustration than is afforded by the worked examples given below.

Plane Angles. In Chapter III. we have made use of the term *cosine*, and in succeeding chapters it will be necessary to make frequent use of the terms *sine* and *tangent*. Hence for the convenience of the reader we shall now explain these terms.

Let D A E represent a plane angle. From any point C, in A E, draw C B perpendicular to A D, and cutting A D in B. Now the length of B C, for a given position of C, evidently depends on the magnitude of the angle D A E, but it gives no indication of this magnitude unless the position of C be defined. For this purpose the ratio $\frac{BC}{AC}$ may be considered, and it can be shown geometrically that wherever C be taken on A E this ratio is constant, and is definitely related to the magnitude of the angle B A C. Similarly the ratio $\frac{AB}{AC}$ is constant and bears a fixed relation to the magnitude of B A C. The ratio $\frac{BC}{AC}$ is called the *sine* of B A C, the ratio $\frac{AB}{AC}$ is called the *cosine* of B A C, and the ratio $\frac{BC}{AB}$ is called the *tangent* of B A C. In the right-angled triangle B A C, *considered with reference to the angle* B A C, the side B C is called the *perpendicular*, the side A B is called the *base*, and A C is called the *hypotenuse*. Hence, in general terms –



$$\text{sine } BAC = \frac{\text{perpendicular}}{\text{hypotenuse}} = \sin BAC.$$

$$\text{cosine } BAC = \frac{\text{base}}{\text{hypotenuse}} = \cos BAC.$$

$$\text{tangent } BAC = \frac{\text{perpendicular}}{\text{base}} = \tan BAC.$$

The reader should deduce geometrically the values of these ratios for angles of 30°, 45°, and 60°. These will be found to be—

$$\sin 30° = \frac{1}{2}. \qquad \cos 30° = \frac{\sqrt{3}}{2}. \qquad \tan 30° = \frac{1}{\sqrt{3}}.$$

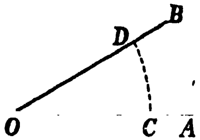
$$\sin 45° = \frac{1}{\sqrt{2}}. \qquad \cos 45° = \frac{1}{\sqrt{2}}. \qquad \tan 45° = 1.$$

$$\sin 60° = \frac{\sqrt{3}}{2}. \qquad \cos 60° = \frac{1}{2}. \qquad \tan 60° = \sqrt{3}.$$

Inverse Functions. The angle B A C is the angle whose *sine* is equal to the fraction $\frac{BC}{AC}$. This is often written $\angle BAC = \sin^{-1}\frac{BC}{AC}$. Similarly $\angle BAC = \cos^{-1}\frac{AB}{AC} = \tan^{-1}\frac{BC}{AB}$. For example, $30° = \sin^{-1}\frac{1}{2} = \cos^{-1}\frac{\sqrt{3}}{2} = \tan^{-1}\frac{1}{\sqrt{3}}$.

The magnitude of a Plane Angle may also be expressed in **Circular Measure.**

Let A O B be a plane angle and C D an arc struck with radius O C.



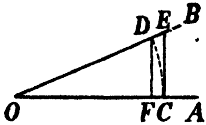
Then the fraction $\frac{\text{arc } CD}{\text{radius } OC}$ is called the circular measure of the angle A O B, and is generally denoted by θ. We have therefore

$$\text{arc} = \text{radius} \times \text{circular measure of angle.}$$
$$= r\,\theta.$$

Thus, the circular measure of the angle subtended by a semicircle at the centre of the circle (*i.e.*, of an angle of 180°)

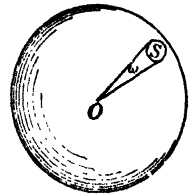
$$= \frac{\text{semi-circumference}}{\text{radius}} = \frac{\pi r}{r} = \pi.$$

Similarly the circular measure of $90° = \frac{\pi}{2}$.



The reader will note that when the angle A O B is *very small*, the ratio $\frac{\text{arc } CD}{OC}$ is very nearly equal to either $\frac{CE}{OC}$ or $\frac{FD}{OC}$, *i.e.*, to either the tangent or sine of the angle A O B. This may be expressed as $\theta = \tan \theta = \sin \theta$, also in the same case $\cos \theta = \frac{OF}{OD}$, and is very nearly equal to unity.

Solid Angle (Art. 12). Let O be the centre of a sphere of radius R. Take an area, S, on its surface. Draw radii from O to the boundary of S. These radii are the generators of a cone, and the solid angle ω of this cone is defined as the same fraction of 4π as the area S is of the surface of the sphere, *i.e.*,



$$\frac{\omega}{4\pi} = \frac{S}{4\pi R^2}.$$

$$\omega = \frac{S}{R^2}.$$

If $R = 1$, then $\omega = S$, so that the solid angle of a cone may be defined as the area of that part of a sphere of unit radius which is included within the cone.

If the given surface does not lie on a spherical surface around the reference point, the second definition must be adopted in all cases when the surface has a finite size. If, however, its area is small we may use the equivalent formula—

$$\omega = \frac{S \cos a}{R^2},$$

where S is the area of the surface, a the angle between a normal of the surface and the axis of the cone (supposed narrow), and R is the radius of the spherical surface passing through the centre of the given surface. (Cf. Fig. 11.)

EXAMPLES I.

1. In a pinhole camera the distance from the aperture in front, to the screen at the back, is 18 inches. Find the relative dimensions of the representation on the screen of an object placed 6 feet in front of the camera.

In Fig. 6, treating the pencils from A and B to A′ and B′ respectively as lines, we see that the triangles A O B and A′ O B′ are equiangular, and therefore similar (Euclid vi. 4).

$$\therefore \frac{A\,B}{A'\,B'} = \frac{C\,O}{O\,C'} = \frac{6}{1\frac{1}{2}} = 4.$$

$$\therefore A\,B = 4A'\,B'.$$

2. A circular uniform source of light, 2 inches in diameter, is placed at a distance of 10 feet from a sphere 2 inches in diameter. Calculate, approximately, the diameters of the umbra and penumbra cast on a screen 5 feet beyond the sphere. *Matric., June* 1889.

Here, in Fig. 8 (*b*)—

S S′ = 2 inches; O O′ = 2 inches; S O = 10 feet; O u = 5 feet.

Diameter of umbra = u u = O O′ = 2 inches.

External diameter of penumbra $= p\,p$; and from the triangles $O\,u\,p$ and $O\,S\,S'$ we have, by Euclid vi. 4—

$$\frac{u\,p}{S\,S'} = \frac{u\,O}{O\,S} = \frac{5}{10} = \frac{1}{2}.$$

$$\therefore u\,p = \frac{S\,S'}{2} = 1 \text{ inch.}$$

$$\therefore p\,p = u\,u + 2\,u\,p = 2 + 2 = 4 \text{ inches.}$$

3. The intensity of illumination of a screen placed 6 feet from a given source of light is denoted by I. Find the intensity when the distance of the screen is increased to 9 feet.

Let I′ denote the required intensity. Then, by Art. **14**—

$$\frac{I'}{I} = \left(\frac{6}{9}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$$

That is, $I' = \frac{4}{9} I$.

4. A small screen is held 6 feet from a source of light, in such a position that the light is incident on it normally. It is then removed to a distance of 10 feet and turned round, so that the light is incident on its surface at an angle of 60°. Compare the intensities of illumination of the screen in the two cases.

Let I and I′ denote the intensities of illumination for the first and second cases respectively. Then, by Arts. **14**, **15**, the intensity of illumination varies *inversely* as the squares of the distances, and *directly* as the cosine of the angles of incidence. That is—

$$\frac{I}{I'} = \left(\frac{10}{6}\right)^2 \cdot \frac{\cos 0^\circ}{\cos 60^\circ}.$$

Now $\cos 0^\circ = 1$ and $\cos 60^\circ = \frac{1}{2}$.

$$\therefore \frac{I}{I'} = \left(\frac{5}{3}\right)^2 \times 2 = \frac{50}{9}.$$

5. Two sources of light, A and B, when placed respectively 8 and 10 feet from a screen, produce the same intensity of illumination of its surface. Compare the illuminating powers of A and B.

Here, by Art. **16**—

$$\frac{\text{Illuminating power of A}}{\text{Illuminating power of B}} = \left(\frac{8}{10}\right)^2 = \frac{16}{25}.$$

6. The intensities of two sources of light, A and B, which are placed 10 feet apart, are as 4 : 9. Find at what points on the line joining them the intensity of illumination is the same.

Let x denote, in feet, the distance of either of the required points from A.

Then—

$$\left(\frac{x}{10-x}\right)^2 = \frac{4}{9} = \left(\pm\frac{2}{3}\right)^2.$$

$$\therefore \frac{x}{10-x} = \pm\frac{2}{3}.$$

That is—

$$\begin{aligned} 3x &= 20-2x, \\ 5x &= 20, \text{ and } x = 4 \text{ feet}; \\ \textit{or } 3x &= -20+2x, \\ \textit{and } \quad x &= -20. \end{aligned}$$

That is, there is equality of illumination at a point between A and B, 4 feet from A and 6 feet from B; also at a point 20 feet from A on the side remote from B. [That is, the line A B is divided internally and externally in the ratio 2 : 3.]

7. A circular uniform source of light, 10 cm. in diameter, is placed 1 metre in front of a spherical opaque body 5 cm. in diameter. Find the shortest distance from the latter at which a screen may be placed so as to have no umbra in the shadow cast upon it; also find the diameter of the penumbra in this position [Fig. 8 (*c*)].

8. A luminous sphere, 5 cm. in diameter, is placed 150 cm. from a disc of wood 2 cm. in diameter. Find the dimensions of the umbra and penumbra cast on a screen 50 cm. behind the disc of wood. The line passing through the centre of the luminous sphere and the disc is perpendicular to the latter and to the screen.

9. In Fig. 6, C O = 3 metres, O C′ = 20 cm., and the diameter of the aperture at O is 1 mm. Find the area of the circular spot of light at C′ due to the pencil of light coming from C. If A B = 2 metres, find also the length of A′ B′.

10. A dark room 10 ft. square, with white walls, has a small hole in one wall. Outside this hole and 56 ft. distant is a stone cross 15 ft. high, and the image appears on the wall of the room. How high will the image be?

11. Under the same circumstances the image of a tree 50 ft. high appears 8 in. high. How far is the tree from the hole?

12. Explain the appearance of the bright circular and elliptical spots seen on the ground in the shadows of trees when the sun and moon are shining.

13. The intensities of two sources of light are in the ratio 9 : 16. Find the ratio of the distances at which they must be placed from a screen, in order to produce on it the same intensity of illumination.

14. The lines joining the points A, B, and C form an equilateral triangle. D is the middle point of B C. A screen is placed at A with its surface parallel to B C. Lights placed at B, C, and D are found to equally illuminate the screen at A; compare their illuminating powers.

15. In Foucault's photometer (Fig. 12) $E L_1 : E L_2 :: a : b$. Find the relative intensities of L_1 and L_2.

16. In Rumford's photometer (Fig. 13) $L_1 s$ is found to be 115 cm., and $L_2 s$ to be 201 cm. Compare the illuminating powers of L_1 and L_2.

17. The intensities of two sources of light are in the ratio 4 : 9. If these sources are 200 cm. apart, where would a Bunsen's photometer be in accurate adjustment between them?

18. The distance between two incandescent lamps of 16 and 25 candle-power respectively is 6 feet. Show that there are two positions, on the line joining the lamps, at which a screen may be placed so as to receive equal illumination from each lamp; and determine these positions.

19. In a Rumford photometer the shadows of the rod thrown by a bat's-wing gas-flame and a Welsbach incandescent gas-light are equally bright when the bat's-wing is 2 ft. from the screen and the Welsbach 4 ft. 3 in. How many times more light does the latter give than the former?

20. The grease spot of a Bunsen photometer disappears when the standard candle-flame is 10 in. from one side and an electric glow lamp 36 in. from the other side. What is the candle-power of the lamp?

21. If a 16-candle-power gas-flame at a distance of 10 ft. illuminates a surface to a particular degree of brightness, at what distance must a 20-candle-power electric glow lamp be placed from that surface to illuminate it to the same degree?

22. Three standard candles are placed 10 in. from one side of the screen of a Bunsen photometer. How far must a 5000-candle-power electric arc be placed from the other side in order to cause the disappearance of the grease spot?

23. A rod is fixed vertically 6 inches in front of a vertical white screen. Three sources of light A B C, of 16, 18, and 48 candle-power, are placed at distances of 2, 3, and 4 feet respectively from the screen, and are so arranged that the three shadows of the pencil thrown by them are close together but do not overlap. Compare the relative degrees of illumination of the shadow?

CHAPTER IV.

REFLECTION AT PLANE SURFACES.

20. WHEN a ray of light travelling in one medium, A, is incident on the surface of another medium, B, it is, in general, broken up into three parts:

1. A portion which is reflected from the surface of B, back into A, according to a certain law. This portion is said to suffer reflection at the surface of B in accordance with the laws of reflection.

2. A second portion passes into B, and travels through that medium in a direction determined by another law. This portion is said to be refracted into the medium B in accordance with the laws of refraction.

3. A third portion is scattered or diffused by the surface both into B and A, of B in an irregular manner. The light thus scattered renders the suface luminous, and it is because of this scattering of light by the surfaces of non-luminous bodies that they become luminous in the presence of a self-luminous body (Arts. **2** and **34**).

When light is incident upon an opaque body no portion of the light is refracted or diffused into it, and the ratio of the quantities reflected and diffused back depends on the nature of the surface of the body and on the angle at which the light falls on the surface. A rough, uneven surface scatters the greater portion of the light falling on it; but a smooth, highly polished surface reflects nearly all the incident light; also the more obliquely light falls upon any reflecting surface the greater is the proportion of reflected light. Since a surface is rendered visible by scattering the light incident upon it, it follows that a perfectly reflecting surface would be invisible.

21. Mirrors. Any good reflecting surface is a mirror. The term is, however, usually confined to polished surfaces of a definite geometrical form—*e.g.*, plane, spherical, cylindrical, etc. The oldest mirrors were of polished metal, and this form of reflector is now much used for optical purposes. The ordinary plane mirror consists of a sheet of plate glass backed by a thin layer of deposited silver, which forms the reflecting surface.* More recently, for scientific purposes, silver *specula* have been employed as mirrors. These are formed of glass surfaces, of the required geometrical form, coated in front with a thin layer of silver which is very highly polished.

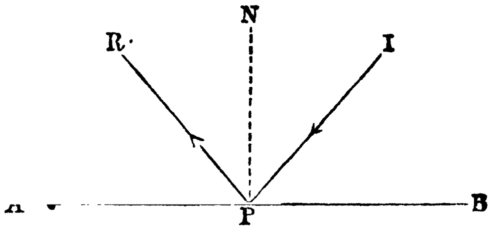
22. Definitions. The *normal* to a reflecting surface at any point is a line drawn at right angles to the tangent plane to the surface at that point. If the surface is plane, then the normal at any point is at right angles to the surface; and if spherical, it is coincident in direction with the radius drawn to that point. The *angle of incidence* of a ray falling on the surface of a medium is the angle between the direction of the ray and the normal to the surface at the point of incidence. The *plane of incidence* is the plane containing the normal and the incident ray. The *angle of reflection* is the angle made by the reflected ray with the normal at the point of incidence. The *plane of reflection* is the plane containing the normal and the reflected rays.

At this point we may mention what is called the *reversibility of light.* It may be stated thus: If by any means light is able to travel from a source at A to a point B, then, if the source is placed at B, light will travel back to the point A by the same path.

* Up to 1840 all glass mirrors were backed with an amalgam of mercury and tin. Since then this process has been almost entirely superseded by the silver process. The silver is deposited on the glass from a solution of silver nitrate, either by the use of tartaric acid (hot process) or by sugar and vinegar (cold process). When dry the surface is brushed over with a dilute solution of mercury cyanide, and then coated with red-lead paint to keep it safe.

23. Laws of Reflection. When a ray of light is incident on a reflecting surface, it is reflected in accordance with two laws, which may be thus stated:

1. The angle of reflection is equal to the angle of incidence. ($\angle$ IPN = $\angle$ NPR, Fig. 17.)



A B, Reflecting surface.
P N, Normal.
I P N, Angle of incidence.
I P, Incident ray.
P R, Reflected ray.
N P R, Angle of reflection.

Fig. 17.

2. The planes of incidence and reflection are coincident.

These may be expressed as one law thus: The angle of incidence and reflection are in the same plane, and are equal to one another.

This law is established by experiment, and may be directly verified by means of the apparatus shown in the figure (Fig. 18). A graduated circle, fixed in a vertical plane, has a small mirror, *m*, attached horizontally at its centre, and carries two tubes, T and T′, having their axes parallel to the plane of the circle and directed towards the central portion of the mirror. These tubes travel round the circumference of the circle, and the position of their axes relative to the graduations is shown by a mark on the slide to which they are attached.

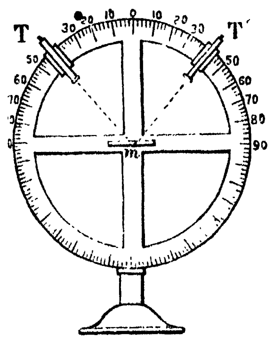


Fig. 18.

The zero of the graduations is placed at the point where the normal to the central portion of the mirror cuts the divided circle. A source of light is placed so as to send a beam of light down one of the tubes on to the mirror; the other tube is then moved round until, on looking through it, the source of light can be seen reflected in the mirror. It will then be found that each of the tubes is at the same angular distance from the zero of the scale —that is, the angle of reflection is equal to the angle of incidence. Moreover, the planes of incidence and reflection are coincident, for both are parallel to, and at the same distance in front of, the plane of the divided circle.

The strongest proof of these laws, however, lies in the fact that in numerous experiments the above laws are assumed, and not once has the assumption led to an inaccurate result.

The laws hold for any smooth surface, whether plane or curved. If curved a small area around the point of incidence will be coincident with the tangent plane at that point, and the normal can be drawn perpendicular to this plane.

24. Images. When a luminous body is viewed directly, pencils of light from every point on the body enter the eye, and thus the body is seen and its form defined. If, however, from any cause these pencils suffer change of direction, such that they actually come from, or appear to come from, an assemblage of luminous points other than the luminous surface of the body, this assemblage of luminous points is called the *image* of the luminous body. An image may be either *real* or *virtual*; in a real image the rays actually do pass through the points of the image, but in a virtual image they only appear to do so, or perhaps it is better to say that a virtual image is such that the rays are straight lines whose directions would pass through it, if produced backward through the reflecting surface. A real image differs from a luminous body in the fact that the latter emits light in all directions, whereas the former transmits light only in the direction taken by the rays involved in its formation.

25. Reflection of light from the luminous point at the surface of a plane mirror. Let L (Fig. 19) denote the position of the luminous point and M M that of the mirror. Consider the reflection of any ray L A. Draw the normal A N at A; then, according to the law of reflection, the reflected ray will lie in the plane L A N, and its direction,

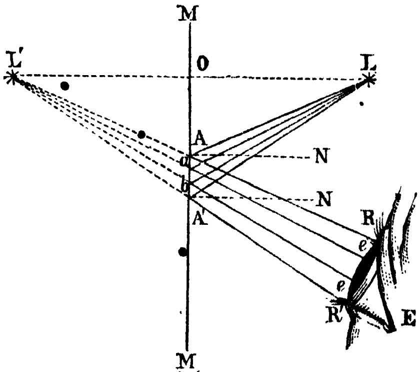


Fig. 19.

A R, will be such that the angle R A N equals the angle L A N. Similarly, for the ray L A′, the reflected ray takes the direction A′ R′, such that R′ A′ N′ is equal to L A′ N′. Now, to an eye placed at R R′, the pencil reflected from the portion A A′ of the mirror will appear to come from a point L′ at the intersection of R A and R′ A′ (Art. 7). It can be shown that this point L′ lies on the normal to the mirror, passing through L and at the same distance behind the mirror as L is in front of it. For, through L draw the normal L O L′, and let R A produced cut it at L′. Then, by Euclid i. 29, $\angle$ L A N = $\angle$ O L A,

and $\angle$ R A N = $\angle$ O L′ A.

But, in accordance with the law of reflection—

$$\angle \text{ L A N} = \angle \text{ R A N};$$

$$\therefore \angle \text{ O L A} = \angle \text{ O L}'\text{A}.$$

∴ in the triangles A O L and A O L′ we have—

the angle A O L = the angle A O L′,
and the angle O L A = the angle O L′ A,
and the side O A common;
∴ O L = O L′ (i. 26).

Similarly, it can be shown that any other reflected ray, if produced backwards, passes through L′; and therefore, to an eye in front of the mirror, a virtual image of L is seen at L′. The image is virtual, because the rays by which it is seen do not actually come from L′, but, owing to the change of direction resulting from the reflection at the surface of the mirror, they appear to do so.

It has been proved, by assuming the truth of the law of reflection, that the image of a luminous point is at the same distance behind the mirror as the point itself is in front of it. Hence, if this can be shown to be true experimentally, we get an indirect experimental proof of the law of reflection (cp. Art. **23**).

Exp. 2. Take a clean polished rectangular plate of thin glass,*

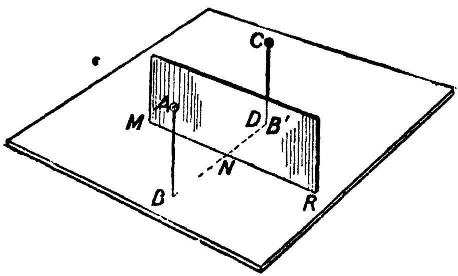


Fig. 20.

stand it in a vertical position upon a sheet of cartridge paper pinned to a drawing-board and mark its trace M R (Fig. 20). Stick a pin, A B, into the paper about four inches in front of it; an image C B′

* Instead of a plate of thin glass the polished back of a piece of ordinary mirror may be used. Dissolve off the backing with methylated spirits and gently polish the metallic surface with a rouge puff. A very good reflecting surface is thus obtained.

of the pin, formed by the nearest polished surface of the glass plate will be clearly seen apparently behind the plate. Place another pin, C D, behind the mirror so as to coincide in position with this image for all positions of the eye; *i.e.*, until all parallax (or side-shifting between image and object when the eye is moved sideways) is eliminated.* Remove mirror and pins, join B D and, by direct measurement with a pair of compasses or a scale, show that the distances of the image and the object from the reflecting surface are equal, and that the line joining them is perpendicular to the reflecting surface.

Exp. 3. With the same apparatus it may also be shown that the angle of incidence is equal to the angle of reflection.

Mount the thin glass mirror as before. Place two pins J and S (Fig. 21), almost anywhere in front of the mirror, and looking into

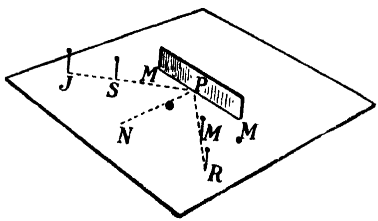


Fig. 21.

the mirror, move the eye about until the images are in line. Place two more pins M and R into the paper so as to be in a line with these images. Remove mirror and pins. Join J S and M R and produce them; they will intersect at a point P on M M. Draw the normal P N at P, and by direct measurement prove the equality of the angles J P N, R P N

26. Reflection of a convergent pencil incident on a plane mirror. The preceding article deals with the reflection of a divergent pencil (L A A′), and shows that, after reflection, it appears to diverge from a point at the same distance behind the mirror as that from which it originally diverged was in front of it. Similarly, if a convergent

* Remember that the further of two objects will appear to move, relatively to the nearer, in the same direction as the eye of the observer.

pencil, P L P′ (Fig. 22), converging to a point L behind the mirror, be incident at A A′, it is reflected so as to converge to a point L′, such that L O L′ is normal to the mirror,

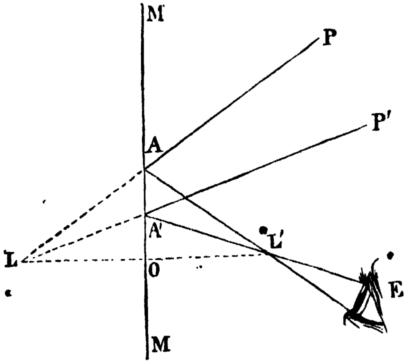


Fig. 22.

and O L = O L′. This can be proved in exactly the same way as the last case. An eye placed at E sees a *real* image at L′.

27. Image of a luminous object formed by a plane mirror. Let A B (Fig. 23) represent a luminous object placed in front of the mirror M M. As in Art. 25 the image of A is formed at A′, such that A O A′ is normal to M M and A′ O equal to A O. Similarly, the image of B is formed at B′, such that B O B′ is normal to M M and B′ O equal to B O. For all points of the object intermediate between A and B images are formed at corresponding points between A′ and B′, and thus a complete image of the object is formed at A′ B′.

A more elaborate construction is sometimes given for determining the position of an image formed by a plane mirror, and, as the method is general and applicable to spherical mirrors, we shall briefly notice it.

It is based on the fact that the intersection of any two reflected rays determine the point on the image from which

they diverge, or appear to diverge. For plane mirrors, the two rays chosen are A O (Fig. 23), incident along a normal to the mirror, and any other ray incident in any other direction, such as A N. The ray A O is reflected back along its original path, and A N is reflected along N R, making the angle of reflection R N *n* equal to the angle of incidence A N *n*, and the image of A is formed at A′, the virtual focus of the reflected rays O A and N R. Similarly, the image of any other point B is obtained, and the images of intermediate points assumed to lie on the line A′ B′, and

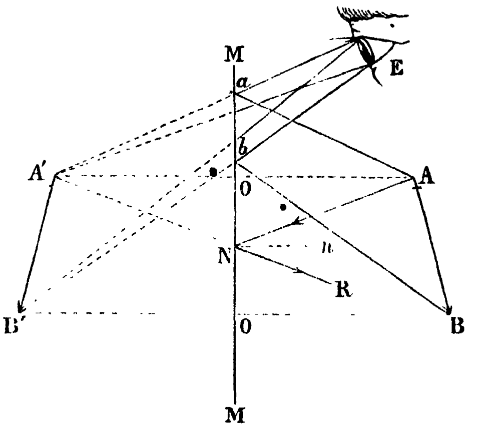


Fig. 23.

hence A′ B′ is said to be the image of A B. When the form of the image is more complex than that considered here, the images of a number of points, sufficient to determine the complex image, must be obtained.

An eye placed at any point E (Fig. 23), in front of the mirror, sees the image A′ B′ by light reflected from the portion *a b* of the surface of the mirror, and the actual path of the *extreme* rays is shown by the lines A *a* E, B *b* E. It is evident from this that, in order that any point of an image may be seen, the line joining this point to the eye must cut the surface of the mirror, and the portion of the

surface at which, by reflection, an image is seen, is that portion which is intercepted by the cone having the eye at its apex and the image at its base.

28. Path of rays by which an image is seen. Let L′ (Fig. 19) represent the image of a luminous point L formed by the mirror M M, and imagine an eye placed at E. Draw lines joining L′ to the extremities *e e* of the aperture of the eye, and cutting the mirror at *a* and *b*; then join L to *a* and *b*, and the lines L *a e* and L *b e* define the pencil of light by which L′ is seen (cp. Art. 27). Each point of the

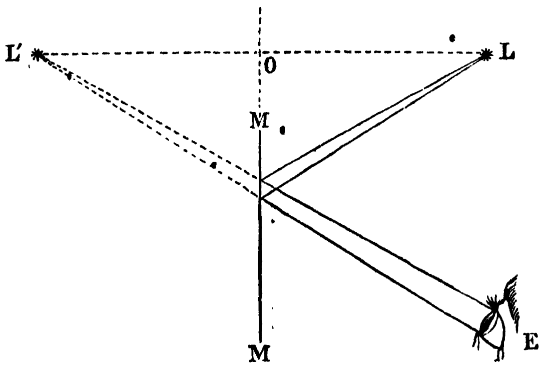


Fig. 24.

image of a luminous object is seen in the way just described, and the extreme rays bounding the collection of pencils reaching the eye are determined in the way indicated at A *a* E, B *b* E in Fig. 23.

In connection with this question it is important to notice what must be the position of an object relative to a mirror in order that an image may be formed by that mirror. Let M M (Fig. 24) represent a mirror; then, if an object L be placed anywhere in front of the plane passing through M M, an image of that object will be formed behind this plane, at a point L′, such that L O L′ is normal to the plane and L O = L′O. This can be proved in the same way as the

proposition of Art. 25; the figure, which corresponds to Fig. 19, shows the necessary construction, and also the path of the rays by which an eye placed at E is able to see the image L′.

29. Lateral inversion. When the image of the face is seen in an ordinary looking-glass, we know that the image of the right eye forms the left eye of the reflected face, while the image of the left eye forms its right eye. This is a particular instance of a result of reflection known as lateral inversion. It does not affect the appearance of objects which are bi-laterally symmetrical; but with non-symmetrical objects, such as printed or written characters, the effect is sufficiently evident and well known.

30. Deviation. When a ray of light is turned out of its original course it is said to suffer deviation, and the

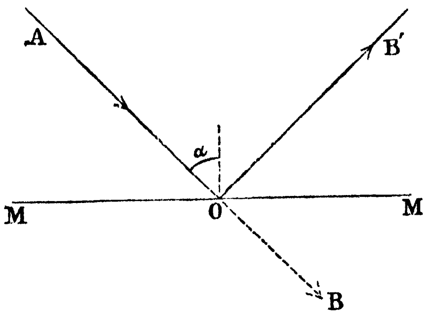


Fig. 25.

angle between its initial and final direction determines the amount of this deviation. The deviation due to a single reflection at a plane surface is easily determined. Let A O (Fig. 25) be incident on the surface M M at an angle a to the normal. Then, since the initial direction of the ray is represented by A B, and its final direction by O B′, the deviation is evidently given by the angle B O B′, which $= 180 - \text{A O B}' = 180 - 2a$.

31. Reflection from a rotating mirror. Let N A (Fig. 26) represent a ray incident normally on the mirror M M. If the position of the mirror remain unchanged, then N A will be reflected back along A N; but if M M be rotated, in the direction shown by the arrows, round an axis at A, into the position M′ M′, then N A will be reflected along A N′ according to the law of reflection. Now the angle N A n = M A M′. ∴ N A N′ = 2N A′ n = 2M A M′. But N A N′ is the angle through which the reflected ray has been rotated by the rotation of the mirror through M A M′.

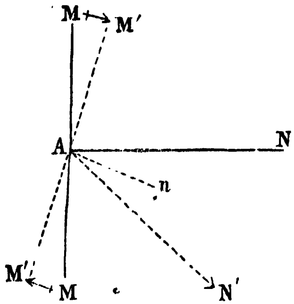


Fig. 26.

Hence, if a mirror be turned through an angle a, the reflected ray is rotated through an angle $2a$; that is, the reflected ray rotates twice as rapidly as the mirror from which it is reflected.

This fact finds important application in the measurement of small angular deflections. The angle is too small to be measured directly by pointer and graduated circle, hence a

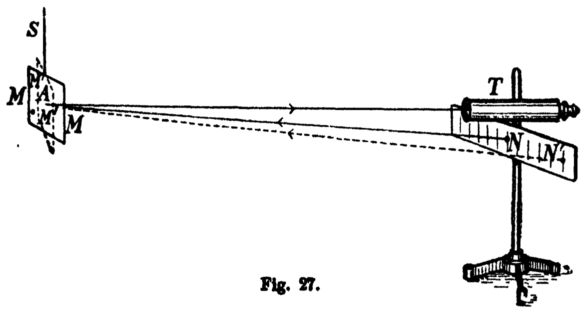


Fig. 27.

mirror M M (Fig. 27) is affixed to the rotating system or suspension wires, and the angle N A N′ (Figs. 26, 27) measured instead.

In practice a telescope and scale (Fig. 27) are employed. When the mirror is perpendicular to A N the scale divisions at N, which is just below the telescope, are in the field of view of T. As M rotates the scale appears to travel across the field of view, and when M has reached M′ the division N′ is seen on the cross wire of the telescope.

$$\text{Now } \angle\, NAN' = 2\angle\, MAM' \text{ (above).}$$

$$\therefore\quad MAM' = \tfrac{1}{2}\angle\, NAN' = \tfrac{1}{2}\tan^{-1}\frac{NN'}{AN}.$$

If the angles are small, the tangent is equal to the circular measure (see Art. 19), and therefore

$$\angle MAM' = \frac{NN'}{2AN}.$$

Since N N′ and A N can be accurately measured, the angle M A M′ is thus easily determined. This is sometimes called Poggendorf's, or the *Subjective*, method, and is largely used on the Continent. The most usual practice in England, however, is to use a concave spherical mirror, in which a real image of the spot of light itself is focussed on the scale (Art. 48, Figs. 51, 52). This is an *Objective* method.

32. Reflection at plane surfaces inclined to each other. Before considering particular cases of special interest, it will great simplify matters to notice the general principles applicable to all cases. Imagine an object A, placed between two mirrors, M_1 and M_2, inclined to each other at any angle. An image of A will be formed by each mirror; and, if the image formed by M_1 lie in front of M_2—that is, if it is anywhere in front of the plane in which this mirror lies (Art. 28)—then an image of this image will be formed by M_2. Similarly, if the image formed by M_2 lie in front of M_1, then an image of this image is formed by M_1. These are said to be images of the second order. In precisely the same way, if this second pair of images are suitably placed, a third pair (of the third order) may be formed, and so on. This multiplication of images stops when a pair is formed in the space behind both mirrors—that is, within the angle

vertically opposite to that in which the object is placed. We shall now consider a few special cases.

(1) **Parallel mirrors.** Let M_1 and M_2 (Fig. 28) represent two parallel mirrors, and A an object placed between them. It is evident that since the mirrors are parallel no image can be formed behind both, and hence every pair of images gives rise to another pair, and thus an infinite series of images may, theoretically, be formed. Through A draw $N_1 N_2$ normal to both the mirrors, and produce it indefinitely on both sides. In obedience to the law of reflection all the images must lie on this line, and their positions on it will depend on the position of A between M_1 and M_2, and on

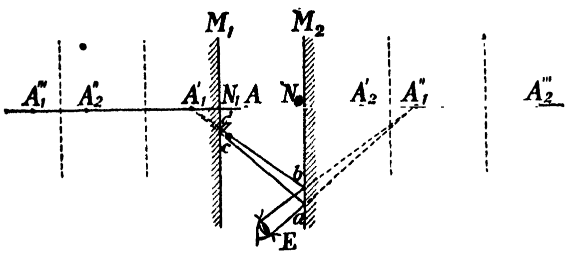


Fig. 28.

the distance between these mirrors. Consider first the reflection from M_1; an image of A is formed at A_1' on the normal through A, and so placed that $A_1' N_1$ equals $A N_1$. Similarly an image of A_1' is formed by M_2 at A_1'' in such a position that $A_1'' N_2 = A_1' N_2$; A_1'' in turn gives rise to A_1''' by reflection at M_1, and so on. In the same way, beginning with the first reflection at M_2, the images A_2', A_2'', etc., are formed by successive reflection at M_2 and M_1. In Fig. 28 the positions of the images up to the third order are shown, and, to distinguish them, the suffix attached to A denotes the mirror at which the *first* reflection took place, and the dashes indicate the order of the image. Thus the series A_1', A_1'', A_1''' . . . is formed by successive reflections from M_1 and M_2, beginning with reflection at M_1; similarly the series A_2', A_2'', A_2''' . . . is formed by successive

reflections from M_2 and M_1, beginning with M_2. The members of each series are so related that any one may be considered as the image of the one immediately preceding it: for example, A_1''' may be considered as the image of A_1'' formed by the mirror M_1, and consequently $A_1''' N_1$ equals $A_1'' N_1$.

To determine the path of rays by which any image is seen, the following construction should be employed. Let it be required to find the path of the rays by which an eye at E sees the image A_1''. First trace this image back to A; A_1'' is an image of A_1', which is itself an image of A. Now join the extremities of the aperture of the eye to A_1'' by lines cutting M_2 at *a* and *b*, and mark the real parts of this path, which, since the rays cannot penetrate the mirror, must lie between the eye and *a b*. Next join *a* and *b* to A_1' by lines cutting M_1 in *c* and *d*, and mark *a c*, *b d* as the real portions of this path. Then finally join *c* and *d* to A, and the twice reflected pencil passing from A to E indicates the required path. From this it is evident that an image of the *second* order, A_1'', is seen by *two* reflections, and similarly an image of the n^{th} order would be seen by n reflections. The mirror from which the last reflection takes place—that is, the mirror in which the image is seen—depends upon whether n is odd or even. In either series of images the *odd* * members are seen by reflection from the mirror at which the *first* reflection takes place, while the even members are seen in the other mirror. At each reflection there is some loss of light, depending in amount on the polish of the reflecting surface; and, as a consequence, the higher the order of any image the fainter it appears, until finally it becomes too faint to be visible.

(2) **Mirrors inclined at right angles.** Let O M_1 and O M_2 (Fig. 29) represent two mirrors at right angles to each other, and A an object placed between them. Then an image A_1' is formed by O M_1 and A_2' by O M_2. But A_1' lies in front of O M_2, and therefore an image A_1'' is formed by that mirror. Also, A_2' is in front of O M_1, and therefore gives rise to the image A_2'', which from the geometry of the figure

* That is, the 1st, 3rd, 5th, etc.

is evidently coincident with A_1''. An eye placed anywhere within the proper limits (Art. 27) sees three images, A_1' A_1'', *or* A_2'', and A_2', at the three corners of the rectangle $A_1' A_2'$. *Both* the images A_1'' and A_2'' cannot be seen at the same time; an eye placed within the angle M_1 O A sees the image A_2''; while one placed in M_2 O A sees A_1''. The figure shows the path of the rays by which the image A_1'

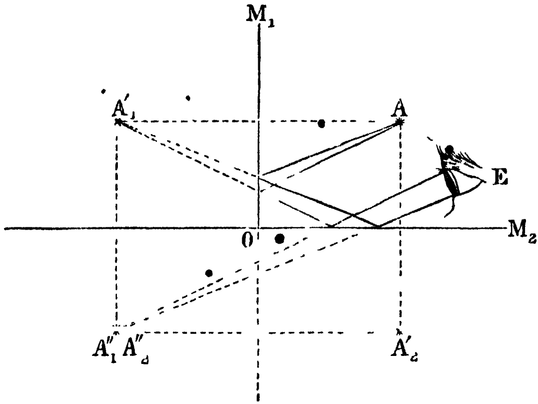


Fig 29.

may be seen by an eye placed at E. The method of determining this path is indicated by the dotted lines, and is similar to that explained above for parallel mirrors. The actual path of the rays necessarily lies within the angle M_1 O M_2.

(3) **Mirrors inclined at any angle.** Let O M_1 and O M_2 (Fig. 30) represent two mirrors inclined at the angle M_1 O M_2 and A an object placed between them. With O as centre, describe a circle passing through A and cutting O M_1 and O M_2 in N_1 and N_2 respectively. Then all the images of A must lie in the circumference of this circle. For consider the image A_1'; according to the law of reflection it is so placed that A n_1 equals A_1' n_1, and A A_1' is at right angles to O M_1. Hence, in the triangles O A n_1 and O A_1' n_1 we have A n_1 equal to A_1' n_1, n_1 O common, and the angle A_1' n_1 O equal to the angle A n_1 O; therefore O A_1' equals O A

(Euc. i. 4), and A_1' lies on the circumference of the circle passing through A. Similarly for any other image. Let us now consider the formation of the images, and the positions they occupy on the circle on which they lie. To simplify matters we shall consider only the series A, A_1',

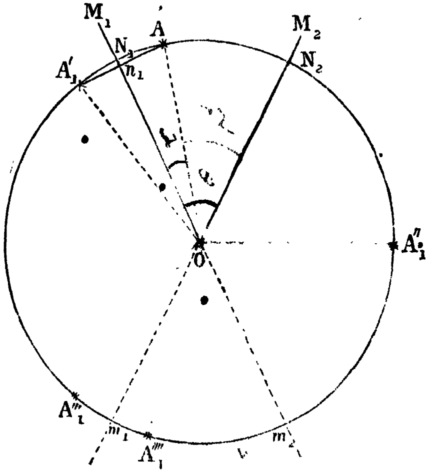


Fig. 30.

A_1'', A_1''', etc. As described above for parallel mirrors, A gives rise to A_1', A_1' to A_1'', and so on, each pair being symmetrically placed with respect to the mirror at which the last reflection takes place. Hence, if $M_1 O M_2 = \theta$, $A O N_1 = \alpha$, and $A O N_2 = \beta$, we have

$$A O A_1' = 2\alpha.$$

$$\begin{aligned} A O A_1'' &= A O N_2 + N_2 O A_1'' \\ &= A O N_2 + N_2 O A_1' \\ &= 2 A O N_2 + A O A_1' \\ &= 2\beta + 2\alpha \\ &= 2(\beta + \alpha). \end{aligned}$$

$$\therefore A O A_1'' = 2\theta.$$

$$\begin{aligned} A O A_1''' &= A O N_1 + N_1 O A_1''' \\ &= A O N_1 + N_1 O A_1'' \\ &= 2 A O N_1 + A O A_1''. \end{aligned}$$

$$\therefore A O A_1''' = 2\alpha + 2\theta.$$

$$\begin{aligned}
A\,O\,A_1^{''''} &= A\,O\,N_2 + N_2\,O\,A_1^{''''} \\
&= A\,O\,N_2 + N_2\,O\,A_1^{'''} \\
&= 2A\,O\,N_2 + A\,O\,A_1^{'''} \\
&= 2\beta + 2a + 2\theta \\
&= 2(\beta + a) + 2\theta.
\end{aligned}$$

$$\therefore\ A\,O\,A_1^{''''} = 4\theta.$$

Thus it can be shown that in general—

$$A\,O\,A_1^{2n+1} = 2n\,\theta + 2a, \text{ and}$$

$$A\,O\,A_1^{2n} = 2n\,\theta.$$

Exactly similar relations can be deduced from the series A, A_2', A_2'' . . . giving the general results—

$$A\,O\,A_2^{2n+1} = 2n\,\theta + 2\beta, \text{ and}$$

$$A\,O\,A_2^{2n} = 2n\,\theta.$$

The number of images that can be formed in this way is limited, the last number of each series being that which is formed within the angle $m_1\,O\,m_2$. To determine this number let A_1^{2n} be the first image to fall on the arc $m_1\,m_2$.

Then—

$$A\,O\,A_1^{2n} > A\,O\,m_2.^*$$

That is—

$$2n\,\theta > \pi - a.$$

That is—

$$2n > \frac{\pi - a}{\theta}.$$

Similarly, if A_1^{2n+1} be the image falling on $m_1\,m_2$, we get—

$$A\,O\,A_1^{2n+1} > A\,O\,m_1.$$

That is—

$$\begin{aligned}
2n\,\theta + 2a &> \pi - \beta \\
2n\,\theta + a + \beta &> \pi - a \\
2n\,\theta + \theta &> \pi - a \\
\theta(2n + 1) &> \pi - a. \\
\therefore\ 2n + 1 &> \frac{\pi - a}{\theta}.
\end{aligned}$$

Hence, in both cases, the number of images in the series A_1', A_1'' . . . is given by the integer next greater than $\frac{\pi - a}{\theta}$.

* $A\,O\,A_1^{2n}$ being an even member of the series A_1', A_1'', etc., will be formed by the mirror $O\,M_2$, and therefore $A\,O\,m_2$ is considered, and not $A\,O\,m_1$ (Art. 32, 1).

Similarly, the number in the series A_2', A_2'', A_2''', etc., is given by the integer next greater than $\frac{\pi-\beta}{\theta}$.

If θ be an exact sub-multiple of π, then $\frac{\pi}{\theta}$ is an integer; and therefore the number of images in each series will be $\frac{\pi}{\theta}$,* and the total number for both series is given by $\frac{2\pi}{\theta}$. It happens, however, in this case (Fig. 31) that the position of

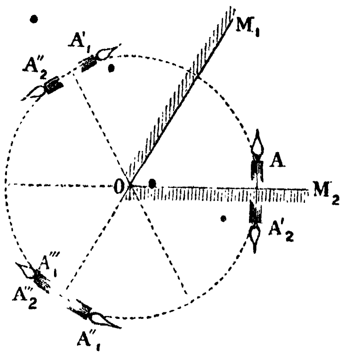


Fig. 31.

the last image of each series on the arc $m_1\ m_2$ is the same,† and hence only $\frac{2\pi}{\theta} - 1$ images can be seen. If the object be included in the number, then $\frac{2\pi}{\theta}$ representations of the same object can be seen arranged round a circle.

Exp. 4. Place two mirrors, $O\ M_1$, $O\ M_2$ (Fig. 32), on a sheet of cartridge paper at an angle of 60° with each other, and between them press a pin A into the paper so that it stands upright. Looking into

---

* For, in this case $\frac{\pi}{\theta}$ is the integer next greater than $\frac{\pi-a}{\theta}$ or $\frac{\pi-\beta}{\theta}$.

† See Ex. II., 2.

the mirrors a series of images A_1', A_2'', A_2''' (A_1'''), A_1'', A_2' will be seen. Locate the positions of these images by other pins as in Exp. 3. Now remove pins and mirrors and prove that (1) the images all lie on a circle whose centre is O and radius O A, (2) the

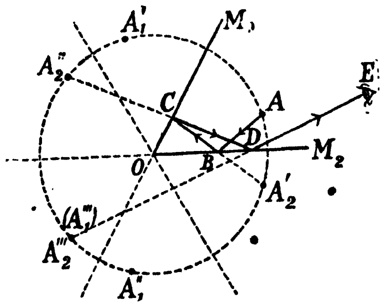


Fig. 32.

angles A O A_1', A_2'' O A_2''', A_2' O A_1'' are equal to one another, and (3) the angles A O A_2', A_1'' O A_2''', A_2'' O A_1' are also equal to one another. Show also that if an eye be placed at E, the path of a ray of light apparently coming from the image A_2''' is A B C D E. The construction is obvious.

The symmetrical distribution of images obtained by repeated reflection between two mirrors when they are inclined at an angle which is an exact submultiple of 180° is the principle of the **kaleidoscope.** Two long narrow mirrors, inclined to each other at an angle 60°, are placed in a slightly longer tube. One end of the tube is closed by a metal disc, pierced at the centre, with a hole through which the observer looks; at the other end a plate of clear glass fits into the tube close up to the mirrors; and a short distance beyond it, at the end of the tube, is a similar plate of ground glass. Between these two glass plates little pieces of coloured glass, etc., are loosely placed, and, with their images, form beautiful and symmetrical patterns visible to an eye placed at the other end of the tube. On rotating the tube the pieces of glass change position, and thus the pattern seen is continually changing. Sometimes three mirrors are employed, the arrangement being such that the cross section of the three is an equilateral

triangle. Each pair of plates acts in the way described above, so that the arrangement gives rise to intricate but symmetrical patterns, which are capable of giving material aid to designers.

33. Deviation produced by successive reflection from two plain mirrors inclined to each other at any angle. Let O M_1 and O M_2 (Fig. 33) represent the two mirrors inclined at an angle a represented by M_1 O M_2, and let A B C D . . . represent a ray reflected successively from O M_1 and O M_2 at

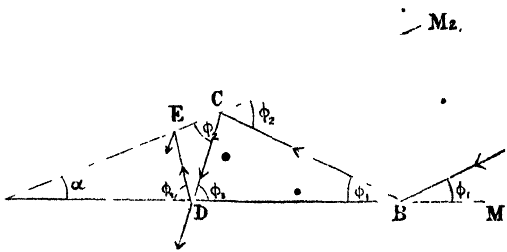


Fig. 33

the points B, C, D . . . Also, let ϕ_1, ϕ_2, ϕ_3 . . . denote the angles which the incident and reflected rays make with the *reflecting surface* at the points B, C, D . . . respectively Then, from the triangle B C O, we have $\phi_2 = \phi_1 + a$; that is, $\phi_2 - \phi_1 = a$, and similarly $\phi_3 - \phi_2 = a$, and so on. Writing these equations in order, we get—

$$\phi_2 - \phi_1 = a$$
$$\phi_3 - \phi_2 = a$$
$$\phi_4 - \phi_3 = a$$
$$. \quad . \quad . \quad . \quad .$$
$$\phi_{n+1} - \phi_n = a;$$

and therefore, by addition—

$$\phi_{n+1} - \phi_1 = na.$$

But, if n be even, then the angles ϕ_{n+1} and ϕ_1 are measured from the *same surface*, and therefore their difference must give the required deviation; for the ray is initially inclined to the reflecting surface at an angle ϕ_1, and finally to the same surface at an angle ϕ_{n+1}.

Hence, when n is even, *the deviation produced by n reflections is equal to $n\alpha$, where α is the angle between the mirrors.* When n is odd, the deviation is that due to the $(n-1)^{th}$ even reflection and the n^{th} odd reflection, and is evidently given by $[(n-1)\alpha-2\phi_n]$*: for example, after the *third* reflection in Fig. 33, the deviation equals $2\alpha-2\phi_3$.

When ϕ_n becomes greater than a right angle, the ray begins to travel outwards from the intersection of the mirrors, not generally following its original path; but if ϕ_n be equal to a right angle, then the ray travels back along the path by which it came. At each reflection the value of ϕ is increased by α; and hence, in order that ϕ_n may equal $\frac{\pi}{2}$, ϕ_1 must be so chosen that α is an exact sub-multiple of $\left(\frac{\pi}{2}-\phi_1\right)$. For example, if $\alpha = 10$ and $\phi_1 = 20$, then the ray will return along its original path after $\frac{90-20}{10} = 7$ reflections after the first.

When a ray is reflected twice, as in Hadley's sextant (Art. 136), the deviation is twice the angle between the mirrors.

34. Irregular or Diffuse Reflection.—When a parallel beam from a magic lantern in a dark room falls on a piece of white card, the light after incidence is not confined to one course, but is scattered in all directions. From anywhere in front the card is brightly visible, and the room is no longer wholly dark. If a mirror had been employed, practically all the light would have been reflected in some definite direction. At first sight there seems to be a great difference between the two phenomena; and it has been sought to explain the behaviour of the card by comparing it to a mirror with many small facets, which reflect the light quite regularly, but in different directions, because they themselves are at different slopes. This phenomenon does in fact occur when light is refracted from water on which are ripples, and accounts for the wide luminous path seen on a lake under the sun and moon; but it is not just what takes place when light is diffused by a card. The fact is that diffusion, and not reflection, is the fundamental phenomenon.

* The minus sign comes in because the deviation ϕ_n is in a direction opposite to that denoted by $(n-1)\alpha$.

Diffusion cannot therefore be explained by reflection; but the latter is a consequence of the former. A full explanation of this would be impossible; but the essential fact is that light consists of a series of waves. Waves striking any obstacle are always scattered in all directions; but this scattering produces a regular reflected wave whenever the obstacles are ranged in an even surface whose inequalities are not large compared to the wave-length. Thus the sound waves from the tick of a watch are about an inch long; they are regularly reflected by a surface whose inequalities are less than, say, half an inch high. Water waves on the sea may be over a hundred feet long; they are reflected easily by a somewhat irregular coast line. Sound waves, six feet long, are reflected so as to produce a true echo from a hedgerow. Light waves have a length of 40 to 75 millionths of a centimetre (one to two millionths of a foot), and are well reflected from polished metal or glass, or the surface of a liquid in which the inequalities are of this order of magnitude, but badly reflected from cardboard, in which they are larger. By means of the cardboard we can, however, prove another consequence of the wave theory, that a ray is reflected fairly regularly if it be very oblique indeed, so as to be almost parallel to the surface, even if the surface be rough (See Art. 35).

Twilight is explicable by diffusion. Clouds, dust, and other floating particles in the atmosphere are illuminated by the sun some time after it has set at any particular place. These scatter the light in all directions, some of the scattered rays of course reaching the earth, illuminating it for some time after sunset. Moreover, some of the scattered light is transmitted to other particles of the atmosphere farther away from the sun, and these reflect the rays a second time; the result of these second reflections is to still further increase the duration of twilight. Twilight is said to end when this scattered light becomes imperceptible. By observation this has been found to occur when the sun is at a depth of about 18° below the horizon.

If the earth had no atmosphere, surfaces on it exposed to the sun's rays would be dazzlingly bright, whilst all other surfaces would be in black shadow, except such parts as might be illuminated by reflection and diffusion from surrounding surfaces. This state of things indeed exists on the moon, where the contrast between light and shade and the sharpness of shadows are extremely great.

It is only by means of the light they scatter that all bodies, except self-luminous ones, are made visible to us. The scattering of the light of the sun by white clouds is the cause of the difference between ordinary daylight with its soft gradations of light and shades, and direct sunlight with its intense lights and deep shadows. For the effect of scattering on the colour of the light see Art. 125.

35. Plane Mirrors. It is extremely difficult to make accurate plane surfaces. The simplest test of flatness that can be applied is that of reflection.

Exp. 5. Make a smooth round hole in a piece of tinfoil and place it in front of a bright light, L (Fig. 34). Fix a telescope, T, in a stand at a distance from the foil and focus the telescope on the

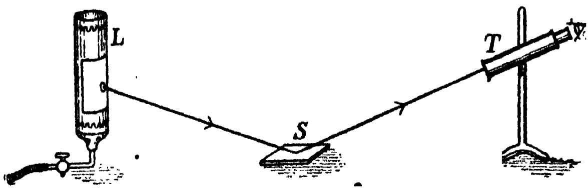


Fig. 34.

hole. On the bench midway between the telescope and lamp lay the surface, S, to be tested, and incline the telescope until the hole is seen by reflection from the surface.

If the surface is plane the image will still be sharp, if irregular the image will be ill-defined and may spread out into a large and irregular blur, like the image of the moon on a lake. Test various kinds of glass from ordinary window glass to optical plane glass.

The experiment may be varied by holding the surface horizontally in the hand, just below the level of the eye, and viewing in it the images formed by reflection of the bars of a well-lighted window frame. If the bars appear sharp and straight the surface is plane, but if wavy and crumpled the surface is irregular.

The proportion of the incident light reflected at a surface depends very much on the nature of the bodies in contact, on the state of polish of the surface and the angle of incidence.

By photometrical experiments it has been found that polished silver reflects about 90 per cent. and a clean mercury surface about 67 per cent. of a direct incident beam in air. Transparent substances reflect much less, a polished glass-air surface reflecting only 4 per cent. and a water-air surface only 2 per cent. of a direct incident beam. When, however, the angle of incidence is $89\frac{1}{2}°$ both water and mercury reflect the same proportion of the incident beam, viz., 72 per cent.

Even with an optically rough surface, such as that of smoked glass, paper, or ordinary coinage, a very good image of an object can be got by reflection at nearly grazing incidence. The student should try this for himself (see also Arts. 34 and 65).

CALCULATIONS.

36. ALL problems on reflection at plane surfaces are, more or less, geometrical deductions, involving a knowledge of the laws of reflection in addition to the usual geometrical propositions.

The results of Art. 32 are not of very great importance, but the simple case where θ is an aliquot part of 180° should be remembered. In this case the number of images formed is $\left(\frac{2\pi}{\theta} - 1\right)$.

In Art. 33 the deviation produced by n reflections from mirrors inclined at an angle a, *when n is even*, should be specially noticed. If D denotes this deviation, then—

$$D = na.$$

Note.—In preparation for the work of the next chapter the reader should notice the following points:—

1. The results of Euclid vi. 3, A, and 4.

2. The meaning of the terms *infinite* and *infinity*. A quantity becomes *infinite* when its value becomes greater than any value we can assign to it. If the value of any quantity q is infinite, this is expressed by writing $q = \infty$.

The term *infinity* will be best understood from its use in the statement that parallel straight lines meet in infinity. If any straight line O A be produced to A′, in the direction O A, until it is of infinite length, the point A′ will be at infinity.

3. Consider the ratio $\frac{a}{x}$. If x becomes infinite, the ratio becomes $\frac{a}{\infty}$, and the value of this expression is zero. That is—

$$\frac{a}{\infty} = 0$$

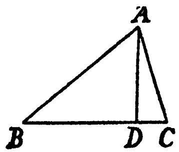
where a is *any* finite quantity.

4. The sine of any angle is equal to the sine of its supplement. That is—

$$\sin a = \sin (180 - a).$$

This is readily seen from a figure.

5. In any triangle the sides are proportional to the sines of the opposite angles.



In the triangle A B C we have—

$$\sin A B C = \frac{A D}{A B} \quad \text{(Art. 19, note.)}$$

$$\sin B C A = \frac{A D}{A C}.$$

$$\therefore \frac{\sin A B C}{\sin B C A} = \frac{A C}{A B}.$$

Similarly—

$$\frac{\sin B C A}{\sin C A B} = \frac{A B}{B C}, \text{ and}$$

$$\frac{\sin C A B}{\sin A B C} = \frac{B C}{C A}.$$

That is—

$$\text{Sin } A B C : \sin B C A : \sin C A B :: C A : A B : B C.$$

Q.E.D.

Examples II.

1. A ray of light starts from A, meets a plane reflecting surface at M, and is reflected to B. Prove that A M B is the shortest possible path from A to B by way of the mirror (Fig. 35).

If A M B be not the shortest path, let *any* other path A M′ B be shorter. Draw A N A′ normal to the mirror, and produce B M to meet A A′ in A′.

Then, since A N = A′ N we have, by Euclid i. 4, A M = A′ M and A M′ = A′ M′.

But A′ M′ + M′B > A′ B > A′ M + M B (Euc. i. 20).

∴ A M′ + M′ B > A M + M B.

Q.E.D.

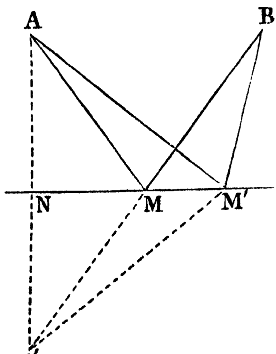


Fig. 35.

2. An object is placed between two mirrors inclined at an angle of 60°; find how many images are formed, and show that the images formed in the angle vertically opposite that contained by the mirrors are coincident. (The conditions of this question are represented in Figs. 31 and 32.)

Since 60° is an aliquot part of 360°, we have, for the number of images formed—

$$n = \left(\frac{2\pi}{\theta} - 1\right) = \left(\frac{360}{60} - 1\right) = 5.$$

Also, A_1''' and A_2''' are the images to be shown coincident. For this purpose we must prove $A\,O\,A_1''' + A\,O\,A_2'''$ (measured in *opposite* directions) equal to 360°.

If $A\,O\,M_1 = \alpha$ and $A\,O\,M_2 = \beta$, then, by the method of Art. 32, we have—

$$\begin{aligned} A\,O\,A_1''' &= 2\alpha + 2(60) = 120 + 2\alpha \\ A\,O\,A_2''' &= 2\beta + 2(60) = 120 + 2\beta. \\ \therefore A\,O\,A_1''' + A\,O\,A_2''' &= 240 + 2(\alpha + \beta) \\ &= 240 + 120 = 360. \end{aligned}$$

3. What must be the angle between two plane mirrors in order that a ray incident parallel to one of them may, after two reflections, be parallel to the other? *Inter. Sc.*, 1872.

Let α denote the angle between the mirrors, then, after two reflections, the deviation produced $= 2\alpha$ (Art. 33). But the deviation required by question $= 180 - \alpha$

$$\begin{aligned} \therefore 2\alpha &= 180 - \alpha; \\ \therefore 3\alpha &= 180; \\ \therefore \alpha &= 60. \end{aligned}$$

4. A small object is placed between two parallel mirrors as in Fig. 28. The distance between the mirrors is 6 inches, and the object is placed 2 inches from one of them. Find the distances between the corresponding members of the two series of images formed; also the distances between the odd members of each series, and between the even members of each series.

5. The sun is 30° above the horizon, and its image is observed in a tranquil pool. What, in this case, is the angle of incidence and reflection?

6. A man, 6 feet high, sees his image in a plane mirror hung vertically. The top of the mirror being 6 feet from the ground, determine its smallest length in order that the man may see his full-length image in it.

7. Find the deviation produced by reflection at a plane mirror, when the angle between the incident and reflected rays is 80°.

8. The angle between two mirrors is 10°. At what angle should a ray of light, travelling towards the intersection of the mirrors, be incident on either mirror in order that it may at the fourth reflection be reversed and travel back along the same course?

9. Show, that if a ray of light be incident at any angle, on one of two mirrors inclined at right angles to each other, then the ray is reflected from the second mirror in a direction parallel to its original direction.

10. A plane mirror which is first one foot from an object is then moved back one foot *parallel to itself*. How far does the image move? Give a diagram in illustration.

11. A mirror, scale, and telescope are used to measure the deflection of a suspended system. The scale is distant one metre from the mirror, and during the movement of the mirror the scale reading alters from 14 cm. to 44 cm. Find approximately the angle of deflection of the system.

12. Smoke the outside of a glass tube. Cover one end with tinfoil and prick a pinhole in the centre of the tinfoil. Look through the other end at a candle. Explain the formation of the concentric circles of light.

13. Make a measured drawing showing the positions of all the images formed by two mirrors inclined to each other at 45°, of an object placed between the mirrors.

14. Two mirrors, M_1 and M_2, are inclined to each other at 50°, and an object is placed between them. Make a measured drawing showing in black the situation of all the images formed where the rays from the object strike M first, and in red the situation of all the images where the rays first strike M_2.

15. An object is placed $\frac{1}{2}$ in. from one plane mirror and 1 in. from another plane mirror parallel to the first, so that the object is between them. Make a measured drawing showing the positions of all the images up to the fourth order.

16. A horizontal narrow strip of plane mirror is hung up against the wall of a room on a level with the eye of the observer. Draw a diagram showing how much of a side wall of the room will be visible by reflection from the mirror.

CHAPTER V.

REFLECTION AT SPHERICAL SURFACES.

37. Preliminary definitions. Mirrors of spherical, parabolic, and cylindrical curvature are used in optical instruments. We shall in this chapter confine the discussion to spherical mirrors.

A spherical mirror, A A′ (Fig. 36), is usually a very small segment of a spherical surface, and may be either

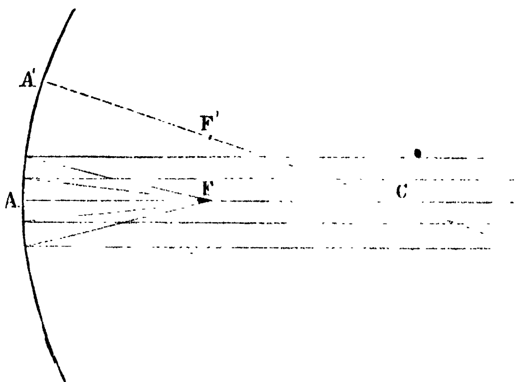


Fig. 36 (A).

concave (Fig. A) or convex (Fig. B). The centre, C, of the spherical surface of which the mirror is a part, is called the *centre of curvature* of the mirror. The line C A, joining the centre of curvature and the central point, A, of the mirror, is the *principal axis* of the mirror, the point A being sometimes called the *pole* or *centre of the face.* Any other

line C·A′ drawn through C and cutting the mirror is called a *secondary axis*, and is, like the principal axis, a normal to the mirror. A section of the mirror by a plane passing through the pole and the centre of curvature is called a *principal section.*

The *aperture* of the mirror is the angle enclosed by two straight lines drawn from the centre of curvature to opposite points in the edge of the mirror. We shall at first limit the discussion to mirrors of small aperture—say not exceeding 10°, though for the sake of clearness the diagrams will show greater apertures.

When a parallel pencil of light is incident on a spherical mirror, in a direction parallel to the principal axis, the

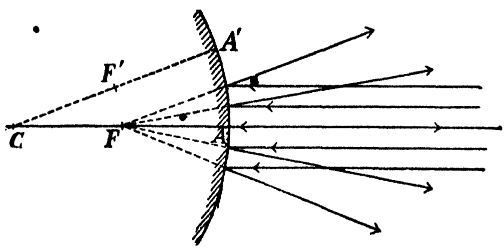


Fig. 36 (B).

reflected pencil converges to or diverges from a point F on the principal axis. This point is called the *principal focus* of the mirror, and the distance, A F, between the principal focus and the pole of the mirror is termed the *focal length* of the mirror. If the pencil is incident parallel to a secondary axis, the reflected rays are, in a similar way, brought to a focus at a point F′ on that axis. In the case of spherical mirrors, it is not strictly true to say that the reflected pencils meet accurately at a point; if the pencil is small, this is approximately the case, but with large pencils the outer rays are reflected to points nearer the mirror than the others. This irregularity of reflection from a spherical surface is called *spherical aberration* (see Art. 44).

38. Construction for reflected ray. Let P Q be any ray incident at Q on a spherical mirror [concave Fig. 37 (A), or convex Fig. 37 (B)]. At Q draw the normal, Q N, to the reflecting surface, by joining C Q and producing it if necessary. Then, in accordance with the laws of reflection, the reflected ray Q P′ is obtained by drawing P′ Q in such a

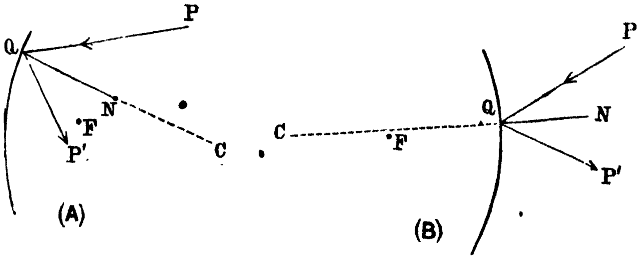


Fig. 37.

direction that the angle of reflection P′ Q N is equal to the angle of incidence P Q N.

From this it is evident that a ray incident along a normal is reflected back along the path by which it came. Also, from Art. **37**, a ray incident parallel to the principal axis is reflected through the principal focus. These two particular cases of reflection should be carefully remembered.

39. Position of principal focus. Let P Q (Fig. 38) be a ray incident on the concave mirror A Q, in a direction parallel to the principal axis C A. Then P Q is reflected through the principal focus F, and the angle P Q C is equal to the angle F Q C. But the angle P Q C is equal to the angle F C Q (Euc. i. 29); therefore F Q C = F C Q, and therefore F Q = F C.

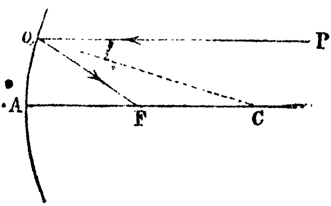


Fig. 38.

Now, if A Q is small, F Q is approximately equal to

F A, and therefore F C equals F A (approximately). That is, the principal focus F is midway between the pole A and the centre of curvature C; and, if A F be denoted by f and A C by r, we have $f=\frac{r}{2}$; or *the focal length of a spherical mirror, for rays incident on a small portion of its surface near the pole, is equal to half the radius of curvature of that mirror.*

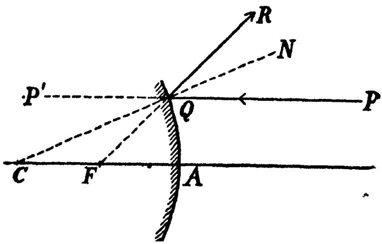


Fig. 39.

An exactly similar proof is applicable in the case of a convex mirror. The principal focus F (Fig. 39) is in this case on the backward prolongation of the reflected ray, and is, of course, *virtual.*

40. Conjugate foci.* Let P (Fig. 40) represent the position of a luminous point on the principal axis of the concave mirror A Q. Then, Art. 27, the image of P will be

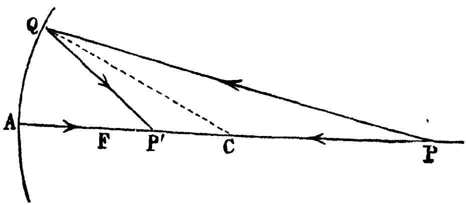


Fig. 40.

formed at the intersection, after reflection, of any two rays coming from P. Consider the rays P A incident along the normal to the mirror, and P Q incident at Q; P A is reflected back along A P and P Q is reflected along Q P′, making the angle of reflection P′ Q C equal to the angle of

* The reader should refer to Euclid vi. 3 and A before reading this article.

incidence P Q C. Let the reflected rays A P and Q P′ intersect at P′; then P′ is the image of P, and lies on the principal axis of the mirror. Also, since P Q C = P′ Q C, then—

$$\frac{QP'}{QP} = \frac{P'C}{CP}. \qquad \text{(Euclid vi. 3.)}$$

But, if A Q is small, then P Q = P A, and P′ Q = P′ A, and therefore—

$$\frac{AP'}{AP} = \frac{P'C}{CP}.$$

If now A C be denoted by r, A P by u, and A P′ by v, then $P'C = AC - AP' = r - v$, and $CP = AP - AC = u - r$. And the above proportion becomes—

$$\frac{v}{u} = \frac{r-v}{u-r}.$$

$$\therefore u(r-v) = v(u-r).$$

$$\therefore ur + vr = 2uv.$$

Dividing by $u\,r\,v$, then—

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

But, by Art. 39—

$$f = \frac{r}{2}. \quad \therefore \frac{2}{r} = \frac{1}{f}, \text{ and}$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \qquad (1).$$

where u denotes the distance of the luminous point P from the pole of the mirror, v denotes the distance of the image of P from the pole of the mirror, r denotes the radius of curvature of the mirror, and $f\left(=\frac{r}{2}\right)$ denotes the focal length of the mirror.

This may be expressed in words by saying that the *algebraical sum of the reciprocals of the distances of the luminous point and its image from the pole equals the reciprocal of the distance of the principal focus from the pole.*

The relation thus obtained is of great importance. It will be noticed that, in the case here considered, all the

distances involved are measured in the same direction from A. When this is not the case, it is necessary to adopt some convention as to sign. The most general convention, and the one adopted throughout this book, is *to consider all distances measured in a direction* **opposed** *to the incident light as* **positive,** *and distances measured in the* **same** *direction as the incident light as* **negative.** With this convention the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ is applicable to all cases of reflection at spherical mirrors.

The points P and P′ connected by this relation are said to be *conjugate foci*, because of the fact that either point may be considered as the image of the other. From the construction it is evident that the image of a luminous point at P′ would be formed at P, just in the same way as the image of P is formed at P′. This may be illustrated experimentally by means of a candle and a concave mirror. If the flame of the candle be placed, at any point beyond C, on the principal axis of the mirror, an image of the flame will be seen between C and the mirror. The position of this image can be marked by adjusting the position of a needle until it appears to coincide with the image. It will then be found that if the candle flame be placed at the point marked by the needle, the image will be seen at the point originally occupied by the flame.* If the luminous point P is not on the principal axis, then its conjugate focus, P′, will be on the secondary axis passing through P, and, distances being measured along the axis, the relation, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, can be established in the way explained above. In fact, the two cases are identical, for the geometrical relations of a secondary axis to a spherical mirror are exactly the same as those of the principal axis.

The same relation for conjugate foci can also be proved for a convex reflecting surface. Let P (Fig. 41) represent a luminous point placed in front of the convex mirror A Q. Then, as in the case of the concave mirror, an image of P is

* This is an instance of the reversibility of light (Art. 22).

formed at P′ on the axis passing through P. In this case P′ is a *virtual focus*, from which the reflected rays A P and Q Q′ appear to diverge. To determine the position of P′ we have—

$$\frac{\text{Q P}'}{\text{Q P}} = \frac{\text{P}'\text{C}}{\text{P C}}. \qquad \text{(Euclid vi. A.)}$$

Also, if A Q be small, this proportion becomes—

$$\frac{\text{A P}'}{\text{A P}} = \frac{\text{P}'\text{C}}{\text{P C}} = \frac{\text{A C} - \text{A P}'}{\text{P A} + \text{A C}}.$$

In this equation, A C, A P, A P′ bear positive numerical values. Now, using the same meanings as above for u, v

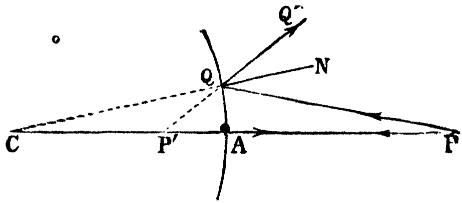


Fig. 41.

and r, it is evident that A C must be replaced by $-r$, A P by u, and A P′ by $-v$. Therefore, substituting, we get—

$$\frac{-v}{u} = \frac{-r-(-v)}{u-r} = -\frac{r-v}{u-r},$$

Cross-multiplying, this becomes—

$$v u - v r = u r - u v.$$

$$\therefore u r + v r = 2 u v.$$

and dividing through by $u\,r\,v$, we get—

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

That is—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

In an exactly similar way this formula may be established for the reflection of a convergent pencil at a spherical surface, and the reader will find it an instructive exercise

to draw the necessary figures, and deduce the formula therefrom.

In proving theorems such as the above in which some quantities may be positive and some negative, students are recommended to take a case in which all the quantities are positive. If this be done, confusion arising from questions of signs will not occur.

41. Relative position of conjugate foci. In the preceding article it has been shown that the formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ holds good for all cases of reflection at a spherical surface. By a general discussion of this formula it is possible to determine the position of P′ for any given position of P. For example, if u be infinite—that is, if the incident light be parallel—then we have—

$$\frac{1}{v}+\frac{1}{\infty}=\frac{2}{r}.$$

Therefore, since $\frac{1}{\infty}=0$,

$$\frac{1}{v}=\frac{2}{r} \text{ or } v=\frac{r}{2}.$$

This means that if a pencil of parallel light be reflected at the spherical surface, its focus, after reflection, is on the axis parallel to the incident light at a point whose distance from the mirror is equal to half the radius of curvature of the mirror (Art. 39).

The general application of this method is, however, somewhat troublesome; we shall therefore consider the question in another way.

The formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ may be written thus—

$$uf+vf-uv=0;$$

or, adding f^2 to both sides of the equation, we get—

$$uv-uf-vf+f^2=f^2.$$
$$\therefore u(v-f)-f(v-f)=f^2.$$
$$\therefore (u-f)(v-f)=f^2.$$

Now, if x and x' denote the distances of P and P′ respectively from F (Fig. 40), we evidently have—

$$FP = x = (u - f),$$
$$FP' = x' = (v - f),$$

and the formula becomes—

$$x x' = f^2.* \qquad (2).$$

From the relation thus obtained we may deduce the following rules:—

(*a*) Since f^2 is always positive, being a square, then x and x' must always have the same sign—that is, **the conjugate foci, P and P′, are always on the same side of F.**

(*b*) If x is greater than, equal to, or less than f, then x' is less than, equal to, or greater than f; or—

$$\text{If } x > = < f, \text{ then } x' < = > f.$$

In addition to the above, the following general rule will be found of great use in determining the motion of the image corresponding to any given motion of the object along an axis of the mirror:—*When an image is formed by reflection, any motion of the object, in a given direction along an axis of the mirror, causes motion of the image in an* **opposite** *direction along the same axis.*†

By the application of these rules we may trace the position of P′, as P travels from infinity on one side of the mirror up to infinity on the other side. Let us consider first the case of a **concave** mirror (Fig. 42). When x is infinite, x' is zero [Art. 36 (3)]; that is, when P is at infinity P′ is at F. As x decreases from $+\infty$ to f, so x' increases from 0 to f. That is, as P travels from infinity to C, P′ travels from F to C. When x equals x', then, since $x x' = f^2$, we have $x = x' = f$. That is, when P is at C, P′ is also at C.

As x decreases from f to 0, then x' increases from f to $+\infty$. That is, as P travels from C to F, P′ travels from C to infinity.

* The same sign convention applies to the measurements of x and from F as for the measurements of u and v from A.

† In the case of a plane mirror, any *normal* to its surface may be considered as an axis.

As x decreases* from 0 to $-f$, x' increases from $-\infty$ to $-f$. That is, as P travels from F to A, P′ travels from infinity† behind the mirror up to A, where P and P′ again coincide as at C.

Now, if P be a real luminous point, or small object, such as a candle flame, it can evidently travel no further than A, and hence we have traced all possible positions of the image of a real luminous point placed anywhere in

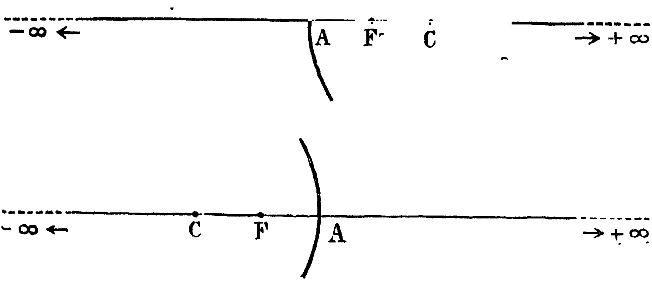


Fig. 42.

front of a concave spherical mirror. The three following positions should be remembered :—

(1) When the object (a luminous point) is anywhere beyond C, the image lies between F and C. $[x > f. \ \therefore x' < f]$.

(2) When the object is between C and F, the image is beyond C. $[x < f. \ \therefore x' > f]$.

(3) When the object is between F and A, the image is behind the mirror, and is virtual. $[x > -f. \ \therefore x' < -f]$.

Also, (4) Image and object coincide at C and at A. $[x = \pm f. \ \therefore x' = \pm f]$.

If a convergent pencil is incident on a concave mirror, then its focus, P, is behind the mirror, and the reflected pencil has its focus P′ in front of the mirror. P′ may, in

* *Decreases*, because $-f$ is *less than* 0.

† When an image disappears at infinity in front of a mirror $(+\infty)$, it, in general, reappears from infinity behind the mirror $(-\infty)$.

this case, be considered as the image of a point P behind the mirror. Hence, as x decreases from $-f$ to $-\infty$, x' increases from $-f$ to 0. That is, as P (the focus of a *convergent* pencil incident on the mirror) travels from A to infinity behind the mirror, P′ travels from A to F.

This case is of little practical importance, as, in all ordinary cases, we have to deal with the reflection of divergent pencils.

Let us now consider the case of a **convex** mirror (Fig. 42). The principal focus, F, is here *behind* the mirror; hence, when P is a real point, x must always be greater than f, and therefore x' is always less than f. That is, as P travels from infinity in front of the mirror, up to the mirror at A, P′ travels from F to A. This is the only case of practical importance.

When a convergent pencil is incident on a convex mirror, its focus, P, is behind the mirror, and the position of the conjugate focus, P′, corresponding to any given position of P is determined, as explained above, by the aid of the relation $xx' = f^2$.

It may be useful to summarise the points of practical importance mentioned in this article.

I. Concave mirror.

1. Luminous point between $+\infty$ and C. Real image between F and C.

2. Luminous point at C. Real image at C.

3. Luminous point between C and F. Real image between C and $+\infty$.

4. Luminous point between F and A. Virtual image between A and $-\infty$.

5. Luminous point at A. Image at A.

II. Convex mirror.

1. Luminous point between A and $+\infty$. Virtual image between A and F.

Each of these cases can be easily deduced from the relation $xx' = f^2$, which expresses in a concise and easily remembered form the whole theory of conjugate foci in connection with reflection at spherical surfaces.

It will be shown later (Art. 94. II., 2, cf. Art. 26) that it is possible to produce what is practically a virtual *object*. Any point on it is a point behind the mirror to which a pencil converges, and through which every ray of that pencil would pass if the mirror were not there. Images and objects are always interchangeable: so we get a case I., 6, by inverting case I., 4, in which a virtual object between A and $-\infty$ has a real image between A and F.

For a convex lens we have, in addition to II., 1, the following cases:

II., 2. Virtual object between A and F gives real image between A and $+\infty$. This is II., 1 inverted.

3. Virtual object between F and C gives virtual image between C and $-\infty$.

4. Virtual object at C coincides with virtual image.

5. Virtual object between C and $-\infty$ gives virtual image between C and F. This is inverse of 3.

42. Formation of images by spherical mirrors. When a luminous object is placed in front of a spherical mirror an image is formed, which may be *real* or *virtual* according to the circumstances of the case. If real, the image is formed in front of the mirror, and can be received on a screen; but if virtual, it appears to be behind the mirror, and cannot be received upon a screen. It may, however, be located by means of a pin as in Exp. 2 (see Exp. 10). The paths of the rays by which these images are seen are described in the next article.

The following is a general construction for determining the image of an object formed by a spherical mirror. Let

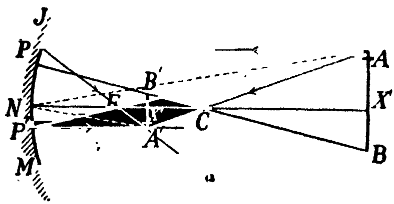


Fig. 43.

A B (Figs. 43, 45, 46) represent an object placed in front of the mirror P M. Consider the ray A M coming from A and incident on the mirror normally at M. Its direction is obtained by joining A C, and, if necessary, producing the line to cut the mirror in M. The reflected ray M A travels

back along the path of the incident ray (Art. 38), and the image of A lies somewhere on this path. Again the ray A P, drawn parallel to the principal axis, is reflected along P F (Art. 38), and the image of A lies on this line also. Hence the image of A is found at A′, the intersection of the lines M A and P F. A third ray A P′ through F may also be drawn. It is reflected along P′ A′ parallel to the principal axis. Similarly, an image of B is formed at B′, and images of points lying between A and B are formed at corresponding points between A′ and B′, and therefore A′ B′ is the complete image of A B.

In connection with the formation of images the following four points have to be considered:—

1. *Relative position of image and object.* This has been fully considered in preceding article; the reasoning there employed is applicable whether the luminous point P be an isolated point, or a point on an object of finite size.

2. *Whether the image is real or virtual.* Whenever the image appears behind the mirror, it must necessarily be virtual; hence, it is only necessary to know the position of an image to decide whether it is real or virtual.

3. *Whether the image is inverted or erect.* It is evident, from Fig. 43, that when object and image are on opposite sides of C, the latter is inverted because of the crossing of the rays passing through C. Hence, if the relative positions of object and image are known, this point is easily decided. It may be remarked that the image of a real object is always inverted if real, and erect if virtual.

4. *Relative size of image and object.* The ratio of the linear dimensions of the image to the corresponding linear dimensions of the object is called the **magnification.** In Fig. 43 the magnification is equal to the ratio of A′B′ to A B. When the image is erect the magnification is taken as positive, and when inverted it is regarded as negative. The magnification is often written as $\frac{\text{image}}{\text{object}}$ or **m**, and we shall now proceed to express **m** in terms u v and f, using Fig. 43 for this purpose, as in that diagram these last three quantities are positive.

(*a*) In the triangles A′ B′ C′ and A B C we have—

$$\frac{A' B'}{A B} = \frac{C X'}{C X} = \frac{c'}{c}.$$

That is—

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from C}}{\text{Distance of object from C}} = \frac{c'}{c} = -\frac{r - v}{u - r},$$

the negative sign being placed in accordance with the above reasons. The negative sign is not needed before $\frac{c'}{c}$, for if the image is inverted c' is of opposite sign to c.

(*b*) A ray A N incident at the pole of the mirror is reflected to A′ since A′ is the image of A. The angles A N X and A′ N X′ are thus equal, and the right-angled triangles A N X, A′ N X′ are similar.

Hence—

$$\frac{A' X'}{A X} = \frac{N X'}{N X}.$$

Similarly—

$$\frac{B' X'}{B X} = \frac{N X'}{N X};$$

therefore by addition—

$$\frac{A' B'}{A B} = \frac{N X'}{N X} = \frac{v}{u}.$$

That is—

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from mirror}}{\text{Distance of object from mirror}} = -\frac{v}{u},$$

the negative sign being inserted, since the image is inverted.

(*c*) The triangle A F X is similar to the approximate triangle P′ F N.

$$\therefore \quad \frac{P' N}{A X} = \frac{N F}{X F} = \frac{f}{u - f}.$$

But—

$$\frac{P' N}{A X} = \frac{A' X'}{A X} = \frac{A' B'}{A B} \text{ (by the above)};$$

$$\therefore \quad \frac{A' B'}{A B} = \frac{f}{u - f}.$$

That is—

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Focal length of mirror}}{\text{Distance of object from focus}} = \frac{f}{u-f},$$

the negative sign being again introduced.

(*d*) The triangle A′ F X′ is similar to the approximate triangle P N F.

$$\therefore \frac{A'X'}{PN} = \frac{FX'}{NF}.$$

But—

$$PN = AX$$

$$\frac{A'X'}{AX} = \frac{FX'}{NF} = \frac{v-f}{f}.$$

That is—

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from focus}}{\text{Focal length of mirror}} = \frac{v-f}{f},$$

since the image is inverted.

In this way we get four different but not independent relations between the linear dimensions of the image and object. These are—

$$m = \frac{\textbf{Image}}{\textbf{Object}} = \frac{c'}{c} = \frac{v}{u} = \frac{f}{u-f} = \frac{v-f}{f} \qquad (3)$$

That these four expressions are equal can also be very simply proved from the general equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

For example, to prove the equality of second and third, multiply the general equation by u

$$\frac{u}{v} + 1 = \frac{u}{f}$$

$$\therefore \frac{u}{v} = \frac{u}{f} - 1 = \frac{u-f}{f}$$

$$i.e. \quad \frac{v}{u} = \frac{f}{u-f}.$$

The proportions expressed by the above equations apply to linear dimensions only; for relative areas we have—

$$\frac{\textbf{Area of image}}{\textbf{Area of object}} = \left(\frac{c'}{c}\right)^2 = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{u-f}\right)^2 = \left(\frac{v-f}{f}\right)^2.$$

It thus appears that if we know the positions of the object of its image we can completely determine the nature of the image. The results of Art. **41**, as there summarised, are therefore of great importance, and for this reason we give below, with figures, the cases for a luminous object of finite size, corresponding to I., 1, 2, 3, 4, and II., 1 of that article.

I. Concave mirror.

(1) Object between C and infinity in front of mirror. The image lies between C and F, and is *real*, *inverted*, and *diminished*. (Fig. 43.)

(2) Object C B at C. The image, C B′, at C, is *real*, *inverted*, and of *same size* as object. (Fig. 44.)

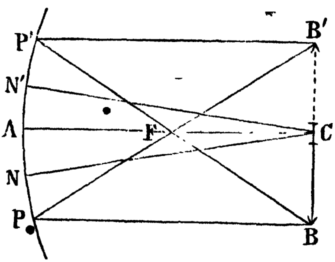


Fig. 44.

The construction for this image should be noticed Any two rays C N, C N′ coming from C are normal to the mirror, and therefore, on reflection, again intersect at C. That is, the image of C is formed at C. B′ the image of B is found in the usual way.

(3) Object between C and F. Image between C and infinity in front of the mirror. If A′ B′ of Fig. 43 be supposed to represent the object, then A B represents its image, and the figure illustrates the case we are now considering. The image is *real*, *inverted*, and *magnified*.

(4) Object between F and the pole. The image is *behind* the mirror, between infinity and the pole, and is *virtual*, *erect*, and *magnified*. (Fig. 45.)

In the limit, when the object is at the pole, the image is

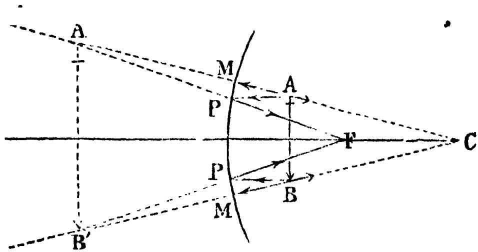


Fig. 45.

also at the pole, and coincides with the object in position and size (Art. 41, I., 5).

II. Convex mirror.

Object in front of mirror between infinity and the pole.

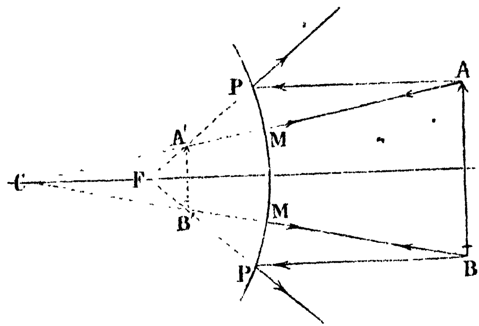


Fig. 46.

The image lies between F and the pole, and is *virtual*, *erect*, and *diminished*. (Fig. 46.)

In addition there are the cases in which the object is virtual. The student can verify these facts for the image.

I. 6. It is erect, diminished, real.

II. 2. Erect, increased, real.

3. Inverted, increased, virtual.

4. Inverted, same size, virtual.

5. Inverted, diminished, virtual.

43. Exercise. To draw the rays by which an eye sees an image of an object formed by reflection at a spherical mirror. If the object is near the principal axis, the image will be near the axis, and, therefore, also the eye must not be far removed from the axis. Let M M (Fig. 47) be a concave mirror, A B an object placed in front

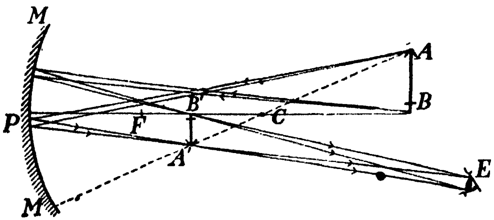


Fig. 47.

of it, A′ B′ the real image of A B, and E the position of the eye. Now this image is only the locus of intersection of reflected rays, and hence is not self luminous, so that it can be seen only by those rays which originally come from the object, and, passing through the image, enter the eye.

Thus, to depict the rays by which E sees A′, draw a pencil of rays diverging from A′ and entering E. Produce the rays backwards to meet the mirror at P, and join the points of intersection to A. The rays by which A is seen are included in the incident pencil A P and the reflected pencil P A′ E. The same construction can be

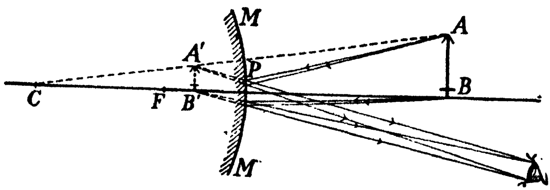


Fig. 48.

applied to other points of the image and object, and a similar construction holds for the visual rays by which the virtual image of an object placed in front of a concave or convex mirror (Fig. 48) is seen.

44. Spherical aberration. In dealing with the laws of reflection from concave spherical mirrors, we have, up to now, limited the discussion to mirrors of small aperture, and we have learnt that parallel rays falling on such

a mirror are all brought together at one point—the principal focus; and that rays diverging from any luminous point are all brought together at one point—the conjugate focus. In order to explain the necessity of this limitation we will now consider the case of a concave mirror of a very large aperture, that is, forming a very large segment of a sphere. (It will well repay the student to do for himself, on a larger scale and drawing many more rays, what has been done in Fig. 49.) Draw a large segment of a circle. Through its centre draw the diameter C A, and parallel with this a number of equidistant straight lines to represent rays of light in a parallel beam falling on a concave mirror. From C draw dotted radii to every point at which a ray is incident on the mirror. These are the normals to those points. Then from those points, and on the inner sides of the normals, set off angles exactly equal to those on the other sides, and draw straight lines at these angles to represent the reflected rays. It will then be seen that the rays nearest the axial ray cut that ray after reflection at a point as near as possible halfway between C and A. A pair of rays a little further from the axis will be found to intersect the axis a trifle behind this point. The next pair of rays intersect the axis after reflection considerably behind F, and the next pair still further behind. Any such wandering of the marginal rays from the focus of the central rays is called *aberration*, and this particular case being due to the form (spherical) of curve employed, is called *spherical aberration*.

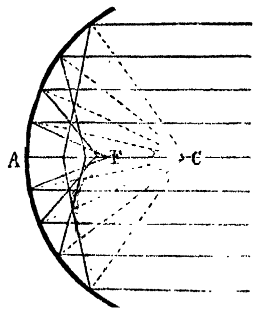


Fig. 49.

45. Caustic. Fig. 49 shows, and the student's diagram with three or four times as many rays does so more clearly, that all the reflected rays lie within an area bounded at the back by the mirror, and in front by a double curve

called the *caustic curve.* This curve is very bright, especially at its vertex, which coincides with the principal focus. Such a curve may be clearly seen on the surface of milk, when a tumbler is about three-quarters filled; a candle being placed so that the inner rim of the glass reflects its light down upon the milk.

46. Diaphragms or Stops. For all optical purposes spherical aberration is an important defect, and the way it is kept within allowable limits is by using mirrors of very small aperture, or by screening the margin of the mirror by an opaque plate called a *diaphragm*, with a central hole in it. Of course the more the mirror is *stopped down* the sharper becomes the definition, but the loss of light becomes at last very serious. It must be noted, too, that these means do not correct, but simply lessen, the defect, which indeed is inseparable from the spherical form.

47. The optical bench. This apparatus is of such frequent use in optical measurements that it is advisable, at this stage, to consider briefly its construction and method of use. In one of its simplest forms the optical bench consists of a thick base board (B B, Fig. 50) of well-seasoned wood,

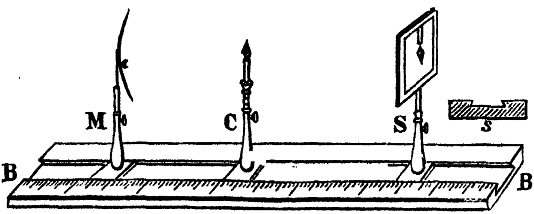


Fig. 50.

about three metres long, and having a deep, wide groove running along the middle of its upper face. The edges of this groove are not vertical, but are obliquely cut in the way shown at *s*, Fig. 50. A scale, showing centimetres and millimetres, is cut parallel to the groove in such a way that the edge of the groove is also the measuring edge of the scale. A set of uprights, constructed to hold suitably mounted*

* The mounting of any object should be effected in such a way that the point, to or from which measurements are to be made, is on the axis of the upright.

lenses, mirrors, candles, screens, etc., are fitted into small base boards, which are so made that they can be pushed into the groove in BB at one end, and moved along to any position on the bench. This position is indicated, with reference to the scale of the bench, by means of a fine index line cut on the base of the upright in the plane of its vertical axis.

The optical bench may be conveniently used for photometric measurements with Bunsen's photometer (Art. 17), which, mounted on an upright placed between two other uprights carrying the lights to be compared, is readily adjusted in the right position, and the required distances are at once read off on the scale.

The optical bench is chiefly used for the experimental determination of the constants of mirrors and lenses. Fig. 50 shows a concave mirror mounted on a stand M, a candle in another upright C, and a screen of thin white unglazed paper mounted on a frame in a third stand S. As shown in the figure an image of the candle is focussed on the screen. The distances between M and S and M and C can be read off on the scale; they are respectively v and u.

A candle flame has, however, large dimensions, and therefore measurements to it cannot be made with great accuracy. A screen with a small hole in it placed in front of a gas flame or incandescent mantle is better, but a still better source of light for laboratory use can be made as follows:—An incandescent electric lamp, or some other bright source, is enclosed in a box, A (Fig. 114), a portion of the front of which is cut away so as to allow the light to shine along the bench. A vertical white cardboard screen, B, containing a large hole, over which a piece of fine wire gauze is fixed, is fastened to the front of the box by india-rubber bands. The strongly illuminated gauze serves as the object, and being in a vertical plane, measurements of u can be accurately made.

C (Fig. 114) shows a suitable carrier for small lenses or mirrors. It consists of a sliding base piece supporting two adjustable arms grooved on the inside. The edges of the lens or mirror are placed within the grooves, and an

elastic band is placed around the arms to keep them in position. In the same figure, D represents a small screen; it consists simply of a piece of white millboard mounted on a sliding base piece.

48. Experimental determination of the radius of curvature, and the focal length of a mirror.

I. Concave mirrors.—1. The simplest method of determining the focal length is to allow a beam of parallel light to be incident on the mirror in a direction parallel to the principal axis, and then to measure the distance of the focus of the reflected beam from the mirror.

Exp. 6. For this purpose mount the mirror in a suitable stand or clip, with its axis parallel to a graduated bar of wood, along which the stand slides. At the zero end of the bar, and at right angles to its length, fix a paper or millboard screen with its centre approximately on the same level as the principal axis of the mirror. Point this arrangement toward the sun, or some other well-illuminated distant object, and adjust the position of the mirror by the method of oscillations (Exp. 1) until a clearly defined image of the object chosen is formed on the screen. The distance between the mirror and screen as indicated by the graduations on the bar is the focal length, for if the object is sufficiently distant the image is practically at the principal focus of the mirror.

2. Art. 42 proves that when an object is placed at the centre of curvature of a concave mirror, the image is also at the centre, but in an inverted position.

Exp. 7. Fix a short polished needle vertically, and point upwards in a wooden stand or clip, and place it in front of the mirror, so that the point of the needle is on the principal axis of the mirror.

Unless the needle be placed too close to the mirror, an inverted image may be seen by an eye placed near the principal axis, at some distance from the mirror (as in Fig. 47). By repeated trials adjust the position of the needle until its point coincides with the point of the inverted image for *several positions of the eyes.*

The point of the needle is now at the centre of the mirror, and hence the radius of curvature is obtained by measuring the distance between the pole of the mirror and the point of the needle. The focal length is equal to half this distance. If the distance is small, measure it by means of a pair of compasses and a scale; but, if large, use a wooden rod, pointed at both ends and of adjustable length, or fix the mirror and needle in two of the uprights of the optical bench, and make the measurements as described in Art. 47.

Exp. 8. Using an optical bench and either of the forms of object mentioned in the latter part of Art. 47, adjust the mirror until an image of gauze is focussed on the screen alongside the gauze itself (Fig. 51). Since the rays thus return very nearly along their former paths, C is practically the centre of the surface A D B. Measure C D as in Exp. 7.

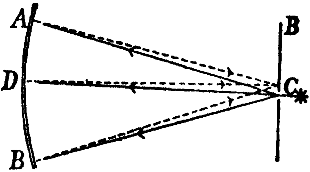


Fig. 51.

3. The general formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, may also be employed.

Exp. 9. Mount the mirror in an optical bench in a slightly slanting position, and receive the image P on a screen placed as in Fig. 52, so as not to interfere too much with the rays from O. Measure u and v, and calculate f. Repeat for different distances.

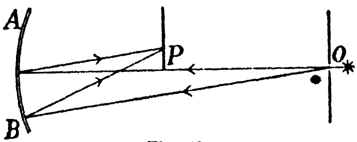


Fig. 52.

Instead of calculating values of f from each pair of values of u and v, these values may be plotted on squared paper and the value of f deduced graphically. For since $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, $\frac{f}{u} + \frac{f}{v} = 1$, which shows that if the points (u, o), (o, v) be joined the line will pass through the point (f, f) for all values of u and v.*

Therefore measure off values of u_1, u_2, u_3 . . . along the horizontal line O U (Fig. 53), and values of v_1, v_2, v_3 . . . along the vertical line O V. Join the corresponding points. The lines u_1v_1, u_2v_2, u_3v_3 . . . should all intersect on the line O F, bisecting the angle V O U. Find the mean point of intersection; its co-ordinates are equal to each other and to the focal length.

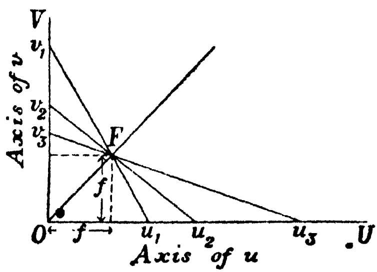


Fig. 53.

* See Briggs and Bryan's *The Right Line and Circle*, § 32.

4. The curvature of a spherical surface may also be measured mechanically by means of a little instrument called a Spherometer.* This consists of a metal frame carrying four pointed legs, three of which are fixed and form the corners of an equilateral triangle, while the fourth, which is central, screws through the frame. In use, the feet of all four legs are made to rest first on a flat surface and then on the spherical surface, and from a knowledge of the distance through which the central leg has been elevated or depressed (for the accurate measurement of which special means are provided), and the distance between the fixed legs, the radius of curvature of the surface can be calculated.

II. Convex mirrors.—1. The radius may be determined by means of a parallax method similar to that of Exp. 7.

Exp. 10. Mount the mirror on the optical bench as usual, and place before it a well-defined object, such as a white thermometer tube. An image can be seen; of course it is virtual. Place a pin on the stand behind the mirror, so that it is just visible over the top, and adjust its distance from the mirror till it stands as nearly as possible over the image of the thermometer tube for all positions of the eye. Then the distance of tube from mirror $= u$, and the distance of pin from mirror $= v$, and f can be found from the general formula.

A similar method can be used for a concave mirror, the tube being placed so near it as to have a virtual image. But unless the mirror has a very large radius of curvature this method is not to be recommended.

2. All other optical methods require the use of lenses, hence a description of the methods employed is postponed to Art. 95.

3. The radius may be determined by means of the spherometer.

* See Bower and Satterly's *Practical Physics*, §§ 27, 28, 30, for further details of this instrument and methods of use.

CALCULATIONS.

43. **The formulæ of importance in the preceding chapter are—**

$$1. \qquad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{r}. \qquad \text{(Art. 40.)}$$

Distances measured from the pole of the mirror.

$$2. \qquad x\,x' = f^2. \qquad \text{(Art. 41.)}$$

Distances measured from the focus.

$$3\ (a). \quad \frac{\text{Image}}{\text{Object}} = \frac{c'}{c}$$

Distances measured from the centre of curvature.

$$3\ (b). \quad \frac{\text{Image}}{\text{Object}} = -\frac{v}{u}$$

Distances measured from the pole of the mirror.

$$3\ (c). \quad \frac{\text{Image}}{\text{Object}} = -\frac{f}{u-f}$$

Distances measured from the pole of the mirror.

$$3\ (d). \quad \frac{\text{Image}}{\text{Object}} = -\frac{v-f}{f}$$

Distances measured from the pole of the mirror.

(Art. 42.)

Distances measured in a direction **opposed** to that of the incident light are considered **positive**, and those measured in the **same** direction as the incident light are considered **negative**.

This convention applies to all cases, wherever the distance considered may be measured from. In applying the above formulæ the following points must be noticed:—

1. On substituting a numerical value for any of the symbols, the sign of the former must always be attached.

For example, if in formula (1), $u = 6$ and $v = -8$, then, on substitution we get—

$$\frac{1}{-8}+\frac{1}{6}=\frac{1}{f}=\frac{2}{r}.$$

$$\therefore \frac{1}{24}=\frac{1}{f};$$

$$\therefore f = 24 \text{ and } r = 48.$$

2. **In applying a formula to determine one of the involved distances, the others being known, no sign must be given to the unknown distance.** Thus, in the above example, no sign is at first given to f; but the result, when worked out, shows it to be positive —that is, the mirror is concave.

3. When distances are measured from the pole of the mirror [formulæ 1, 3 (b), 3 (c), and 3 (d)], the radius of curvature and focal length are **positive** for a **concave** mirror, and **negative** for a **convex** mirror. This is in accordance with the sign convention given above, and needs special notice only as a reminder.

4. **Always draw a fairly accurate figure** representing given conditions. This prevents mistakes as to sign.

Formulæ 1 and 3 (b) are the most important.

Formulæ 3 (a) and 3 (b) should be learnt *in words* (Art. **42**); 3 (c) and 3 (d) are not important, but are sometimes very convenient. The different forms of formula 3 may be remembered by noticing that "image" and "v" are associated, as are also "object" and "u."

5. Sign need not be considered in connection with formulæ 3 if the ratios be learnt *in words.* But if learnt as formulæ involving u, v, f, c, and c', then the signs must be considered, just as in any other case, and the interpretations of the results given in Arts. **42-4** must be remembered.

When the magnification is one of the data of a problem, great attention must be paid to this point. See Ex. III. 4.

If on substitution $\dfrac{\text{image}}{\text{object}}$ is found to be positive the image is erect, and, if negative, inverted.

A convex mirror gives only virtual images, and these are always erect. A concave mirror gives inverted real images and erect virtual images.

EXAMPLES III.

1. An object is placed 15 cm. in front of a concave mirror of 30 cm. focal length. Find the position of the image and the ratio of its size to that of the object.

Here we have given—

$$u = 15;\ f = 30.$$

Hence, substituting in the general formula we have—

$$\frac{1}{v} + \frac{1}{15} = \frac{1}{30}. \quad \therefore \frac{1}{v} = -\frac{1}{30}.$$

$$\therefore v = -30.$$

That is, the image is 30 cm. *behind* the mirror, and is therefore

virtual. Also, image and object are on the same side of C; therefore image is *erect.*

Also—

$$\frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = -\frac{-30}{15} = +2.$$

That is, image is *virtual,* and twice the size of the object.

This problem may also be solved by the application of 2 and 3 (*c*), thus :—

From data—

$$x = -(30 - 15) = -15$$
$$f = 30.$$

Therefore, substituting in $x\,x' = f^2$, we have—

$$-15\,x' = (30)^2$$

or—

$$x' = -60.$$

That is, the image is 60 cm. from the focus, in the same direction as the mirror, or 30 cm. *behind* the mirror.

Also—

$$\frac{\text{Image}}{\text{Object}} = -\frac{f}{u-f} = -\frac{30}{15-30} = +2.$$

That is, the image is *virtual,* and twice the size of the object.

2. A pencil of rays, converging to a point 20 cm. behind a mirror, is brought to focus, by reflection from its surface, at a point 10 cm. in front of the mirror. Determine whether the mirror is convex or concave, and find its radius of curvature.

$$\text{Here } u = -20,\ v = 10,$$
$$\therefore \frac{1}{10} - \frac{1}{20} = \frac{2}{r} \quad \therefore \frac{2}{r} = \frac{1}{20},$$
$$\text{or } r = 40 \text{ and } f = 20.$$

That is, the mirror is concave, and its radius of curvature is 40 cm.

3. An object, 3 cm. in length, is placed 20 cm. in front of a convex mirror of 12 cm. focal length. Find the nature and position of the image.

$$\text{Here } u = 20,\ f = -12.$$
$$\therefore \frac{1}{v} + \frac{1}{20} = -\frac{1}{12}.$$
$$\therefore \frac{1}{v} = -\frac{1}{20} - \frac{1}{12} = -\frac{2}{15}.$$
$$\therefore v = -7{\cdot}5.$$

That is, the image is 7·5 cm. *behind* the mirror, and is therefore *virtual.*

Also—

$$\frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = +\frac{7\cdot5}{20} = +\frac{3}{8}.$$

That is, image is *virtual*; and—

$$\text{Length of image} = \frac{3}{8} \text{ of } 3 \text{ cm.} = 1\cdot125 \text{ cm.}$$

Applying formulæ 2 and 3 (c) to this problem we get—

$$x = 20 + 12 = 32, \text{ and } f = 12.$$
$$\therefore 32\,x' = (12)^2.$$
$$\therefore x' = \frac{12 \times 12}{32} = 4\cdot5.$$

That is, the image is 4·5 cm. from the principal focus in the positive direction, or 7·5 cm. *behind* the mirror.

Also—

$$\frac{\text{Image}}{\text{Object}} = -\frac{f}{u-f} = -\frac{-12}{20-(-12)} = \frac{+12}{32}.$$
$$\therefore \frac{\text{Image}}{\text{Object}} = +\frac{3}{8},$$

the same result arrived at above.

4. A gas flame is placed at a distance of 8 feet from the wall of a room. Find the radius of curvature of a concave spherical mirror, and where it must be placed in order that it may produce, on the wall, an image of the gas flame magnified threefold.

Here, if x denote the distance of the mirror from the gas flame, we have—

$$u = x;\ v = x + 8$$

And—

$$\frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = -\frac{x+8}{x} = -3 \ (\textit{i.e.}, \text{ Image is inverted}).$$
$$\therefore 3\,x = x + 8.$$
$$\therefore x = 4.$$

And—

$$\frac{\text{Image}}{\text{Object}} = -\frac{f}{u-f}. \qquad \therefore -3 = -\frac{f}{4-f}, \text{ or}$$
$$12 - 3f = f.$$
$$\therefore f = 3 \text{ and } r = 6.$$

Or, after determining $x = 4$, we may employ 1 instead of 3 (c), thus—

$$\frac{2}{r} = \frac{1}{12} + \frac{1}{4} = \frac{1}{3}.$$
$$\therefore r = 6,$$

Hence, the mirror must be placed 4 feet from the gas flame—that is, 12 feet from the wall—and its radius of curvature should be 6 feet.

5. A square piece of cardboard of 1 inch side is placed at right angles to the principal axis of a concave mirror of 18 inches focal length. At what distance from the mirror must it be placed in order that an image, 9 square inches in area, may be formed?

$$\frac{\text{Area of image}}{\text{Area of object}} = \left(\frac{f}{u-f}\right)^2.$$

$$\therefore \frac{9}{1} = \left(\frac{18}{u-18}\right)^2.$$

$$\therefore 3 = \pm \frac{18}{u-18}.$$

$$\therefore u = 24 \text{ or } 12.$$

That is, the object may be placed 24 inches in front of the mirror, or 12 inches in front of the mirror. In the former case the image is *real* and *inverted*; in the latter it is *virtual* and *erect*.

This problem may also be solved by means of formulæ 1 and 3 (*b*).

6. An object is placed 16 inches from the centre of curvature, and 12 inches from the focus of a convex mirror. Find the nature and position of the image.

Here, the distances between the focus and centre of curvature $=(16-12) = 4$ inches.

$$\therefore r = -8 \text{ and } f = -4,$$
$$\text{and } u = 16 - 8 = 8.$$

$$\therefore \frac{1}{v} + \frac{1}{8} = \frac{1}{-4}$$

$$\frac{1}{v} = -\frac{3}{8} \text{ or } v = -2\tfrac{2}{3}.$$

That is, the image is $2\frac{2}{3}$ inches behind the mirror, and is ***virtual***, *erect*, and ***diminished*** (Art. **42**, II.).

[***Virtual*** and ***diminished*** shown by ratio—

$$-\frac{v}{u}\left(= -\frac{-2\frac{2}{3}}{8} = +\frac{1}{3}\right);$$

erect and ***diminished*** shown by ratio—

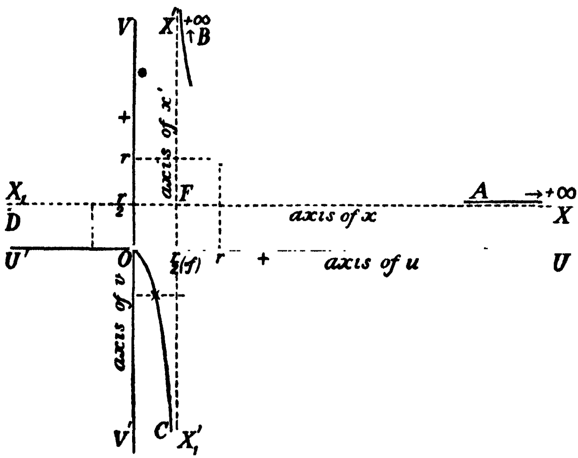
$$\frac{c'}{c}\left(=\frac{5\frac{1}{3}}{16}=\frac{1}{3}\right).]$$

7. Draw a curve showing the relation between the distance of an object, and that of its image as measured from a concave mirror as the distance of the object is progressively varied. Art. **41** gives us

the relative distances, and it is best to begin by putting these in tabular form.

u	v
∞	f
$<\infty$ but $>r$	$>f$ but $<r$
r	r
$<r$ but $>f$	$>r$ but $<\infty$
f	$\pm\infty$
$<f$ but $>\frac{f}{2}$	$>-\infty$ but $<-f$
$\frac{f}{2}$	$-f$
o	o
$-f$	$\frac{f}{2}$
$-\infty$	f

Plotting these on squared paper we obtain the following curves. The curve A B is a rectangular hyperbola. This is also easily seen



to be the case from a consideration of the equation $xx' = f^2$, where x, x' are the co-ordinates measured from the axes F X, F X'.

The curve C D is the rectangular hyperbola $x x' = f^2$, where both x and x' are negative. Note that a curve which is asymptotic to a line goes to infinity on one side of the line, and reappears at an infinite distance in the opposite direction on the other side of the line.

8. A concave spherical mirror is so placed that a candle flame is situated on its principal axis at a distance of 18 inches from its surface. An inverted image, three times as long as the candle flame itself, is seen sharply defined on the wall. What is the focal length of the mirror?

9. Prove that if an object is placed at a distance of $3f$ in front of a concave mirror (of focal length f), then the image is one-half the size of the object.

10. A small object on the axis of a concave mirror, at a distance of 16 inches from it, produces a *real* image which is three times its own size. Find the focal length of the mirror.

11. A small object 0·1 inch long is placed at a distance of 3 feet from a convex mirror of 12 inches focal length. What is the length of the image and its distance from the mirror?

12. A gas flame is placed at a distance of 10 feet from the wall of the room. What must be the radius of curvature of a concave spherical mirror, and where must it be placed in order that it may produce on the wall an image of the gas flame magnified (linearly) fourfold?

13. A penny is held 8 inches in front of a convex mirror of 1 foot radius. Where will its image be, and what will be its diameter compared with that of the penny?

14. How far from a concave mirror of radius 3 feet would you place an object to give an image magnified three times? Would the image be real or virtual?

15. An object 6 cm. long is placed 1 metre in front of a concave mirror of 10 cm. focal length. Find the nature and size of the image.

16. Prove that when an object is placed midway between a concave mirror and its principal focus the image is twice as large as the object.

17. An object is held in front of a convex mirror, at a distance equal to the focal length of the mirror. Determine the size, nature, and position of the image.

18. A gas jet is placed on the principal axis of a spherical mirror 10 cm. in front of it. A real and inverted image is produced on a screen held in front of the mirror. If the length of the image is three times that of the flame, find the focal length of the mirror and the position of the screen.

19. An image produced by a convex mirror of focal length f is $1/r$th the size of the object. Show that the distance of the object from the mirror is $(r-1)f$.

20. A plane mirror is placed 6 feet in front of a concave mirror of 2 feet focal length. Find where an object must be placed between the two mirrors in order that images and object may coincide.

21. Trace the changes in the position of the image formed by a convex spherical mirror as the object is moved from a great distance up to the surface of the mirror.

22. An object 1 inch high is placed on the axis of a concave mirror of 1 foot focal length at a distance of two feet from the mirror. Draw a diagram showing the position and size of the image, explaining all necessary lines in the construction.

23. Describe what you see when you look at yourself in a concave mirror, and slowly move backwards from the mirror.

24. Explain, giving a drawing, how it is that you see yourself as you do in a polished metal ball.

25. A luminous point is situated 30 feet in front of a concave mirror of 1 foot radius, and on the principal axis. Show *by a scale drawing* what will become of the rays after reflection from the mirror.

26. Show, *by two other scale drawings*, what will become of the rays after reflection, when the luminous point is brought up, first to 7 inches, then to 4 inches from the mirror.

27. A luminous point is 108 feet in front of a concave mirror of 10 inches focal length. Where is its image formed? And is it real or virtual, erect or inverted, enlarged, reduced, or same size as the object?

28. A luminous point is 80 feet in front of a concave mirror of 24 inches radius. Calculate the position of its conjugate focus.

29. A luminous point is 12 inches in front of a concave mirror of 7 inches focal length. Calculate the position of the conjugate point.

30. If with the same mirror the luminous point be brought up to 4 inches from the mirror, calculate the position of its conjugate focus.

31. A luminous point is successively 80 feet, 25 inches, and 3 inches in front of a convex mirror of 8 inches focal length. Calculate the corresponding position of the conjugate point

32. A luminous point is 9 inches in front of a mirror of 6 inches focal length. Show, *by a scale drawing*, the course of the rays after reflection, and the position of the focus.

33. An object 6 inches high is 10 feet in front of a concave mirror of 18 inches focal length. Calculate the position and size of the image, and state whether it is real or virtual, erect or inverted.

EXAMINATION QUESTIONS.

QUESTIONS SET AT VARIOUS UNIVERSITY EXAMINATIONS.

1. Rays of light from a bright gas flame pass through a small pinhole in a black screen, and are received on a sheet of ground glass. Describe by the help of a picture the image seen on the glass. What would be the effect of making the pinhole square instead of round?

2. What do you understand by the intensity of illumination at a point, and how would you show that the intensity of illumination at a point, due to a given source, is inversely proportional to the square of the distance of the point from the source?

3. State the optical law on which photometric measurements are based. A gas flame and a candle are eight feet apart, the former giving out nine times as much light as the latter. Show that there are two positions in which a screen may be placed so as to be equally illuminated by the two sources, and find these positions.

4. How would you determine experimentally the quantity of light reflected at different angles by a piece of plane glass?

5. How do you account for the fact that a large mass of red-hot iron appears equally bright when viewed from points at different distances?

6. A ten-candle lamp is placed one metre from a surface. At what distances must gas flames of 14 and 16 candle-power respectively be placed so as to produce an equal illumination of the surface? Mention any one method that is commonly used to enable equality to be judged.

7. State the laws of the Reflection of Light by plane-polished surfaces, and explain fully an accurate method of proving them by experiment.

8. Two plane mirrors are inclined at an angle of 60°: trace the path of a pencil of rays proceeding from a luminous point between the mirrors to the eye, after undergoing one reflection at the surface of each mirror.

9. A plane mirror revolves about an axis. Explain a method of ascertaining experimentally whether or not the axis is perpendicular to the surface of the mirror.

10. A ray of light is reflected successively by two plane mirrors, the plane of incidence being perpendicular to the line of intersection of the mirrors: prove that when the mirrors are at right angles to each other the final direction of the ray is parallel to its original direction.

11. Assuming the laws of the ordinary Reflection of Light, find the position of the image of an object placed in front of a plane mirror. What are the limits of position of the object (the mirror being supposed fixed) so that an image of it may be formed by the mirror?

12. A plane mirror revolves about a vertical central axis. A fixed horizontal ray of light falls upon its centre and is there reflected. Prove generally that if the mirror move through any angle the reflected ray will appear to have moved through double that angle.

13. Give a careful sketch of the arrangement of a lamp, slit, lens, and scale, by means of which the image of the slit formed by the lens and reflected by a plane mirror may be thrown on to the scale. Trace in your sketch the course of pencil of the rays.

14. On a moonlight night when the surface of the sea is covered with small ripples, instead of an *image* of the moon being seen in the sea, a long band of light is observed on the surface of the sea extending towards the point which is vertically beneath the moon. Account for this phenomenon in accordance with the laws of reflection, illustrating your explanation by a figure.

15. Two mirrors are inclined to each other at right angles. Show that three images of an object placed in the angle between the mirrors are formed, and draw the pencil of rays by which the second image can be seen by an eye looking at one mirror.

16. A bright object is placed between two plane mirrors inclined at 45°. Draw a picture showing the path of a ray of light proceeding from the object and reaching the observer's eye after four reflections.

17. State the laws of Reflection of Light. Two mirrors are placed parallel to one another, and a luminous point is placed midway between them. Show how to draw accurately the path of a ray of light, which, after undergoing 3 reflections at one mirror and 4 at the other, enters an eye also placed midway between the mirrors, but at some distance from the source of light.

18. How could you arrange two mirrors so as to be able to see the side of your head when looking straight forward? Give a drawing showing the complete course of a ray.

19. Completely enunciate in two statements the law of Reflection of Light, and show how to find the position of the principal focus of a concave spherical mirror.

20. Sketch a concave spherical mirror facing a distant luminous object, and showing the position and nature of the image of this object given by the mirror.

21. Given a concave spherical mirror, how could you find its radius of curvature by optical means alone, and without resorting to geometrical operations?

22. A candle flame is placed at a distance of three feet from a concave mirror formed of a portion of a sphere the diameter of which is three feet. Determine the nature and position of the image of the candle flame produced by the mirror, and state whether it is erect or inverted.

23. An object 6 inches long is placed symmetrically on the axis of a convex spherical mirror, and at a distance of 12 inches from it. The image formed is found to be 2 inches long. What is the focal length of the mirror?

24. Show how to find the position of the image of an arrow placed in front of a concave spherical mirror. Explain when it is an erect, and when an inverted image.

25. Explain the formation of images by a concave cylindrical mirror. Find the relation between the distances of the two conjugate foci from the mirror. What is the position of the image of a point which is at the distance of the diameter from the reflecting surface of the cylinder?

26. A small object is placed in front of a concave spherical mirror of 6 inches radius at a distance of four inches from the surface of the mirror. Where will its image be situated? will it be erect or inverted? and what will its dimensions be compared with those of the object? Where must the object be such that the image may be of the same size?

27. Explain the formation of an image by a convex mirror.

The radius of a convex mirror is 6 inches. If the linear dimensions of an object be twice those of its image, where must each be situated?

CHAPTER VI.

REFRACTION AT PLANE AND SPHERICAL SURFACES.

50. Refraction. We have seen that a ray of light travels in a straight line so long as its course lies in the same homogeneous medium, but when it passes from one medium into another it undergoes a change of direction at the surface of separation of the two media. This change of direction is called **refraction.** In illustration of this phenomenon the following simple experiments are frequently adduced:—

(1) When a piece of stick is partly immersed in water in an oblique position, it appears bent at the surface of the

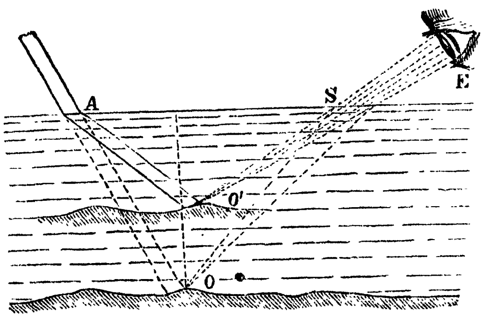


Fig. 54.

water (Fig. 54). This is due to the refraction of the rays coming from points of the stick below the surface of the water. For example, rays coming from O (Fig. 54) are refracted at S in passing from the water to the air, and appear to come from O′. Similarly, other points between

O and A appear, at E, to lie between O′ and A, and thus the portion O A of the stick appears bent into the position O′ A.

(2) If a coin be placed at the bottom of a vessel with opaque sides, in such a position as to be just out of the range of vision of an observer stationed a short distance off, it will be found that on pouring water into the vessel the coin soon becomes visible. Thus, if the coin be placed at S′ (Fig. 3), it will be invisible to an eye at E, until, on pouring a sufficient quantity of water into the vessel, a small pencil of rays coming from S′ and refracted at O, in passing from the water to the air, reaches the eye by the bent course S′ O A E.

For similar reasons a pool of water appears shallower than it really is, and small air bubbles in solid glass objects appear nearer the surface than they actually are (Art. 62).

51. Angles of incidence and refraction. Let A O (Fig. 55) represent a ray of light incident at O on the surface of separation of the media M and M′, and let O B represent the refracted ray. Then, if N O N′ be the normal to the surface at O, the angle A O N is the *angle of incidence*, and B O N′ is the corresponding *angle of refraction*. The laws of refraction, as established by experiment, refer to the relative position and magnitude of these angles, and may be stated thus:—

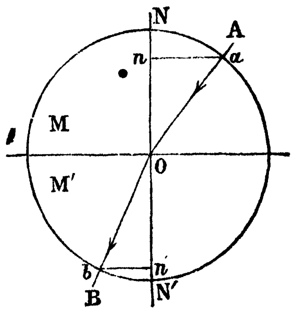


Fig. 55.

(i) *The angles of incidence and refraction lie in the same plane—that is, the incident and refracted rays, and the normal at the point of incidence, all lie in the same plane.*

(ii) *For the same two media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is always the same.*

7

(This law is generally known as the law of sines.) Without employing the term *sine*, this law may be explained by a geometrical construction. With centre O (Fig. 55) and any radius O a describe the circle a N b N′, cutting O A and O B in a and b. From a and b drop perpendiculars $a\,n$ and $b\,n'$ on the normal N N′. Then the law may be expressed by stating that, for the same two media, the ratio $\frac{a\,n}{b\,n'}$ is constant.

52. Experimental verification of the laws of refraction. The law stated in the preceding article may be roughly verified by means of the apparatus shown in Fig. 56. A cylindrical glass vessel, V V, is fixed, in a suitable stand, with its axis horizontal and its circular section vertical. A circular scale, divided into degrees, is fitted or engraved round its circumference. This vessel is half filled with water, holding a small quantity of freshly precipitated silver chloride in suspension, and the surface of the water is accurately adjusted on a level with the centre C of the circular scale. A pencil of parallel light is now reflected from a small mirror, m, so as to be incident in the plane of the scale on the surface of the water at C.* The path of the refracted ray is rendered visible by the light scattered by the particles of silver chloride, and is seen to be deviated from the direction of the incident ray immediately on entering the water. If the scale is accurately vertical, its plane will contain the normal to the surface of the water at the point of incidence,

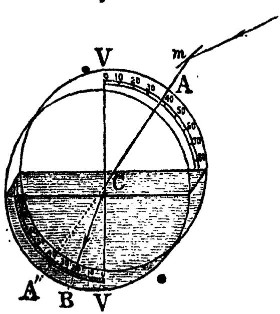


Fig. 56.

* This adjustment should be made before the water is placed in V V. The position of the mirror m is adjusted until the reflected beam, A C A′, cuts the scale at points A and A′, which are equally distant from the zero. It is now evident that the beam passes through the centre of the circular scale.

and the refracted pencil will be seen to lie in this plane, thus verifying the first law of refraction. Also, if the magnitudes of the angles of incidence and refraction be read off on the circular scale for several different values of each, it will be found that, in accordance with the law of sines, the ratio of the sine of the angle of incidence to the sine of the corresponding angle of refraction is constant.

A simpler verification of the law may be performed with a rectangular block of glass and a number of pins.

Exp. 11. Fix the block, A B (Fig. 57), upon a sheet of cartridge paper, and mark in the outline with a sharp pencil. Fix a pin, P,

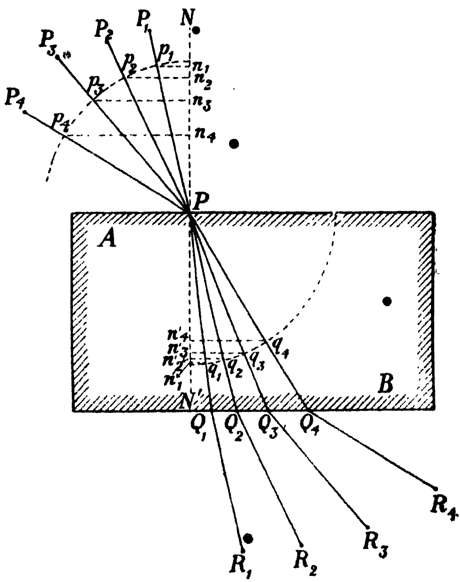


Fig. 57.

into the paper, and close up to the glass. Arrange other pins, $P_1 P_2 P_3 P_4$, at convenient distances from one another and from P. With the eye on a level with the block and looking through it towards P P_1, place two pins Q_1, R_1, Q_1 being in contact with the glass and R_1 some distance out, so that $R_1 Q_1 P P_1$ appear in a straight line. Do the same with $R_2 Q_2 P P_2$, $R_3 Q_3 P P_3$, etc.

Remove A B and join up the pinpricks by straight lines. Draw the normal NN^1 at P, and measure the angles P_1PN, P_2PN, P_3PN . . . Q_1PN^1, Q_2PN^1, Q_3PN^1 . . . with a protractor. Look up the values of the sines of these angles in a book of tables.

Show that—

$$\frac{\sin P_1PN}{\sin Q_1PN^1} = \frac{\sin P_2PN}{\sin Q_2PN^1} = \frac{\sin P_3PN}{\sin Q_3PN^1} = \ldots = \text{a constant, } \mu \text{ say.}$$

If a protractor is not available describe a circle with P as centre cutting the rays in $p_1 p_2 p_3$ . . . $q_1 q_2 q_3$ . . . Draw perpendiculars p_1n_1, p_2n_2, p_3n_3 . . . $q_1n_1^1, q_2n_2^1, q_3n_3^1$ . . . to NN^1. Measure these with a pair of dividers and a diagonal scale and show that

$$\frac{p_1n_1}{q_1n_1^1} = \frac{p_2n_2}{q_2n_2^1} = \frac{p_3n_3}{q_3n_3^1} = \ldots = \text{the same constant, } \mu.$$

Also observe that P_1P, P_2P, P_3P . . . are parallel to Q_1R_1, Q_2R_2, Q_3R_3 . . . showing that the directions of the rays have not been changed but only that the rays have been shifted laterally by a distance which increases with the angle of incidence.

53. Refractive indices. We have seen that, when a ray of light is refracted from one medium, *a*, into another, *b*, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant. This ratio is the **relative index of refraction** from the medium *a* into the medium *b*. That is, if ${}_a\mu_b$ represent this index, and if ϕ and ϕ' denote respectively the angles of incidence and refraction, we may write—

$${}_a\mu_b = \frac{\sin \phi}{\sin \phi'}. \qquad (a).$$

It has been established by experiment that the path of a ray of light is reversible—that is, if, in Fig. 55, B O be taken to represent the incident ray, then O A will be the path of the refracted ray. This fact is evidently expressed by writing—

$${}_b\mu_a = \frac{\sin \phi'}{\sin \phi} \qquad (b).$$

From (*a*) and (*b*) we have—

$${}_a\mu_b \cdot {}_b\mu_a = \frac{\sin \phi}{\sin \phi'} \cdot \frac{\sin \phi'}{\sin \phi} = 1.$$

That is—

$${}_a\mu_b = \frac{1}{{}_b\mu_a}, \text{ or } {}_b\mu_a = \frac{1}{{}_a\mu_b}. \qquad (1).$$

This result may be stated in words by saying, that if ${}_a\mu_b$ denotes the index of refraction from a to b, then $\frac{1}{{}_a\mu_b}$ denotes the index of refraction from b to a. For example, if the index of refraction from air to water be $\frac{4}{3}$, then the index of refraction from water to air is $\frac{3}{4}$.

When a ray of light is refracted from vacuum into any other medium, the index of refraction from vacuum into that medium is called the absolute refractive index or the refractive index of the medium.

If a ray of light pass from a given medium, through a layer of another medium bounded by parallel planes, into the medium in which it was originally travelling, it is known, from experiment, that the initial and final directions of the ray are parallel. This may either be taken as an experimental fact, or deduced from results already obtained from experimental data (*see* Exp. 11).

Thus, let A A and B B (Fig. 58) represent the parallel surfaces of separation of a layer of the medium b from a, and let R N, R′ N′ represent the path of a ray travelling from a through b into a again. Then, it is evident that the angles R′ N′ n' and R N n are equal, for they have respectively the same relation to the equal angles O′ N′ N and O N N′. Hence, R′ N′ is parallel to R N, but is not in the same straight line with it. It follows from this that when a ray of light passes from one medium through any number of layers of different media, having parallel surfaces of separation, back into the same medium, then the initial and final directions of the ray are parallel.

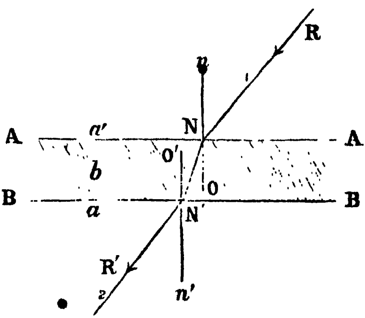


Fig. 58.

Consider the case for the three media *a*, *b*, *c*, shown in Fig. 59. Here—

$$_a\mu_b = \frac{\sin\phi_1}{\sin\phi_2}$$

$$_b\mu_c = \frac{\sin\phi_2}{\sin\phi_3}$$

$$_c\mu_a = \frac{\sin\phi_3}{\sin\phi_1}$$

$$\therefore\ _a\mu_b \cdot {}_b\mu_c \cdot {}_c\mu_a = 1.$$

$$\therefore\ _a\mu_b \cdot {}_b\mu_c = \frac{1}{_c\mu_a} = {}_a\mu_c \text{ [by (1) above].}$$

That is—

$$_a\mu_c = {}_a\mu_b \cdot {}_b\mu_c. \qquad (2).^*$$

This is an important relation, and enables us to determine the relative index of refraction from *a* to *c*, given the indices

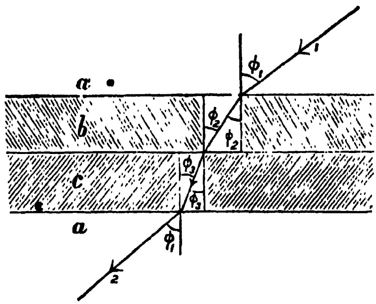


Fig. 59.

of refraction from *a* to *b* and from *b* to *c*. For example, if the index of refraction from air to glass, $_a\mu_g$, is $\frac{3}{2}$, and that from air to water, $_a\mu_w$, is $\frac{4}{3}$, then the index of refraction from water to glass, $_w\mu_g$, is given by—

$$_w\mu_g = {}_w\mu_a \cdot {}_a\mu_g = \frac{1}{_a\mu_w} \times {}_a\mu_g = \frac{_a\mu_g}{_a\mu_w}.$$

That is—

$$_w\mu_g = \tfrac{3}{2} \times \tfrac{3}{4} = \tfrac{9}{8}.$$

* This formula is easily remembered by noticing that *a* and *c* are the initial and final suffixes on *each* side. Compare the suffixes *w*, *g*, below.

This formula also enables us to establish a relation between the **relative refractive index** for any two media and the **absolute refractive indices** of those media. Thus, if ${}_v\mu_a$ denote the absolute refractive index of the medium *a*, and ${}_v\mu_b$ that of *b*, then—

$$ {}_a\mu_b = {}_a\mu_v \cdot {}_v\mu_b $$

or—

$$ {}_a\mu_b = \frac{{}_v\mu_b}{{}_v\mu_a}. \qquad (3). $$

That is, *the relative index of refraction from a to b is the ratio of the absolute refractive index of b to the absolute refractive index of a.*

It should here be noticed that, as a general rule, a ray of light in passing from one medium into a denser one is bent towards the normal, while in passing into a rarer medium it is bent away from the normal. This is equivalent to stating, that if the medium *b* is denser than *a*, then—

$$ {}_a\mu_b > 1, \text{ and} $$

$$ \therefore \ {}_a\mu_b = \frac{1}{{}_a\mu_b} < 1, $$

which expresses the case for refraction from *b* into the rarer medium *a*. Since all media are denser than vacuum, it follows that all absolute refractive indices are greater than unity.*

So far we have considered the index of refraction merely as a geometrical relation, established by experiment, between the directions of the incident and refracted rays. When considered in connection with the undulatory theory of light, a definite physical meaning can, however, be attached to this constant. It can be shown that the index of refraction from any medium *a* into another medium *b* is the ratio of the velocity of light in *a* to its velocity in *b*.

That is—

$$ {}_a\mu_b = \frac{V_a}{V_b}, $$

* This law holds for familiar transparent media, but not for all media without exception. It has been shown by methods similar to that of Art. 99 that certain metals have refractive indices considerably less than 1. See the Table.

where V_a denotes the velocity of light in *a*, and V_b denotes the velocity of light in *b*.

This ratio differs for waves of different wave-length, being *greater* the *shorter* the wave-length; and, as difference of wave-length, in waves of light, corresponds to difference of *colour*, it follows that the value of the refractive index depends on the colour of the light which suffers refraction. The light having the *greatest wave-length* and *lowest refractive index* is of a deep *red* colour, and that of the *shortest wave-length* and *highest refractive index* is coloured *violet*. Between these two extremes the refractive index increases as the wave-length decreases, and the colour of the light shades off from red through orange, yellow, green, and blue to violet.

Table of Refractive Indices.*

(Mean Values.)

Diamond	2·60	Hydrochloric acid	1·41
Iceland Spar	1·65	Alcohol	1·37
Flint glass (Heavy)	1·62	Ether	1·36
Rock-crystal	1·55	Water (at 20°C.)	1·33
Rock-salt	1·54	Chlorine	1·00078
Canada balsam	1·53	Carbonic acid gas	1·00045
Crown glass (Heavy)	1·53	Ammonia	1·00039
Plate glass	1·52	Nitrogen	1·00030
Alum	1·45	Air	1·00029
Ice	1·31	Oxygen	1·00027
Carbon disulphide (at 20°C.)	1·63	Hydrogen	1·00014
Olive oil	1·47	Iron	1·73
Oil of turpentine	1·47	Copper	0·65
Sulphuric acid	1·43	Sodium	0·12

54. Simple construction for incident and refracted rays. An easy construction for the directions of the incident and refracted rays is afforded by the fact that angles in a semicircle are right angles.

* The student will find it convenient to remember the following approximate values :—

$$\left.\begin{array}{l}\text{Refractive index for air and glass} = \tfrac{3}{2}.\\ \quad ,, \qquad ,, \qquad ,, \qquad \text{water} = \tfrac{4}{3}.\end{array}\right\}$$

Let A B (Fig. 60) be the surface of separation between two media a and b, and let M O be a ray in the medium a incident on the surface at O. Required to find its path through the second medium.

Draw N O L, the normal at O, and on any convenient length O L as diameter describe the circle O P R L. Produce M O to cut the circle at P. Divide L P at Q so that ${}_a\mu_b \,.\, \mathrm{LQ} = \mathrm{LP}$. With L as centre describe the arc Q R cutting the circle in R; draw O R, it is the refracted ray.

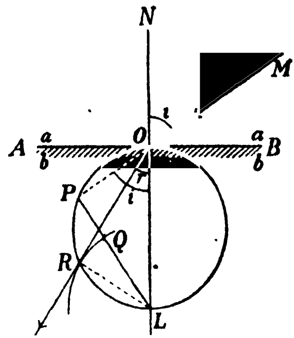


Fig. 60.

$$\text{For } \sin i = \sin \mathrm{LOP} = \frac{\mathrm{LP}}{\mathrm{LO}} = {}_a\mu_b \cdot \frac{\mathrm{LQ}}{\mathrm{LO}} = {}_a\mu_b \frac{\mathrm{LR}}{\mathrm{LO}} = {}_a\mu_b \sin \mathrm{LOR}.$$

Another construction is as follows:—Let A B (Fig. 72) be any incident ray on a surface $m\ m$. Draw any normal, N A A′, cutting the incident ray in A, and choose A′ on it so that $\mathrm{BA'} = {}_a\mu_b\,\mathrm{BA}$. Then join A′B and produce it to C. B C is the refracted ray. For evidently

$$\frac{\sin\phi}{\sin\phi'} = \frac{\sin \mathrm{BAN}}{\sin \mathrm{BA'N}} = \frac{\mathrm{BN/BA}}{\mathrm{BN/BA'}} = \frac{\mathrm{BA'}}{\mathrm{BA}} = \mu, \text{ by construction.}$$

Hence the law is satisfied.

55. Critical angle. All possible values of an angle of incidence or refraction must evidently lie between 0° and 90°. Now, when a ray of light passes from a rarer into a denser medium, it is bent towards the normal—that is, the angle of refraction, ϕ', is less than the angle of incidence, ϕ, and therefore, whatever be the value of ϕ, between 0° and 90°, that of ϕ' must also lie between 0° and 90°, and consequently refraction is always possible. But, if a ray of light pass from a denser into a rarer medium, it is bent away from the normal, and the angle of refraction, ϕ', is greater

than the angle of incidence, ϕ; so that, when ϕ passes a certain limit at which ϕ' becomes equal to 90°, refraction is no longer possible, and the incident ray is **totally reflected** at the surface of the rarer medium.

Let *m m* (Fig. 61) represent the surface of separation of any two media *a* and *b*, of which *b* is the rarer, and let I O represent a ray incident, at O, at a small angle I O N, and refracted along O R. As the angle of incidence increases and the incident ray takes the positions *a*, *b*, *c*, the angle of refraction also increases, and the refracted ray takes successively the corresponding position *a′*, *b′*, *c′*. The angle of

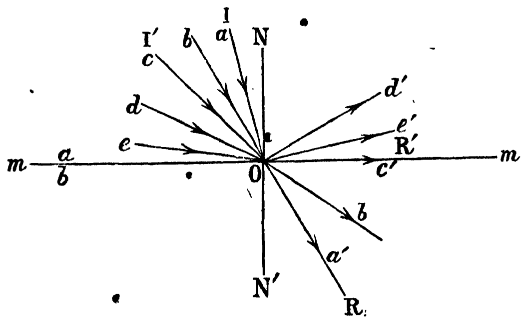


Fig. 61.

refraction being, however, greater than the angle of incidence, a position is reached at *c* where the angle of refraction R′ O N′ becomes equal to 90°, and the refracted ray, O R′, travels along the surface of separation of the media. The angle of incidence, I′ O N, at which this takes place is the **critical angle** for the media *a* and *b*. As the angle of incidence becomes greater than I′ O N the ray is no longer refracted into *b*, but is **totally reflected** from the surface *m m* in accordance with the ordinary laws of reflection. Hence, as the incident ray passes through the positions *d*, *e*, it is reflected from *m m* along the corresponding paths, *d′*, *e′*.

Hence, *when refraction takes place from a denser into a rarer medium, the angle of incidence, which corresponds to an angle of refraction of* 90°, *is called the critical angle for*

the given media. At this angle refraction ceases and total reflection from the surface of separation of the media begins.

It should be noticed that for angles of incidence between 0° and the critical angle, only *a portion* of the light incident on the surface of the rarer medium is reflected at that surface, the remainder being refracted and scattered (Art. 20); but, for angles of incidence greater than the critical angle, the incident light is almost totally reflected, no portion of it being refracted.

The value of the critical angle is readily determined for any media when the relative index of refraction for those media is given. Thus, let ${}_a\mu_b$ denote the index of refraction from a to b, then, in notation used above, if θ denote the critical angle, we have—

$${}_a\mu_b = \frac{\sin\phi}{\sin\phi'} = \frac{\sin\theta}{\sin 90^\circ} = \frac{\sin\theta}{1} = \sin\theta.$$

That is, the critical angle for refraction from a medium a into a rarer medium b is the angle whose sine is the relative index of refraction from a to b, or—

$${}_a\theta_b = \sin^{-1}{}_a\mu_b. \qquad (4).$$

This value of θ for air and water is about $48^\circ\ 30'$, and for air and glass it ranges from 38° to 41° according to the nature of the glass.

56. Total reflection. As we have seen in the preceding article, total reflection takes place when a ray of light, travelling in the denser of two media, is incident on the surface of separation at an angle greater than the critical angle of the media.

This phenomenon is readily exhibited by means of the apparatus shown in Fig. 56. The position of the mirror *m* is changed, and adjusted so as to reflect a beam of light upwards through the water into the air—for example, along the path B C A. As the angle of incidence is slowly increased the refracted ray gradually approaches the surface of the water, and finally, when the critical angle is passed, suffers total reflection at the surface of separation of the air and water, and is seen in the water as if reflected from a mirror coincident with this surface.

Simple illustrations of total reflection are often met with. For example, if a glass vessel containing water be held above the level of the eye, and the surface of separation of the water from the air be looked at from below, it appears, when seen by total reflection, as a brilliant reflecting surface.

Similarly, the edge of a crack in a pane of glass, seen obliquely, exhibits the same effect, as does also the surface of a glass tube held obliquely in a beaker of water when looked at through the sides of the beaker, especially if the surface of the tube has been very lightly smoked in a flame.

The brilliancy of many precious stones is due to their large refractive indices and therefore small critical angles. When light enters a cut diamond at any face it finds it very difficult to get out at most of the other faces, and hence bright beams issue at the one or two faces which are available for emergence.

57. The mirage. This is a phenomenon, on a large scale, due to total reflection from layers of air. In hot sandy deserts inverted

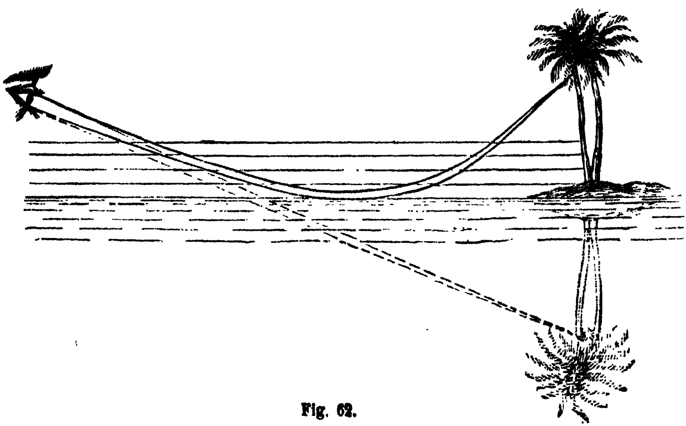


Fig. 62.

images of distant objects and of the sky are often seen reflected as from a lake. By contact with the hot sand the lower layers of air become so heated that up to the height of a few feet the density increases upwards. Rays of light from a distant object entering

these layers of rarefied air obliquely downwards become bent up more and more by refraction the lower they penetrate, and at last fall on a stratum at an angle greater than the critical angle (Fig. 62). Here total reflection takes place, and the rays becoming bent up more and more in traversing the denser layers above, at last reach the observer's eye as if they came from a point as far below the reflecting layer as the object is above it, while at the same time he sees the object direct by rays which do not pass down into the reflecting layer. Thus in the figure the appearance is that of a palm-tree standing by a tranquil pool of water.

A similar mirage can often be seen across lakes on tranquil autumn mornings. In this case it is the water that heats the lower layers of air.

In the Arctic regions inverted images of ships and other objects are sometimes seen in the air, even though the objects themselves

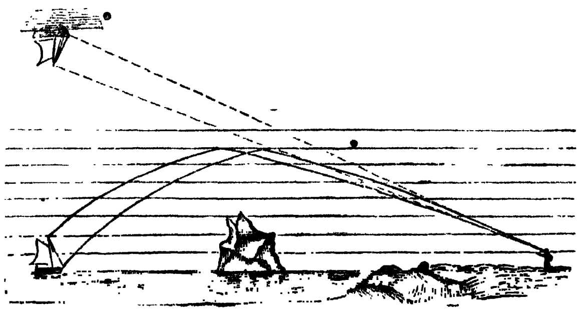


Fig. 63.

may be below the horizon. This is due to the very low temperature of the ice and sea cooling the lower atmospheric strata so much that their density increases rapidly downwards. Then rays passing obliquely upwards from the objects into the rarer layers become more and more bent down until they suffer a total reflection as in the other case, and as shown in Fig. 63. A well-known instance of this happened some years ago at Dover. Ships close in to the French coast were distinctly seen from the English side of the Straits.

58. Geometrical construction for critical angle. Let O be a point in the plane of separation A B (Fig. 64) of two media a and b. It is required to find the direction of a ray which, passing through the medium b and incident on A B at the point O, is just totally reflected.

Draw N O L, a normal at O, and on any convenient length O L as diameter describe a circle L R O. With L as centre

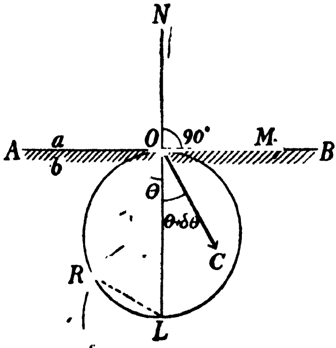


Fig. 64.

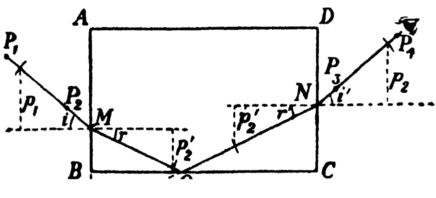
and radius equal to $\frac{\text{O L}}{{}_a\mu_b}$ strike an arc cutting the circle in R. Then R O is the direction of the critical incident ray; for, since the angle O R L is a right angle,

$$\sin \text{R O L} = \frac{\text{R L}}{\text{O L}} = \frac{1}{{}_a\mu_b} = {}_b\mu_a,$$

therefore R O L is equal to θ, the critical angle The refracted ray O M just skims the surface. If R O L exceed θ by ever so little, all the light is reflected, taking the direction O C.

59. The glass block of Art. 52 may also be used to determine the index of refraction, by an experiment in which the light is totally reflected from one face.

Exp. 12. Place the block upon a sheet of paper as before. With the eye in the neighbourhood of P_4 (Fig. 65) sight a pin placed at P_1 by observing its reflection in the face B C.



M′ Fig. 65.

Now insert pins at P_2, P_3 so that P_1 P_2 P_3 P_4 appear in the same straight line. Remove the glass, and produce P_1 P_2 and P_4 P_3 to meet A B and D C at M and N. Between M and N the ray has been reflected at the surface B C. To find its path produce A B to M′, making B M′ equal to B M. Join N M′, cutting B C in Q. Then P_1 P_2 M Q N P_3 P_4 is the complete path of the ray from P_1 to the eye. Draw normals at M and N and from the values of i, r, i', r' or p_1, p_1', p_2, p_2' calculate two values of μ. They should agree very closely.

60. Wollaston's method for the determination of the refractive index of a liquid by total reflection. Let R A (Fig. 66) be a ray of light passing through a liquid l and

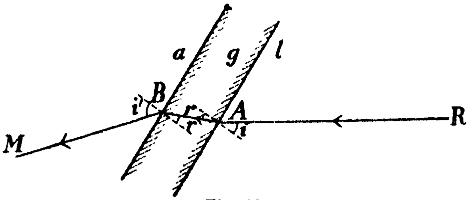


Fig. 66.

incident at A on a parallel plate of glass g. If i is the angle of incidence the angle of refraction, r, is given by—

$$\frac{\sin i}{\sin r} = {}_l\mu_g = \frac{{}_a\mu_g}{{}_a\mu_l}.$$

The ray now meets the second surface at B, and if air be the medium on the other side of the plate the angle of refraction i' is given by—

$$\frac{\sin i'}{\sin r} = {}_a\mu_g.$$

i' is always greater than i, and hence if i be gradually increased a time will arise when the light will be totally reflected at B. If θ and ϕ be the values of i and r when this occurs, we have—

$$\frac{\sin \theta}{\sin \phi} = \frac{{}_a\mu_g}{{}_a\mu_l}$$

$$\sin \phi = \frac{1}{{}_a\mu_g}.$$

$$\therefore \sin \theta = \frac{1}{{}_a\mu};$$

so that θ is the critical angle from the liquid to air.

The problem therefore resolves itself into an accurate determination of this angle. When i is less than the critical angle the ray R A will travel through the plates, but when i is equal to or greater than the critical angle the ray is totally reflected at the air-glass surface, and no light gets through.

Exp. 13. Two fairly thin glass plates about 4 cm. long and 3 cm. wide are taken, separated by pieces of microscope cover slips placed at the corners, and then cemented together at the edges by bicycle cement or wax, thus forming a glass cell A (Fig. 67), containing a thin film of air. A is then fixed in a metal clip, B, which is supported by a vertical spindle, C, working in the centre of the top of a flat wooden box (Fig. 68). By means of a head, D, the glass cell can be rotated, the amount of rotation being indicated by the motion of a pointer, E, over a graduated circle, F. The wood in the middle of two opposite sides of the box is cut away and replaced by two glass plates on which sheets of tinfoil have been pasted. Two narrow rectangles of tinfoil are removed from the sheets and these serve as slits M and R. The liquid whose index of refraction is to be determined is placed in a glass vessel, G, whose sides must be

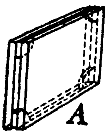


Fig. 67

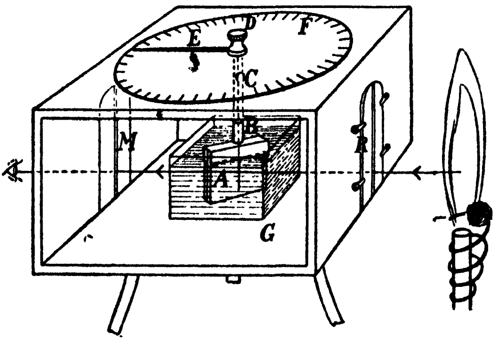


Fig. 68.

plane and parallel; G is then placed in the position shown in the diagram. A source of monochromatic light (a sodium flame is usually employed *) is then placed in front of R.

Placing the eye beyond M and looking along M R, turn D so that A is nearly broadside on to the light. R is then easily seen. Now

---

* To set up a sodium flame, take about a foot of iron wire and wind one end in a conical spiral. Wind the other end round the tube of a Bunsen burner and place a piece of salt on the upper spiral so that the flame comes in contact with it.

Another way is to soak a piece of asbestos paper in strong brine, wrap it around the top of the burner, and fasten it in position with wire.

turn D gradually to the right or left until R just becomes invisible. The ray R A is now making an angle θ with the normal. Take the reading of E. Now rotate the plate back to its original position, and then beyond it until R becomes once more just invisible. The two positions of the plate are represented by $A_1 A_1$, $A_2 A_2$ in Fig. 69, and it is easily seen that the angle through which the cell has been rotated is equal to twice the critical angle (for sodium light) of the liquid contained in G. When F is graduated to read to 20′, values of the critical angle can be determined quickly and accurately, yielding values of μ correct to one-half per cent.

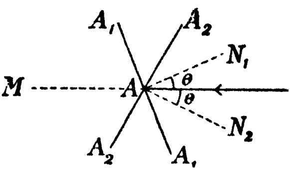


Fig. 69.

For a more accurate method for the determination of μ by total reflection, see Art. 168. A still simpler method is that in which the slits are replaced by pins and the glass cell is carried by a small wooden stool, the position of the stool being given by pins stuck into its legs. The experiment can then be carried out on a sheet of paper.

61. Deviation produced by refraction. Let A O (Fig. 70) represent the incident ray and O B the refracted ray, then the deviation produced by the refraction at O is expressed by—

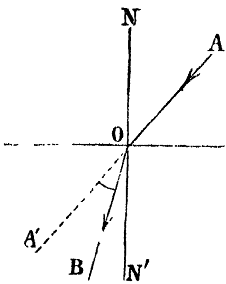


Fig. 70.

$$D = A'OB = A'ON' - BON'.$$
$$= AON - BON'$$

That is—

$$D = \phi - \phi', \qquad (5).$$

where ϕ denotes the angle of incidence and ϕ' the angle of refraction. As $\sin \phi = \mu \sin \phi'$, it is evident that if ϕ gradually increases, ϕ' also increases, but at a slower rate; hence the deviation increases as the angle of incidence increases.

When the angle of incidence is zero, then the angle of refraction is also zero, and therefore no deviation is produced—that is, when a ray is incident along the normal to the surface of separation of two media it does not suffer deviation, but continues its course in the same straight line.

It is most interesting to note that just as the path taken by light when reflected is that which makes the time of passage least (see Ex. II., 1), so likewise does the path taken on refraction. That is, the

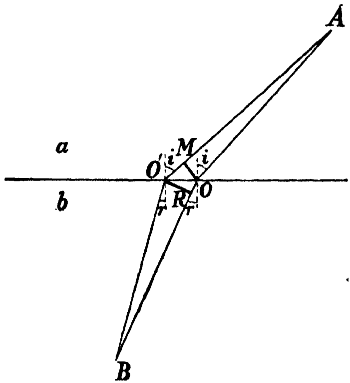


Fig. 71.

time taken by light to travel from A to B by the path A O B (Fig. 71), namely, $\frac{\text{A O}}{v_a} + \frac{\text{O B}}{v_b}$, is less than that taken along any other path. For the purposes of proof consider a ray A O′ B very near A O B, so that the angles of incidence and refraction are approximately the same for both rays. Let O M be drawn perpendicular to A O′, and O′ R perpendicular to O B.

$$\text{Now} \sin i = \sin \text{O}'\text{O M} = \frac{\text{O}'\text{M}}{\text{O O}'}, \sin r = \sin \text{O O}'\text{R} = \frac{\text{O R}}{\text{O O}'},$$

$$\therefore \quad {}_a\mu_b = \frac{\sin i}{\sin r} = \frac{\text{O}'\text{M}}{\text{O O}'} \cdot \frac{\text{O O}'}{\text{O R}} = \frac{\text{O}'\text{M}}{\text{O R}};$$

$$\text{but} \quad {}_a\mu_b = \frac{v_a}{v_b}, \quad \therefore \quad \frac{\text{O}'\text{M}}{v_a} = \frac{\text{O R}}{v_b}.$$

That is, the time taken by light in travelling from M to O′ in the medium *a* is the same as that taken in travelling from O to R in the medium *b*. Also A O is very nearly equal to A M and B O′ to B R, hence the light takes the same time to travel from A to B along the two paths A O B, A O′ B.

Now by mathematics it can be proved that, when a function is gradually varying, its variation is zero when near a maximum or a minimum. In the case in question the time is certainly not a maximum, hence it must be a minimum. (*Cf.* with Art. 75).

62. Refraction at a single plane surface. So far we have dealt only with the refraction of a single ray; we shall now consider the refraction of small pencils *directly* * incident on the surface of separation of the media. Let $m\ m$ (Fig. 72) represent the surface of separation of two media, a and b, of which b is the denser, and let A B represent one of the extreme rays of a diverging pencil of light directly incident on $m\ m$ along A N. A B is refracted at B along B C, while A N, being

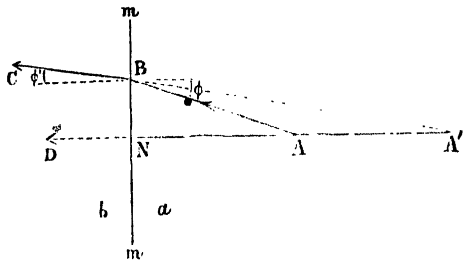


Fig. 72.

normal to $m\ m$, passes on along N D without suffering deviation. The focus of the refracted pencil will now be found at the point A′ from which B C and N D apparently diverge. It thus appears that A′ and A are conjugate foci, and that A′ may be considered as the image of A formed by refraction at the surface $m\ m$. It now remains to determine the relation between the distances of A and A′ from that surface. Let ϕ and ϕ' denote the angles of incidence and refraction, and μ the refractive index † for the case considered. Then, since ϕ and ϕ' are respectively equal to the angles B A N and B A′ N (Euc. i. 29), we have—

$$\mu = \frac{\sin \phi}{\sin \phi'} = \frac{\sin \mathrm{B\,A\,N}}{\sin \mathrm{B\,A'N}} = \frac{\mathrm{B\,N\,.\,B\,A'}}{\mathrm{B\,A\,.\,B\,N}} = \frac{\mathrm{B\,A'}}{\mathrm{B\,A}}.$$

* A pencil of light is *directly* incident on a surface when the axis of the pencil is perpendicular to that surface.

† In what follows, μ always denotes the refractive index for refraction in the direction in which the light is supposed to be travelling.

But if B N is small—that is, if the incident pencil is small—then. B A and B A′ are approximately equal to N A and N A′, and we have—

$$\mu = \frac{N A'}{N A}.$$

Now, adopting the notation of Chapter V., and retaining the sign convention there explained, let N A be denoted by u and N A′ by v. Then—

$$\mu = \frac{v}{u}.$$

$$\therefore v = \mu u. \qquad (6).$$

That is, the distance of the image from the plane refracting surface is μ times that of the object.

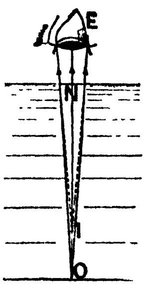


Fig. 73.

This explains why, on looking vertically downwards, the depth of a pond of water appears to be only three-fourths of what it really is. Let O (Fig. 73) represent an object at the bottom of the pond; then, after refraction at the surface of the water, the small direct pencil incident along O N appears to diverge from I—that is, the object O is seen at I.* From relation (6) obtained above we get—

$$I N = \mu . O N.$$

Now μ from water to air $= \frac{3}{4}$.

$$\therefore I N = \tfrac{3}{4} O N.$$

In exactly the same way the apparent thickness of a plate of glass, or other transparent medium, as seen by an eye looking along a normal to the surface of the plate, is less than its actual thickness. For if O (Fig. 73) represent an object close to the face of the plate remote from the eye, then its apparent position is at I—that is, I N is the apparent thickness of a plate of actual thickness O N. Hence, if t denotes the thickness of the plate, its apparent thickness is given by $\mu\, t$, where μ is the index of refraction *from the medium into air.*

The result is true only in the case of small direct pencils.

* In Fig. 73 the dotted lines below N should be in direct continuation of the full lines above N

Exp. 14. To determine the refractive index of a solid (or liquid). Cut a thin slip of stamp-paper and stick it in a vertical position at O to the edge of the glass block A B (Fig. 74) already used in Exps. 11 and 12. Place the block on the table and draw in the normal O N P. With the eye on this normal, look at O through the glass; observe that it appears nearer. It appears at I, and to locate I place a pin on the normal and with the eye in several positions not far distant from the normal adjust it until its image by reflection coincides with I. Note its final position, P. Remove the block, make I N equal to N P, then I is the position of the image of O and ON/IN $= \mu$.

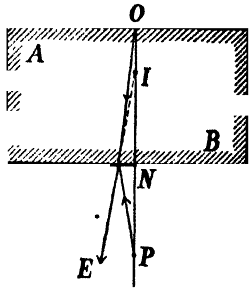


Fig. 74.

A more accurate method is to use a low-power microscope which has a vertical adjustment and a fine scale by which the vertical motion can be measured. Cut a small star of paper, place it on the table and focus the microscope on it. Read the scale (1). Place the block on the paper-star. Elevate the microscope until it is again in focus. Again read the scale (2). Lay another small star of paper on the top of the block and again elevate the microscope until this is in focus. Again read the scale (3); the difference between (2) and (3) is equal to I N, and that between (1) and (3) is O N. Again read the scale (3); the differences in the readings between (1) and (3), (2) and (3) are equal to O N and I N respectively.

If the refractive index of a liquid is required, the liquid must be placed in a glass cell, whose walls are thin, plane and parallel. The operations are then the same as for the solid block, the refraction of the glass walls being neglected.

It is also possible to work at the end of the block. Place the slip of paper, O, at the corner; I can then be located by a pin, which is moved along the end of the block. When the pin occupies the position of the image of O, it can be pressed into the paper.

Strictly, I is the conjugate focus of O only when the angle of the pencil diverging from O is infinitely small; as this angle increases, the focus I approaches nearer and nearer the surface (Fig. 75), until, when it is equal to twice the critical angle for the media, the point I coincides with N. If the angle of the pencil be greater than 2 θ (where θ denotes the critical angle), then all the rays making angles greater than θ with the normals at the points of incidence are totally reflected, and do not emerge from the water.

Hence, remembering the reversibility of the direction of the passage of the light travelling along any path, it follows that an eye placed at O and looking upwards will see all external objects, most of them greatly distorted, comprised within a vertical cone whose semi-vertical angle is equal to the critical angle. The water surface outside this cone acts as a mirror, reflecting rays from objects lying below it.

If now we consider an oblique conical pencil of O *a b c d* emanating from O and entering an eye placed at E, it does not diverge from a definite point after refraction. Two special cases may be considered. If the pencil be much narrower horizontally than vertically, it may be regarded as a number of rays all lying in (or close to) the vertical plane in which the figure is drawn. This will diverge after

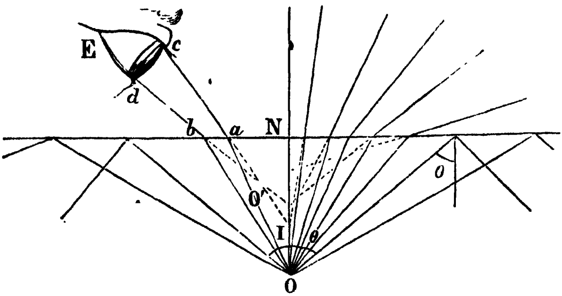


Fig. 75.

refraction from O′, almost exactly. If the pencil be wider horizontally than in plane of figure, it can be regarded as an aggregate of rays all equally inclined to O N, and these diverge after refraction from I. For other forms of pencil neither of these results are true; but the emergent pencil consists of rays, every one of which cuts (very nearly) two "*focal lines*," one at I in the plane of the figure, and one at O′ perpendicular to it.

If we are observing the point O with two eyes situated on a level and at the same distance from O N, these receive two narrow pencils which appear to come from the same point I *on the normal O N*. If we are observing with two

eyes in the same vertical plane through O N, *i.e.*, the plane of figure, the rays received appear to diverge from a point O′ *not on the normal O N but on the observer's side of it.** If the eyes are held obliquely, the image seen is confused and cannot be located at a definite point.

Thus the apparent thickness of the medium becomes less and less as it is looked at more and more obliquely, and finally becomes zero when the direction of vision is parallel to its surface. This explains why the flat bottom of a vessel full of water appears slightly concave; the points vertically below the eye are seen by direct pencils, but the surrounding points by slightly oblique pencils, so that the water appears shallower as the range of vision travels outwards from the point vertically below the eye. If the eye be moved along parallel to the surface of the water, this appearance of concavity moves along with it, and thus an apparent wave motion is given to the bottom. For the same reason the depth of a pool of water appears to increase as we approach it and to diminish as we recede from it.

63. Image of a point seen by direct refraction through a plate. When a plate of glass or any transparent substance is interposed between the eyes and a near object, the distance of the eyes from the latter is apparently diminished.

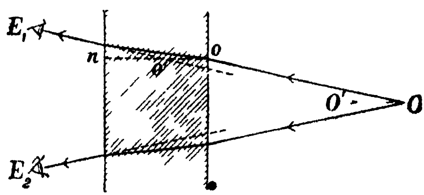


Fig. 76

This is evidently due to the apparent diminution in the thickness of the plate; and, if t denotes the actual thickness, then the apparent thickness, t', is given by $t' = \mu t$, and the position of the object is apparently nearer the eye by a

* Fig. 75 well represent this case if c and d are taken to be positions of the eyes of the observer.

distance $(t - \mu t)$ or $t\ (1 - \mu)$, where μ denotes the index of refraction *from the plate to air*. Fig. 76 shows how this apparent change of position is effected. An object at O is seen at O′, the virtual focus of the refracted pencils which enter the eyes at E_1, E_2. O O′ represents the apparent change of position, and being equal to $o\,o'$ is apparently equal to $o\,n - o'\,n$; that is, if O O′ be denoted by d, we have—

$$d = t\,(1 - \mu). \qquad (7).$$

If the refractive index *from air to the plate* be used, the formula becomes—

$$d = t\,\frac{\mu - 1}{\mu}$$

64. Caustics by refraction. On the right-hand side of Fig. 75 it will be noticed that the backward prolongations of the refracted rays are tangential to a curve which is known as a *virtual caustic by refraction* (see also Art. 91).

65. Images produced by a plate with parallel faces. Let O (Fig. 77) represent an object placed in front of the plate. Rays reach the nearest face of the plate in all directions from O. Consider the ray O a. It is partially reflected from the first face at a, and an image due to this reflection is seen at I. But a portion of the light incident at a is refracted into the plate along $a\,b$, and, on incidence, at b, on the second face of the plate, a portion is reflected along $b\,c$, and the remainder refracted out into the air. The first portion, travelling along $b\,c$, again suffers partial reflection and refraction at c, and the emergent ray, $c\,f$, gives rise to another image I′, fainter than the first at I, because of the loss of light at b and c. Similarly, after reflection at d, and refraction at e, the light emergent along $e\,g$ gives rise to another image I″ fainter than that at I′. In this way, by continued reflection and refraction, a series of images are formed on the line I O′; each member of the series becoming fainter and fainter as the number of reflections by which it is produced are increased.

When we stand in front of a thick plate-glass mirror and examine our reflection in it, there is no apparent

confusion, becauses the images formed by the two surfaces are almost exactly superposed, and still more because the second image (that due to the silver) quite overpowers by its brilliancy the feeble first image formed by the front surface of glass. But if we hold a finger, or better, a candle-flame, near the glass, and look obliquely at its reflection, we at once see both these images partly overlapping

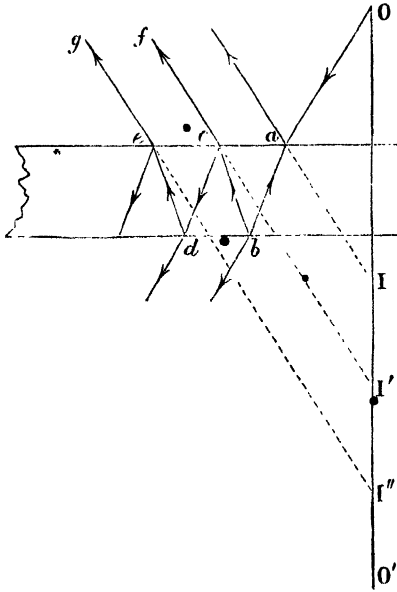


Fig. 77.

each other, and the second much brighter than the first. On looking more obliquely the images separate more widely, the first becoming brighter and the second less bright, and a third image, fainter than either, appears. On looking still more obliquely, a fourth, and perhaps a fifth, image will be seen; and now the first image is the brightest, and the others show a gradual diminution of brightness.

To explain this, a modification of Fig. 77 must be

employed. In Fig. 77, the parallel rays from *a*, *c*, and *e* are too widely separated to enter the eye at the same time; and, even if they did, they would all blend together to give an image at infinity. Let O and E (Fig. 78) represent object and eye; then all the rays (or, rather, the axes of the narrow diverging pencils) by which O is seen by E must on

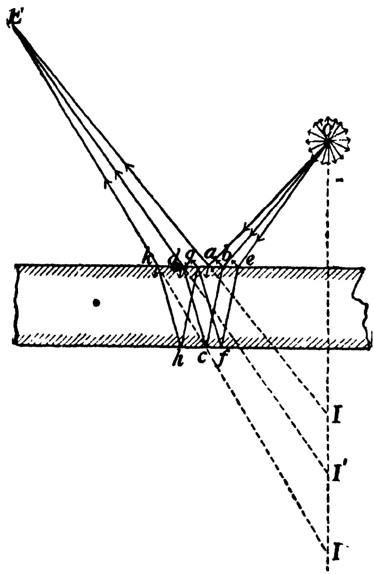


Fig. 78.

emergence from the plate converge to the eye. O emits light in all directions. Of the rays emitted one—*viz.*, O *a*—is partly reflected at the front surface of the plate, and the reflected part passes to the eye along *a* E. The other part penetrates the glass; but, since none of this on emergence will enter E, we need consider it no further.

Another ray, O *b*, suffers one internal reflection at *c*, and enters the eye by the path O *b c d* E. Parts of O *b* are reflected at *b* and at *d*, but they can be neglected.

A third ray, O *e*, reaches the eye by means of the path O *e f h k* E. Portions of it reflected at *e* and *k* and emergent at *g* need not be considered. Other rays can be treated in a similar manner, and, if we consider each of these rays as the axial ray of a small conical pencil, it will be obvious that these pencils will, to an eye at E, be apparently coming from images I, I′, I″, which are on the normal through O, but which are not quite equidistant from each other.

The number of images seen depends upon the polish of the reflecting surface, for, after a certain number of reflections and refractions, the quantity of light reaching the eye becomes too small to excite the sensation of vision, and the loss of light by reflection at any surface depends upon the degree of polish of that surface. In performing this experiment it will be noticed that the first image increases in intensity as the angle at which it is seen is increased, and at very oblique incidence it becomes much the brightest. This shows that the quantity of light reflected from a glass surface increases as the angle of incidence increases.

In ordinary looking-glasses these multiple images are scarcely ever noticed, and are of no importance; but in optical instruments they would be most inconvenient, and their formation is prevented by silvering the glass on the *front* surface and polishing the silver deposit as highly as possible. When great brilliancy is not necessary a very fair single-image reflector may be obtained by coating the back of a piece of plate glass with lampblack, which absorbs all the light except that going to form the first image.

If the luminous point O is at a great distance away, so that the rays O *a*, O *b*, O *c*, etc., may be considered parallel to each other, only one beam, and that of parallel rays, will enter the eye; and hence only one image is seen in the plate or mirror. If, however, the plate be not exactly uniform, and its faces plane and parallel, more than one image of a distant point will be seen; and hence this experiment affords a severe test of the *goodness* of a plate.

66. Refraction at a single spherical surface. Let $m\,m$ (Fig. 79) represent the spherical surface of separation of two media a and b, of which b is the denser, and let A B represent an extreme ray of a small pencil of light, diverging from A, and incident *directly* on $m\,m$ along A N. A B is refracted at B along B C, while A N, being normal to $m\,m$, passes on along N D without undergoing deviation.

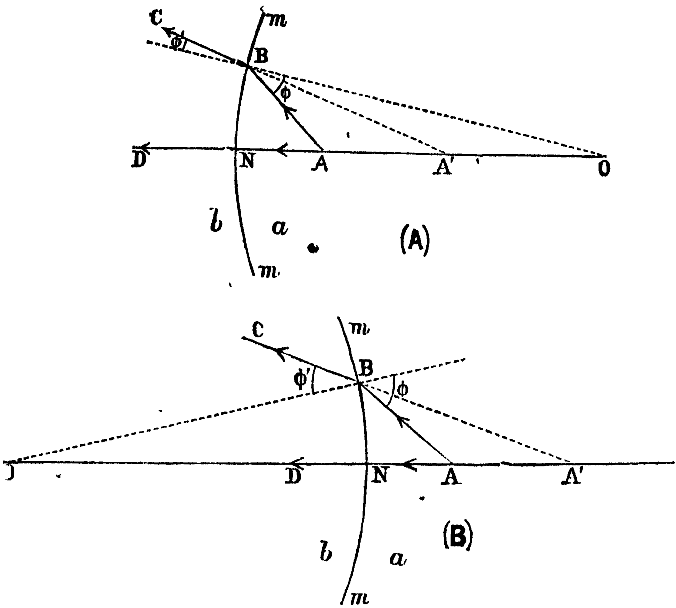


Fig. 79.

The virtual focus of the refracted pencil will now be at the point A′, from which B C and N D apparently diverge. That is, A′ is the focus conjugate to A, and may be considered as the image of A formed by refraction at the spherical surface $m\,m$.

Let O represent the centre of curvature of $m\,m$, then O B is the normal at B, and the angles of incidence and refraction, ϕ and ϕ', are respectively equal to, or supplementary

to, A B O and A′ B O (Euc. i. 15). Hence, if μ denote the index of refraction from a to b, we have—

$$\mu = \frac{\sin \phi}{\sin \phi'}$$

$$= \frac{\sin \phi}{\sin \mathrm{BOA}} \cdot \frac{\sin \mathrm{BOA}}{\sin \phi'} \text{ (identically)}$$

$$= \frac{\sin \phi}{\sin \mathrm{BOA}} \cdot \frac{\sin \mathrm{BOA'}}{\sin \phi'}.$$

But, by applying Art. **36** (4) and (5) to the triangles B O A and B O A′, we get— $\dfrac{\sin \phi}{\sin \phi'} = \dfrac{\mathrm{AO}}{\mathrm{BA}} \cdot \dfrac{\mathrm{BA'}}{\mathrm{A'O}}$.

$$\therefore \mu = \frac{\mathrm{AO}}{\mathrm{BA}} \cdot \frac{\mathrm{BA'}}{\mathrm{A'O}}.$$

If B N be sufficiently small, then B A and B A′ are approximately equal to N A and N A′ respectively.

$$\therefore \mu = \frac{\mathrm{AO}}{\mathrm{NA}} \cdot \frac{\mathrm{NA'}}{\mathrm{A'O}}.$$

As before, let N A be represented by u, N A′ by v, and N O by r. Then, with the usual sign convention, we have—

$$\mu = \frac{\mathrm{AO}}{\mathrm{NA}} \cdot \frac{\mathrm{NA'}}{\mathrm{A'O}} = \frac{(r-u)v}{u(r-v)}.^*$$

Therefore, whether $m\, m$ be concave or convex, we have—

$$\mu = \frac{(r-u)v}{(r-v)u}.$$

$$\therefore \mu r u - \mu u v = r v - u v.$$

$$\therefore \mu r u - r v = u v(\mu - 1).$$

Therefore, dividing through by $r\,u\,v$, we get—

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}. \qquad (8).$$

This is a general relation, applicable to all cases of refraction of small direct pencils at a *single* plane † or spherical surface of separation of two media, whose relative index of refraction *for the direction in which the light is travelling* is denoted by μ.

* In Fig. 79 (B) A O = A N + N O = $u + (-r) = u - r$.
and A′ O = A′ N + N O = $v + (-r) = v - r$.

† When $m\, m$ is a *plane* surface (Art. **62**), then r is infinite and $\dfrac{\mu - 1}{r} = \dfrac{\mu - 1}{\infty} = 0$, and the formula reduces to $\dfrac{\mu}{v} - \dfrac{1}{u} = 0$ or $v = \mu\, u$, a result identical with relation (6) above.

67. Positions of the principal foci of a spherical refracting surface. When v is infinite the value of u is called the *first principal focal distance.* Denoting it by f_1, we have

$$f_1 = -\frac{r}{\mu - 1}.$$

The point on the principal axis at a distance f_1 from the pole is called the *first principal focus*, and rays proceeding from it (f_1 positive), or to it (f_1 negative), are refracted, so as to proceed parallel to the axis.

Similarly, by making u infinite we obtain a value of v called the *second principal focal distance.* Denoting it by f_2 we obtain

$$f_2 = \frac{\mu r}{\mu - 1}.$$

The point on the principal axis at a distance f_2 from the pole is called the *second principal focus*, and rays originally proceeding parallel to the axis are refracted so as to diverge from it (f_2 positive), or converge to it (f_2 negative).

68. Exercise. Construction of the refracted ray at a spherical surface. Let M J (Fig. 80) be any ray incident on the surface of radius O J. With centre O describe two spheres of radii $\mu \cdot$ O J and $\frac{\text{O J}}{\mu}$. Produce M J to cut the outer sphere in R. Join O R, cutting the inner sphere in S. Then J S is the refracted ray. For since O S : O J :: O J : O R the triangles O S J, O J R are similar, hence $\angle$ O J S = $\angle$ O R J, and therefore

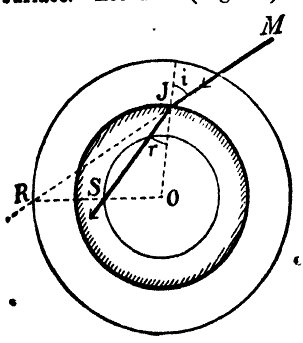


Fig. 80.

$$\frac{\sin \text{O J R}}{\sin \text{O J S}} = \frac{\sin \text{O J R}}{\sin \text{O R J}} = \frac{\text{O R}}{\text{O J}} = \mu,$$

from which we see that the angle O J S is the angle of refraction.

It is to be noted here that all rays directed towards R are refracted so as to pass through S. Similarly all rays coming from S are refracted so as to appear to come from R. R and S are conjugate foci for *all* rays and the surface is said to be *Aplanatic* (See also Art. 78.)

69. Opacity of mixtures of transparent substances. If some paraffin oil and water, or any other two liquids which will not mingle, and which have no chemical action on each other, be shaken in a test-tube the mixture becomes opaque like milk. The result of the agitation is to break up each liquid into a multitude of minute drops, each one of which retains all the transparency that the oil and the water in bulk possessed. But when a ray of light falls on the mixture, and encounters first a drop, let us say, of water, a certain proportion of the light will be reflected from the first surface of the drop, the rest passing through the drop until it encounters a neighbouring drop of oil. Here another reflection takes place, and the weakened ray passes through the drop of oil till it encounters a drop of water, when further reflection and further weakening takes place. Since there must be scores of such reflecting surfaces in every tenth of an inch of the mixture, it will be apparent that the light will be unable to penetrate directly to any considerable depth, and the *opacity* is at once explained; and the milky whiteness also, for the mixture reflects the light freely instead of allowing it to pass freely through it away from the eye. Foam is white and opaque for a similar reason—it being a mixture of minute particles of air and water, both of which are separately transparent. Milk also owes its whiteness to the same cause, for it consists of a multitude of minute globules of transparent fat floating in a transparent watery liquid. Snow and crushed glass are white and opaque for similar reasons. If two transparent liquids of precisely the same absolute index of refraction were shaken together, no such results would follow, for there would be no internal reflections at the bounding surfaces.

Again, if a colourless transparent solid were immersed in a colourless transparent liquid of the same refractive index, the solid would be invisible.* An approach to this condition

* As a matter of fact, Lord Rayleigh has shown that in a field of uniform illumination any transparent body would be invisible, even if the body and the surrounding medium were of very different refractive index.

may be made by immersing a glass rod in glycerine, when it will be found that the existence of the rod might be easily overlooked on a casual glance. A fibre of cotton when seen under the microscope is nearly transparent, but paper, which is a feltwork of such fibres, is opaque, because the interstices between the fibres are occupied by air, which has a very different refractive index from the cotton. In the manufacture of tracing-paper, the air is replaced by greasy substances, whose index of refraction approaches much more nearly to that of the cotton fibres than air does, consequently there is much less internal reflection and more transparency than in the untreated paper. The increased transparency of a linen lantern screen on wetting it is similarly explicable.

70. Atmospheric Refraction and its Effects. If the earth had no atmosphere the rays of light proceeding from a celestial body would travel in straight lines right up to the observer's eye or telescope, and we should see the body in its actual direction. But when a ray *Sa* (Fig. 81) meets the uppermost layer *AA'* of the earth's atmosphere, it is refracted or bent out of its course and its direction changed to *a b*. On passing into a denser stratum of air at *BB'*, it is further bent into the direction *b c*, and so on; thus, on reaching the observer the ray is travelling in a direction *OT*, different from its original direction, but in the same vertical plane.

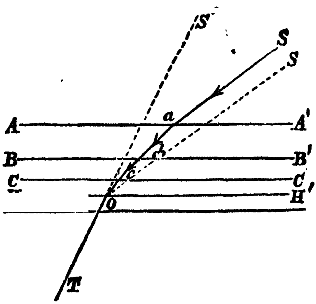


Fig. 81.

The body is therefore seen in the direction *OS'*, although its real direction is *aS* or *OS*. Also, since the successive horizontal layers of air *AA'*, *BB'*, *CC'*, . . . are of increasing density, the effect of refraction is to bend the ray towards the perpendicular to the surfaces of separation, that is, toward the vertical.

Hence *the apparent altitudes of the stars are increased by refraction.*

In reality, the density of the atmosphere increases *gradually* as we approach the earth, instead of changing abruptly at the planes *AA'*, *BB'*, *CC'*. . . . Consequently the ray, instead of describing the polygonal path *Sabc O*, describes a curved path, but the general effect is the same.

The elevation due to refraction increases as we go from the zenith to the horizon. At the latter place the elevation is about 33′; consequently a celestial body appears to rise or set when it is 33′ below the horizon. Thus the effect of refraction is to accelerate the time of rising, and to retard, by an equal amount, the time of setting of a celestial body. In particular the sun and moon, whose angular diameters are 32′ and 31′ respectively, appear to be just above the horizon when they are really just below.

When the sun or moon is near the horizon, it appears distorted into a somewhat oval shape. This effect is due to refraction. The whole disc is raised by refraction, but the refraction increases as the altitude diminishes, so that the lower limb is raised more than the upper limb, and the vertical diameter appears contracted. The horizontal diameter is unaffected by refraction, since its two extremities are simply raised. Hence the disc appears somewhat flattened or elliptical, instead of truly circular.

The apparent position of terrestrial objects also suffers from the same influence, distant bodies appearing elevated above their true position.

The duration of **twilight** (Art. 34), is also increased by refraction.

The twinkling of the stars. The air near the ground is more or less disturbed by convection currents, and thus the refractive index will vary from point to point even on the same level. The rays of light from a star, besides being bent as described above, will deviate now and then from their position in an atmosphere at rest, and so the light from a star will sometimes be concentrated at a point and sometimes decreased in intensity. Therefore, to a fixed observer, the star begins to twinkle or scintillate. That planets do not twinkle as much as stars is due to the angular size of the former; the directions of the rays from different parts of the planetary discs may vary very much, but the sum of the number of rays received by any given area—even if small—is very nearly constant, and thus uniform illumination results.

The above explanation is borne out by the fact that a star seen through a large telescope does not twinkle, the average amount of light falling on such a large area as an object glass being approximately constant.

CALCULATIONS.

71. In the preceding chapter several important relations have been established. For convenience of reference we shall here summarise the formulated expressions of these relations :—

(1) $$_b\mu_a = \frac{1}{_a\mu_b}.$$ (Art. 53.)

That is, the index of refraction from *b* to *a* is the reciprocal of that from *a* to *b*.

(2) $$_a\mu_c = {_a\mu_b} \cdot {_b\mu_c}$$ (Art. 53.)

(3) $$_a\mu_b = \frac{_v\mu_b}{_v\mu_a}$$ (Art. 53.)

(4) $$_a\theta_b = \sin^{-1} {_a\mu_b}.$$ (Art. 55.)

The relations (3) and (4) should be learnt in words.

(5) $$D = (\phi - \phi')$$ (Art. 61.)

(6) $$v = \mu u$$ (Art. 62.)

(7) $$\begin{cases} t' = \mu t \\ d = t(1 - \mu) \end{cases}$$ (Art. 63.)

(8) $$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}.$$ (Art. 66.)

In formulæ which involve *v* and *u*, distances are measured from the surface of separation of the media, and the usual sign convention (Art. 40) is adopted.

In all cases μ denotes the index of refraction in the direction in which the light is travelling.

Examples IV.

1. The absolute refractive indices of diamond and glass are respectively $\frac{5}{2}$ and $\frac{3}{2}$. Find the relative indices of refraction from glass to diamond, and from diamond to glass.

Here, if $_g\mu_d$ denote the relative index of refraction from glass to diamond we have, from (3)—

$$_g\mu_d = \frac{_v\mu_d}{_v\mu_g} = \frac{5}{2} \div \frac{3}{2} = \frac{5}{2} \times \frac{2}{3} = \frac{5}{3}.$$

$$\therefore {_g\mu_d} = \tfrac{5}{3}, \text{ and by (1)}$$

$$_d\mu_g = \tfrac{3}{5}.$$

2. Find the critical angle for water and glass, given that the index of refraction from air to glass is $\frac{3}{2}$, and that from air to water $\frac{4}{3}$.

Of the media water and glass, glass is the denser, and by (1) and (2) we have—

$$_g\mu_w = {_g\mu_a} \cdot {_a\mu_w} = \tfrac{2}{3} \cdot \tfrac{4}{3} = \tfrac{8}{9}.$$

Now, if $_g\theta_w$ denote the critical angle for glass and water, then—

$$_g\theta_w = \sin^{-1} {_g\mu_w} = \sin^{-1} \tfrac{8}{9}.$$

That is, the critical angle for glass and water is an angle whose sine is $\frac{8}{9}$. Reference to a table of sines show this to be 66° 44′.

3. A small air bubble in a piece of glass with a plane surface is 3 inches below that surface; find its apparent distance from an eye looking at it, along a normal to the surface, from a point 8 inches from the surface. (Index of refraction from air to glass $\frac{3}{2}$.)

Here, applying $t' = \mu t$ (7), and remembering that the light is supposed to be travelling from glass to air, and that therefore $\mu = \frac{2}{3}$, we have—

$$t' = \tfrac{2}{3} \times 3 = 2 \text{ inches}.$$

Therefore the apparent distance of the bubble from the eye $= 8 + 2 = 10$ inches.

4. A gold-fish globe of 6 inches radius is filled with water. Determine the apparent position of a point inside the globe, 4 inches from its surface, when seen by an eye looking along the radius of the globe.

Here, the surface at which refraction takes place is spherical and, neglecting the action of the glass of the globe, we have—

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}, \qquad (8).$$

and $\mu = \frac{3}{4}$ (water to air)
$u = 4$ inches
$r = 6$ inches
v is required.

$$\frac{3}{4\,v} = \frac{1}{4} - \frac{1}{24} = \frac{5}{24}.$$

$$\therefore\ 20\,v = 72.$$

$$\therefore\ v = 3{\cdot}6 \text{ inches}.$$

That is, the apparent position of the point is inside the globe on the radius passing through its real position, and 3·6 inches from the surface.

5. A piece of plate glass, 5 inches thick (refractive index 1·6), is placed between the eye and an object. Find what alteration will take place in the apparent distance of the object from the eye.

6. Find the relative index of refraction from Canada balsam to air. (Refer to the table of refractive indices for data.)

7. The sine of the critical angle for two media is $\frac{7}{8}$. What is the index of refraction from the rarer to the denser of the two?

8. *Find the absolute refractive index of carbon disulphide, given that the relative index of refraction from carbon disulphide to glass is 0·9, and the absolute refractive index of glass is 1·512.*

9. If a ray of light passes from one medium to a second, making the angle of incidence = 45°, and the angle of refraction equal to 30°, show that the refractive index for the media is $\sqrt{2}$.

10. The critical angle of a given medium is 60°. What is its refractive index?

Note.—When *the critical angle* or *the refractive index* of any medium is referred to simply, it must be understood that the other medium involved is vacuum.

11. A vessel, 6 inches deep, is filled with alcohol. What is the apparent depth of the liquid?

12. The refractive index of water is 1·33, and the velocity of light in air is 300,000,000 metres per second. Find the velocity of light in water.

13. A small air bubble in a sphere of glass, 4 inches in diameter, appears, when looked at so that the bubble and the centre of the sphere are in a line with the eye, to be 1 inch from the surface. What is its true distance from the surface? ($\mu = 1·5$.)

14. A small air bubble at the centre of a glass sphere is seen from a point outside the sphere. What is the apparent position of the bubble? Explain.

15. A brass sphere of 2 cm. radius is surrounded by a glass shell of 6 cm. external radius. What is the apparent thickness of this shell?

16. A block of transparent jelly of refractive index 1·33 is bounded on one side by part of the convex surface of a sphere of radius 8 millimetres. Find the position of the principal focus within the mass of the material.

17. Draw three parallel straight lines, an inch apart, in the plane of the paper to represent rays of light incident upon a glass sphere of radius 2 inches, with its centre upon the last of the series, and trace, by a geometrical construction, the paths of the refracted rays within and beyond the sphere. (μ of glass = 1·5.)

18. In an experiment, as described in Exp. 11, the following readings were taken:—

values of i,	10° 30′,	24° 24′,	39° 0′,	58° 0′.
Corresponding values of r,	6 48	15 48	25 45	34 12.

Find the mean value of μ.

19. In an experiment, as described in Exp. 12, the following readings were taken:

$$p_1 = 3 \cdot 05, p'_1 = 2 \cdot 05. \quad p_2 = 2 \cdot 90, p'_2 = 1 \cdot 94.$$

Find the mean value of μ.

20. *Describe Wollaston's method for the determination of the index of refraction of liquids by means of total reflection.* In an experiment with a certain liquid the angular distance between the two positions of extinction was 97° 0′. Find μ, and the liquid.

21. Explain the quivery appearance seen above hot rocks or bricks, and the streaky appearance of water in which ice, sugar, or acid is being dissolved.

22. A piece of a colourless mineral is dropped into a colourless liquid; the mineral is invisible in the liquid. How are the refractive indices of the liquid and of the mineral related?

23. A hollow watertight prism containing air, with flat glass sides, is immersed in a glass tank full of water. Draw and explain a diagram showing the path of a ray of light passing through the water and the prism.

24. Light incident at an angle of 60° on one face of an equilateral glass prism is deviated 30° at the first face. Draw a diagram showing the path of the rays through and out of the prism, and find the refractive index of the glass.

25. Explain by the aid of a diagram what occurs when light is incident on a glass plate. Explain why a transparent substance such as glass is opaque when finely powdered

26. If you hold a glass of water with a spoon in it a little above the level of the eye, and look upwards at the under surface of the water, you will find that you are unable to see that part of the spoon which is above the water. Explain this.

27. A ray of light passes from alcohol to a parallel plate of Iceland spar 1 in. thick, and then into air. The ray is incident on the Iceland spar at 45°. Make a scale drawing showing the exact path of the ray. The index of refraction of Iceland spar is $\frac{7}{4}$, and of alcohol

CHAPTER VII.

REFRACTION THROUGH PRISMS AND LENSES.

72. In this chapter we shall not consider dispersion, and must therefore be understood to deal with the refraction of rays and pencils of light *of definite wave-length*, and therefore of definite refractive index and *colour*. Such light is sometimes referred to as *monochromatic* or *homogeneous* light, and is conveniently obtained, of a yellow colour, from a flame coloured by the presence of a salt of sodium (as in Fig. 68).

Prisms.

73. Prisms. From an optical point of view, **a prism** is any portion of a medium lying between two plane faces inclined to each other at any angle. The line of intersection of these faces is known as the *edge* of the prism, and a section of the prism at any point in its length, perpendicular to this edge, is called a *principal section.* The *refracting angle* of the prism is the angle between its faces, as measured by the corresponding plane angle of the principal section. The prisms generally used for experiments are triangular prisms, in the geometrical sense of the term. The principal sections of such prisms are equilateral, isosceles, or scalene triangles, according to the purpose for which the prism is intended. When the section is equilateral the angle at each edge is equal to 60°, and thus there is no gain in having three edges; with an isosceles section there are two different angles available, and with a scalene section the angle at each edge is different, and thus the prism is equivalent to three prisms considered in the optical sense.

74. Refraction through a prism. In dealing with refraction through a prism, we shall consider only the case where the plane of incidence and refraction is coincident with a principal section of the prism.

Let A B C (Fig. 82) represent the principal section of a prism, and B A C the refracting angle considered; then, if the material of the prism be of higher refractive power than the external medium, a ray R N incident on the face A C, at N, is bent towards the normal on entering the prism, and, taking the course N N′, is incident on the face A B at

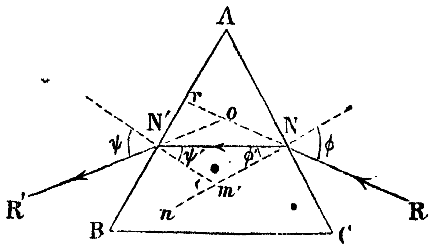


Fig. 82

N′, where it is bent away from the normal, and leaves the prism by the path N′ R′.

The ray R N is thus, after refraction through the prism, *deviated* from its original direction R N, and finally travels along N′ R′. The deviation, as in Art. 61, is evidently measured by the angle $r\,o\,N'$; its *magnitude* is found to depend on the path of the ray through the prism, but its *direction* is always away from the refracting edge. There is one position for which this deviation is a minimum; when the prism is so placed that the incident and emergent rays make equal angles with the normals at their respective faces, then the deviation is a minimum, and the prism is said to be in the position of minimum deviation. This can be proved theoretically; but as the proof is beyond the scope of this work, we must, for the present, consider it as a fact established by experiment. (See Art. 76.)

The minimum deviation produced by any prism depends

on the angle of the prism and the refractive index of its material relative to the external medium. We shall now proceed to establish an important relation between these quantities. In Fig. 82 let ϕ and ϕ' denote the angles of incidence and refraction at N, and ψ' and ψ the corresponding angles at N'.*

Then, if D denote the deviation produced, we have—

$$D = r\,o\,N' = o\,N\,N' + o\,N'\,N. \qquad \text{(Euc. i. 32).}$$
$$= (\phi - \phi') + (\psi - \psi'). \quad \text{[Cp. Art. 61, Formula (5).]}$$
$$\therefore D = \phi + \psi - (\phi' + \psi'). \qquad (1).$$

But, since the angle contained between any two lines is equal to that contained by lines perpendicular to them, we have, if A denote the angle of the prism—

$$n\,m\,N' = B\,A\,C = A.$$
$$\text{But } n\,m\,N' = \phi' + \psi'. \qquad \text{(Euc. i. 32.)}$$
$$\therefore A = (\phi' + \psi'). \qquad (2).$$

Substituting this value of $(\phi' + \psi')$ in (1), we get—

$$D = \phi + \psi - A. \qquad (3).$$

Now, when the prism is in the position of minimum deviation, the ray passes *symmetrically* through the prism, and we must therefore have $\phi = \psi$ and $\phi' = \psi'$.

Therefore, from (3)—

$$D = 2\,\phi - A. \qquad \therefore \phi = \frac{D + A}{2}. \qquad (4).$$

And from (2)—

$$2\,\phi' = A. \qquad \therefore \phi' = \frac{A}{2}. \qquad (5).$$

But, if μ denote the refractive index of the material of the prism, relative to the external medium, then—

$$\mu = \frac{\sin\phi}{\sin\phi'}.$$

Therefore, substituting from (4) and (5), we have—

$$\mu = \frac{\sin\frac{1}{2}(D + A)}{\sin\frac{1}{2}A}. \qquad (1).$$

---

* At N' the angle N N' m is the angle of incidence; but, for the sake of symmetry with ϕ', it is here denoted by ψ' and not by ψ.

This result, in connection with refraction through a prism in the position of minimum deviation, is of great practical importance.

When the angle of the prism is small, a very convenient expression for D may be obtained from the formula just established. Thus we have—

$$\mu = \frac{\sin \frac{1}{2}(D + A)}{\sin \frac{1}{2} A}.$$

Now, if D and A be so small that the angles $\frac{1}{2}(D + A)$ and $\frac{1}{2} A$ may be considered as approximately equal to the sines of these angles, we have—

$$\mu = \frac{D + A}{A}, \text{ or, } D = (\mu - 1) A. \qquad (2).$$

75. Conjugate foci in the case of refraction through a prism in the position of minimum deviation. It is a general law that, when any quantity is passing through its maximum or minimum value, a small change in the variable concerned produces very little effect on the magnitude of the quantity itself. For example, the magnitude of the deviation produced by refraction through a prism depends upon the path of the rays; but when the prism is in the

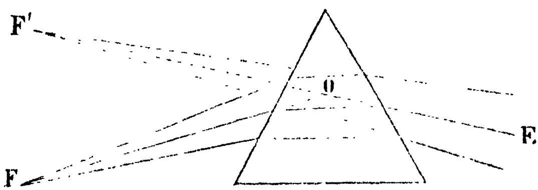


Fig. 83.

position of minimum deviation, any small change in the path produces but little change in the magnitude of the deviation. Hence, for rays passing through a prism, by paths near to that of minimum deviation, the deviation which each undergoes is practically the same, and very nearly equal to the minimum value.

Hence, if a *small* pencil of rays coming from F (Fig. 83) be incident on a prism at such an angle that the axis passes

along the path of minimum deviation, then all the rays will be deviated to an approximately equal extent, and will therefore, on emergence, be inclined to one another at nearly the same angle as before incidence. Hence, if produced backwards, the rays of the pencil appear to come from a point F′ such that F′O = FO. Similarly, if we imagine the path of the pencil to be reversed, we see that a convergent pencil having its focus at F′ would, after refraction through the prism, converge to the point F.

F and F′ are thus conjugate foci; and, if the pencil be incident near the refracting edge of a prism of small angle, placed in the position of minimum deviation for the axis of the pencil, we may neglect the thickness of the prism, and state that conjugate foci are on the same side of the prism and equidistant from its edge.

It follows, from what has been said above that, if an object

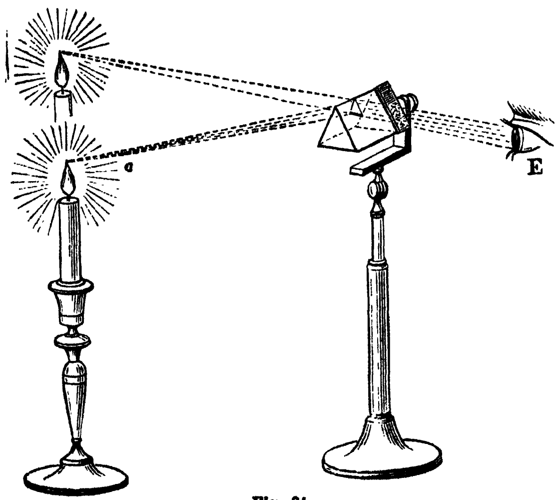


Fig. 84.

be placed at F, its image is seen at F′ by an eye placed at E. This image is evidently virtual and displaced from the position of the object towards the edge of the prism (Fig. 84).

76. Practical illustration of minimum deviation. Exp. 15. Take a prism, P (Fig. 85), and cut out a circular piece of cardboard, C, so that when the prism is mounted on it its edges stand vertically over the circumference of the disc. Stick P to C by means of soft wax. Fasten a piece of cartridge (drawing) paper to a board and in the middle of the sheet describe a circle of the same size as C. Rule a straight line B D across the paper and erect two pins at B and D. Place P and C in position, one face of P being nearly normal to B D. Look through the prism from the other side and erect two pins E_1 and F_1 in the paper on the same side of the eye, so that E_1, F_1, and the images of B and D appear in a straight line. The images of B and D will appear a trifle indistinct and coloured, and the alignment must be effected by means of their centres. Test the observation by placing the eye beyond B and looking along B D E_1 F_1. Mark the position of A on the paper with a pencil point 1,

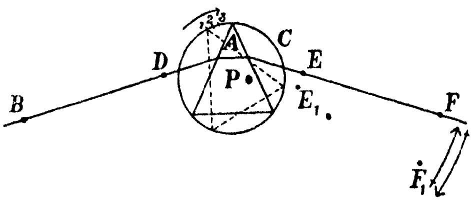


Fig. 85.

remove E_1 F_1, and rule in the line E_1 F_1. Still keeping C exactly over the pencilled circle, turn it around so that A moves a little to the right. Repeat the observations. Denote the new position of A, E_1 and F_1 by 2, E_2 and F_2 respectively. Repeat for several positions of A until the angle of incidence is large and the images of the pins very indistinct. It will be found that as P is turned around, the line of pins on the eye side of the prism moves up to a position E F and then retreats. Remove the prism and card. Mark in the position of the prism which gave E F, produce B D, F E to meet the prism faces and show that the angle of incidence is equal to the angle of emergence. Now measure the angle between the prolongation of B D and E F. It is equal to the D of Art. 74. Measure the angle A of the prism by a protractor and find the refractive index of the material composing the prism by means of the formula

$$\mu = \frac{\sin \frac{D + A}{2}}{\sin \frac{A}{2}}.$$

77. Assuming that experiment proves that there is only one position of the prism with regard to the incident beam which makes the deviation a minimum, it is easy to show theoretically that this position occurs when the incident and emergent rays occupy symmetrical positions with respect to the prism. For if the incident beam A B (Fig. 86) is fixed in direction, and the position of the prism M is such as to make the deviation of the ray A B C D a minimum when the angles of incidence and emergence are ϕ and ψ respectively, ϕ and ψ being unequal

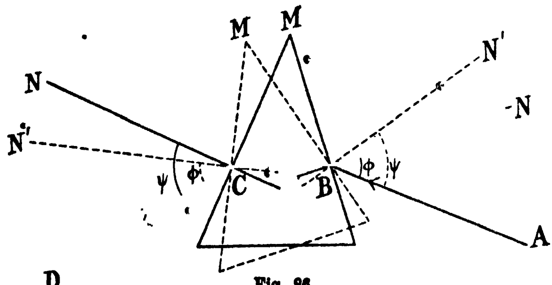


Fig. 86.

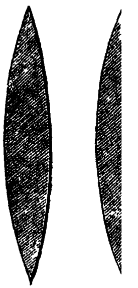
to each other, then also will the deviation be a minimum when the angles of incidence are respectively ψ and ϕ. So that there are two positions of the prism, namely, M and M′, which render the deviation a minimum. **Exp. 15.** shows this to be false, and it will be noted that the two cases only merge into one when ϕ and ψ are equal to each other.

The following is an approximate theoretical proof: It is easy to prove that an angle increases faster than its sine does (a glance at a table of sines will show this). From this it follows that as an angle of incidence increases, the angle of refraction also increases but at a lesser rate; and therefore that the angle of deviation increases (cf. Art. 61). Consider now a ray passing symmetrically through a prism as in Fig. 82 For this ray $\phi = \psi$ and $\phi' = \psi'$. Now suppose another ray passes for which ϕ' is slightly increased. Since $\phi' + \psi' = A$, ψ' will decrease by an equal amount. ϕ will increase and ψ will decrease, but $\phi - \phi'$ will be greater than $\psi - \psi'$, *i.e.*, the increment of ϕ will be greater than the decrement of ψ, hence the total deviation of this new ray which equals $\phi + \psi - A$ will be greater than the deviation of the symmetrical ray. That is, the deviation is least when $\phi = \psi$.

LENSES.

78. Lenses. A lens may be generally defined as a portion of a medium enclosed between two surfaces of definite geometrical form and having a common normal. Usually these surfaces are portions of spheres or plane surfaces, and the medium most generally employed is glass Lenses of this form may be considered as solids of revolution. For example, if any one of the sections shown in Figs. 87 and 88 be supposed to revolve round a central horizontal axis in the plane of the paper, the solid described by such revolution determines the form of the lens corresponding to that section.

It is usual to divide lenses into two classes :—



(a) (b) (c)

Fig. 87.

1. Convex lenses (Fig. 87). Of these there are three chief forms—

(*a*) Double **convex.**

(*b*) Plano-**convex.**

(*c*) Concavo-**convex** [*converging meniscus*].

The distinguishing characteristic of these lenses is that they are *thicker at the centre than at the edges.*

2. Concave lenses (Fig. 88). Corresponding to the three forms of convex lenses we have—

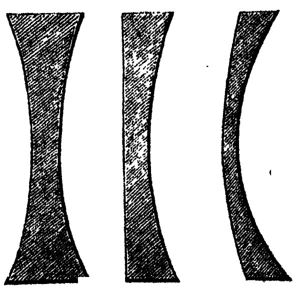
(*a*) Double **concave.**

(*b*) Plano-**concave.**

(*c*) Convexo **concave** [*diverging meniscus*].

The distinguishing characteristic of this class is that the lenses *are thinner at the centre than at the edges.*

The action of any of these forms of lenses on a pencil of rays passing through them depends on the refractive index of the medium of which



(a) (b) (c)

Fig. 88.

they are made, relative to the surrounding medium. Usually we have to deal with glass lenses surrounded by air, that is, the medium of the lens is of higher refractive power than the surrounding medium. In this case, **convex** lenses cause the rays of a pencil to become more convergent, or less divergent after passing through them, and for this reason are sometimes called **converging** lenses. Similarly, **concave** lenses are called **diverging** lenses because the rays of a pencil are always more divergent or less convergent after refraction through them than before.*

This action of convex and concave lenses may be explained in the following way. The section of a double convex lens may be considered as rather similar to that of two prisms placed base to base as in Fig. 89. Consider the rays P A and P B incident on the prisms at A and B. As explained in Art. 74, these rays are deviated away from the edges of the prisms on which they are incident, and are thus less

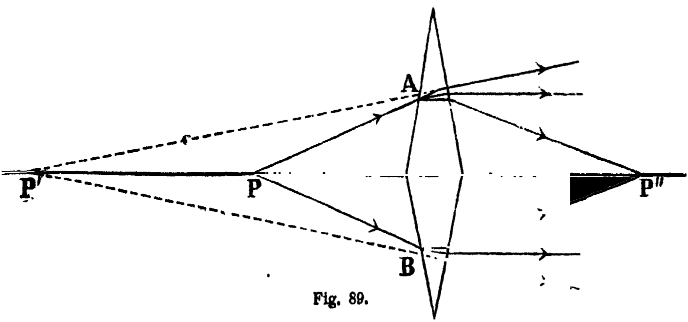


Fig. 89.

divergent after refraction. The path of the rays P A and P B, after passing through the lens, depends on the magnitude of the deviation produced; they may either diverge from P′, run parallel, or, if the deviation be sufficiently great, converge to a point, P″.

Similarly, the section of a double concave lens may be

* When the refractive index of the substance of the lens is less than that of the surrounding medium, then a *convex* lens acts as a *diverging* lens, and a *concave* lens as a *converging* lens.

considered as rather similar to that of the two prisms placed apex to apex, as in Fig. 90. In this case the rays P A and P B are refracted away from the edges of the prisms, that is, from the centre of the lens, and, after refraction, appear to diverge from the point P′; the rays are thus more divergent after passing through the lens than before.

In the case of the prisms shown in Figs. 89 and 90, the positions of P′ and P″ will depend on the positions of A and B, but in the case of a lens, owing to the curvature of the surface, all rays coming from P would, after refraction, pass

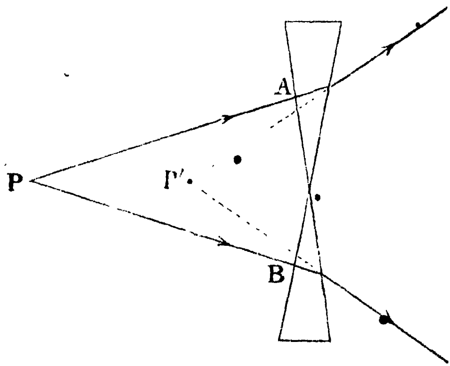


Fig. 90.

through the same point. When this is accurately the case the curvatures of the surfaces of the lens are specially adapted to the existing conditions, and the lens is said to be *aplanatic*; but, for ordinary lenses, with spherical or plane surfaces, this is only approximately the case, and the defect resulting from this want of accuracy is known as *spherical aberration.** (*See* Art. 90.)

A lens, being a solid of revolution, is symmetrical about its centre, and hence all sections passing through the axis of revolution are similar. It thus follows, that what has

* When the surfaces of the lenses are only very small portions of spherical surfaces, spherical aberration is almost negligible, and the lens is, for all practical purposes, *aplanatic*.

been explained above, for one section, is true for all similar sections, and consequently, if a *pencil* of light, diverging from P, be refracted through a lens, all the rays are symmetrically deviated, and, after refraction, pass through the same point.

79. Influence of curvature of surfaces of lens on deviation. Consider the refraction of the rays P *a b* P′ and P *c d* P′ through the lens L (Fig. 91). It is evident from the figure that, in order that the rays may pass

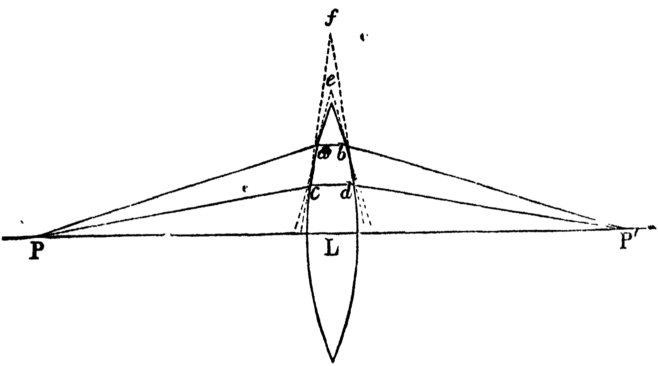


Fig. 91.

through P′, the deviation of P *a b* P′ must be greater than that of P *c d* P′. At *a* and *b* draw tangent planes to the surfaces of the lens meeting in *e*, and at *c* and *d* draw tangent planes meeting in *f*. Now the deviation in the case of the ray P *a b* P′ is that due to the prism of refracting angle *a e b*, and the deviation for the ray P *c d* P′ is that due to a prism of angle *c f d*. But, when the angle of the prism is small, the deviation produced is approximately proportional to the angle of the prism (Art. 74). Therefore, in this case the deviation for the ray P *a b* P′ is greater than that for P *c d* P, and thus it is possible for both rays to pass through P′.

80. Definitions. The *principal axis* of a lens coincides with its axis of revolution, and when the surfaces of the lens are spherical, passes through the centres of curvature of these surfaces. When one surface is plane, and the other spherical, the axis passes through the centre of curvature of the spherical surface and is normal to the plane surface. The **optical centre of a lens** *is that point, on the principal axis, through which pass all rays, or, in certain cases, the prolongations of the portion of such rays which is within the lens, having their paths parallel before and after refraction through the lens.*

Let C and C′ (Fig. 92) be the centres of the two spherical

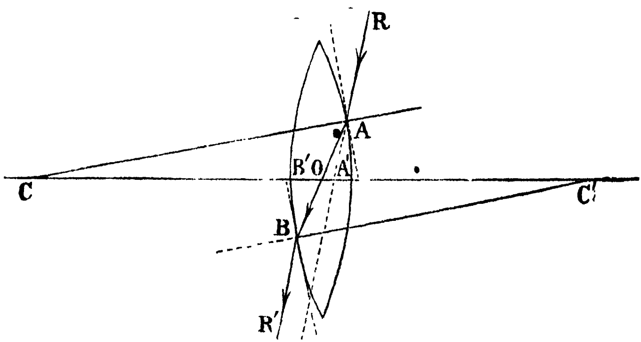


Fig. 92.

surfaces of a lens. Draw any radius C A, and through C′ draw the radius C′B parallel to C A. Join A B cutting the principal axis C C′ in O. Then O is the optical centre of the lens. For, if A B represent the path, *through the lens*, of the ray R A B R′, then, by construction, A B makes equal angles, at A and B, with the normals C A and C′B, and consequently the incident and emergent rays R A and B R′ also make equal angles with these normals and are therefore parallel.* This is true for *any* two parallel radii, C A and C′B, and hence O is the optical centre of the lens

* The action of the lens is, under the conditions considered, exactly similar to that of a plate enclosed by the parallel tangent planes at A and B (cf. Art. 53).

as defined above. To determine the position of O, we have, from the triangles A O C and B O C′ that—

$$\frac{C\,O}{C'O} = \frac{C\,A}{C'B} = \frac{C\,A'}{C'B'}.$$

$$\therefore \frac{C\,A'}{C'B'} = \frac{C\;O}{C'\,O} = \frac{C\,A' - C\,O}{C'B' - C'O} = \frac{O\,A'}{O\,B'}.$$

That is, the point O divides the thickness of the lens into segments proportional to the radii of curvature of the adjacent faces. In the case of double convex and double concave lenses the optical centre lies in the interior of the lens; in plano-convex and plano-concave lenses it is situated on the spherical surfaces, and in a converging or diverging meniscus it lies outside the lens on the same side as the surface of lesser radius of curvature.

Although the incident and emergent rays R A and B R′ are parallel, they are not in the same straight line; but, if the thickness of the lens be small, the displacement produced is negligible, and it may be stated that all rays passing through the optical centre of the lens suffer no deviation, but continue their course in the same straight line.

Any line, other than the principal axis, passing through the optical centre is called a *secondary axis*.

When a parallel pencil of light is incident on a lens in a direction parallel to the principal axis of the lens, the rays, after refraction through the lens, converge to or diverge from a point on the principal axis. This point is the **principal focus** of the lens, and its distance from the optical centre of the lens, measured along the principal axis, is the **focal length** of the lens.

In the case of a convex lens, of any form, the parallel pencil of rays is made to *converge* to a point F (Fig. 93) on

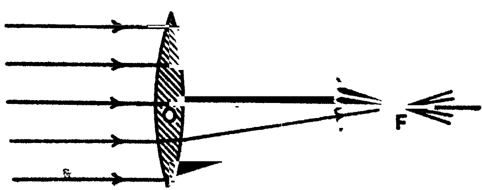


Fig. 93.

the other side of the lens. A concave lens (Fig. 94) causes the rays to *diverge* from a point F on the same side of the lens as the incident pencil. In both cases O F represents

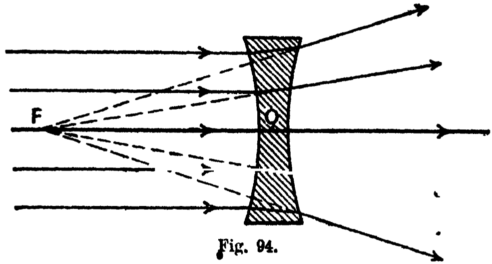


Fig. 94.

the focal length, and, applying the usual convention of sign, it is evident from Figs. 93 and 94 that, if distances be measured from O, the focal length of a *concave* lens is *positive* and that of a *convex* lens *negative*.

If the pencil of parallel light is incident in a direction parallel to a secondary axis A F_1, inclined at a *small* angle

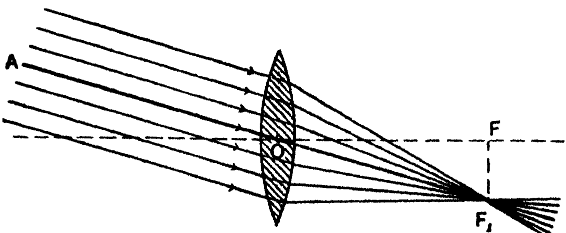


Fig. 95.

to the principal axis (Fig. 95), the focus of the refracted pencil is on the secondary axis at a point F_1, such that O F_1 is approximately equal to the focal length of the lens.

81. Path of a ray through a lens. Let C and C′ (Fig. 96) denote the centres of curvature of the faces of the lens A B, and let the ray P A be incident on the surface of the lens at A. Join C A and produce it to N; then C A N is the normal at A, and the ray P A is refracted

into the lens along A B, making an angle B A C with the normal such that—

$$\frac{\sin P A N}{\sin B A C} = \mu$$

where μ denotes the refractive index of the material of the lens relative to the surrounding medium. Similarly at B, the ray is incident on the second face of the lens and is refracted along B P′ in such a direction that

$$\frac{\sin P' B N'}{\sin C' B A} = \mu.$$

To determine, by this construction, the path of any given ray would be a very troublesome process; and it is therefore

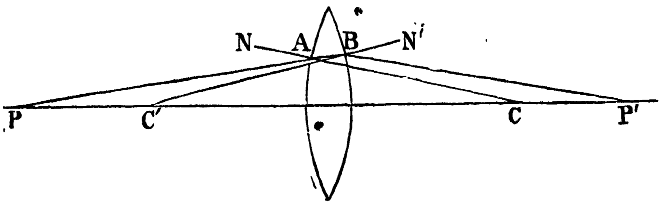


Fig. 96.

important to notice two particular cases in which the path is readily determined.

1. Any ray passing through the optical centre * of a lens continues its course in the same straight line (Art. 80).

2. When the incident ray is parallel to the principal axis, the refracted ray passes through the principal focus (Art. 80).

82. Conjugate foci.† When rays of light, diverging from a point P (Fig. 97) on the principal axis of a lens, are refracted through the lens, the focus of the refracted pencil

* For ordinary purposes the optical centre of a *thin* lens may be taken at any point in its thickness, on the principal axis.

† It should be noticed that if the conjugate foci are both *real* the image of an object placed at either focus is formed at the other; but if one of the foci is *virtual*, then the image of an object placed at that focus is not formed at the other, but rays converging to the *virtual* focus are refracted through the conjugate focus. That is, the optical relation between conjugate foci assumes reversal of the direction of the light.

is another point P′, also on the principal axis. These points, P and P′, are called *conjugate foci*. When the point P is on any secondary axis *inclined at a small angle to the principal axis*, the point P′ is also on that secondary axis; but it is important to notice that secondary axes have not the same relation to lenses as they have to mirrors. In the case of mirrors, the secondary axes have exactly the same geometrical relation to the spherical reflecting surface as the principal axis, but for lenses this is not the case, and refraction along secondary axes involves several compli-

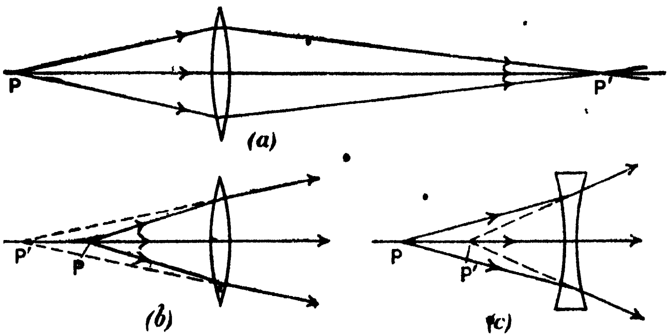


Fig. 97.

cations which we cannot now consider. When, however, the angle between a secondary axis and the principal axis is *small* the laws applicable to refraction along the principal axis may be applied with approximately correct results.

The relation between the distances of *conjugate foci* from the centre of the lens, and the focal length of the lens, will be deduced in a later Article by a simple geometrical method. We shall now, however, establish a relation between these distances by the application of the formula—

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r},$$

deduced in Art. 66.

This formula applies to refraction at a *single* spherical surface, and μ represents the relative index of refraction from one medium to the other in the direction taken by the rays of light. Let $a\,c\,b\,d$ (Fig. 98) represent a lens and let r denote the radius of curvature of the face $a\,d$, and s the radius of curvature of the face $c\,b$. Then, considering

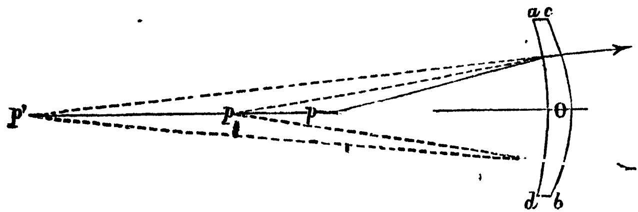


Fig. 98

first refraction at the face $a\,d$, the pencil of rays diverging from p_1 appears, after refraction, to come from p, and if $O\,p_1$* be denoted by u, $O\,p$ by v', and the index of refraction from air into the lens by μ, we have—

$$\frac{\mu}{v'} - \frac{1}{u} = \frac{\mu - 1}{r}. \qquad (1)$$

After this first refraction at the face $a\,d$, the pencil diverging from p may be supposed to be incident on the face $c\,b$ and to suffer refraction at that surface *from the lens into the air*, the focus of the emergent rays being at p'. Now, if μ denote the index of refraction from air into the lens, then $1/\mu$ denotes the index of refraction from the lens into air (Art. 53). Hence, if $O\,p'$ be denoted by v, we have—

$$\frac{\frac{1}{\mu}}{v} - \frac{1}{v'} = \frac{\frac{1}{\mu} - 1}{s}.$$

$$\therefore \frac{1}{v} - \frac{\mu}{v'} = \frac{1 - \mu}{s} = -\frac{\mu - 1}{s}. \qquad (2)$$

---

* In this investigation the thickness of the lens is supposed to be negligibly small compared with u and v, so that O may be taken as a point on either of the spherical surfaces $a\,d$ and $c\,b$.

Adding the equations (1) and (2), we get—

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right). \qquad (3)$$

This establishes a relation between u and v, the distances of the conjugate foci from the lens, and the radii of the spherical surfaces.

If u be infinite, that is, if the incidental rays are parallel, then, by Art. 80, the emergent pencil passes through the *principal focus*, and v becomes equal to the *focal length* of the lens. Therefore, if the focal length be denoted by f, we have—

$$\frac{1}{f} - \frac{1}{\infty} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right).$$

But—

$$\frac{1}{\infty} = 0. \qquad \text{(Art. 26, 3)}$$

$$\therefore \quad \frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

Substituting this value of $(\mu - 1)\left(\dfrac{1}{r} - \dfrac{1}{s}\right)$ in (3) we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \qquad (4)$$

which is the most important formula relating to lenses.

For the sake of clearness the details of Fig. 98 have been so chosen that all the distances involved are positive,* but the different formulæ established in this article may, if due regard be paid to sign, be obtained in the same form for all cases of refraction through a lens. The general formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

is therefore applicable to all cases, and, if P′ be considered as the image of P, establishes a relation between the distances of the object and image from the centre of the lens and the focal length of the lens.

To trace the path of P′ along the principal axis of the lens as P moves from an infinite distance on one side of the

* Wherever reference is made to sign it must be understood that the convention explained in Art. 40 is the one adopted.

lens to an infinite distance on the other side is best done as follows :—

The formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ may be treated as in Art. 41.

Multiplying up, $uf - vf = uv$.

$$i.e., \; f^2 + uf - vf - uv = f^2$$

$$\therefore \; (f + u)(f - v) = f^2.$$

Let x and x' denote the distances of object, B, and image, B′, measured from F′ and F respectively, the ordinary convention being used. Then, using Fig. 105,

$$x = BF' = u + f, \; x' = FB' = -(f - v), \text{ and } \therefore \; xx' = -f^2.$$

Note that f^2 being positive, x and x' always are of different sign; *i.e.*, the object is on the other side of F′ that the image is of F.

This leads to the following results :—

I.—**For Convex lenses** (Figs. 93 and 99).

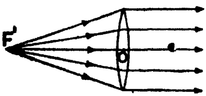


Fig. 99.

1. When P is at infinity, that is, when the incident rays are parallel to the principal axis, P′ is at the principal focus F (*i.e.*, when $x = +\infty$, $x' = o$) (Fig. 93).

2. When P is in front of the lens at a distance numerically equal to twice the focal length, then P′ is an equal distance behind the lens (*i.e.*, $x = -f$, $x' = +f$).

3. When P is at F′, that is, when the incident rays diverge from a point in front of the lens, and at a distance from its centre equal to the focal length, then P′ is at infinity on the other side of the lens, that is, the refracted rays are parallel to the principal axis ($x = o, x' = \infty$) (Fig. 99).

4. When P is at O, P′ is also at O (*i.e.*, $x = +f$, $x = -f$).

5. When P is at F, that is, when the incident rays converge to the principal focus behind the lens, then P′ is halfway between O and F (*i.e.*, when $x = +2f$, $x = -f/2$).

6. When P is at infinity behind the lens, the incident rays are parallel, and, as in (1), P′ is at F (*i.e.*, $x = -\infty$, $x' = o$).

In cases such as 5 and 6, when the incident rays converge towards a point P and do not actually pass through it, the point of convergence can be described as a virtual object. Similar definitions apply to mirrors. Thus in Art. 26 a plane mirror produces a real image of a virtual object. See also Art. 41.

II.—For Concave lenses (Figs. 94 and 100).

1. When P is at infinity, that is, when the incident light is parallel, P′ is at F (*i.e.*, when $x = +\infty$, $x' = o$) (Fig. 94).

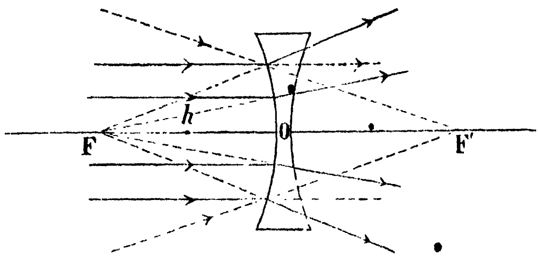


Fig. 100.

2. When P is at F, then P′ is midway between F and O (*i.e.*, when $x = 2f$, $x' = -f/2$).

3. When P is at O, P′ is also at O ($x = +f$, $x' = -f$).

4. When P is at F′, that is, when the incident rays converge to a point behind the lens at a distance from its centre equal to the focal length of the lens, then P′ is at infinity (*i.e.* when $x = o$, $x' = -\infty$).

5. When P is at a distance behind the lens numerically equal to twice the focal length, P′ is the same distance in front of the lens (*i.e.*, $x = -f$, $x' = +f$).

6. When P is at infinity behind the lens the incident rays are parallel, and, as in 1, P′ is at F (*i.e.*, $x = -\infty$, $x' = o$).

Of the above results I., 1, 3, 6 and II., 1, 4, 6 are summed up in the statement that, when a parallel pencil of light is incident on a lens, the focus of the refracted pencil is at the principal focus of the lens. I., 4 and II., 3 are evident and easily remembered. I., 2, 5 and II., 2, 5 may be deduced from the general formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, and should be carefully remembered.

General Rule. When an image is formed by *refraction through a lens*, any motion of the object along the principal axis produces a corresponding motion of the image in the **same** direction along the axis.

The application of this rule is simple. For example, from I., 1 and 3, we see that, as P moves from infinity to F'; P' moves in the same direction from F to infinity behind the lens. Similarly from II., 2 and 3, as P moves from F to O, P' moves in the same direction from h to O.

83. When drawing to scale diagrams of lenses of known focal lengths it is not sufficient, as in the case of mirrors, to know the curvatures of its faces, since the focal length depends upon the refractive index of the lens as well as upon these curvatures. But in the case of lenses of *crown glass* there is a simple rule which expresses with a near approach to accuracy the relation of focal length to curvature. The refractive index of crown glass is very nearly equal to 1·5 (Art. 53), therefore on substitution in the general formula—

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) \qquad \text{(Art. 82. 3.)}$$

we obtain—

$$\frac{1}{f} \backsimeq \frac{1}{2}\left(\frac{1}{r} - \frac{1}{s}\right).$$

In the case of a double convex lens whose faces are of the same curvature $s = -r$ and therefore—

$$\frac{1}{f} \backsimeq \frac{1}{2} \cdot \frac{2}{r} \backsimeq \frac{1}{r};$$

* $\backsimeq$ means "is approximately equal to."

that is—

$$f \approx r$$

or the principal focus of the lens is approximately at the

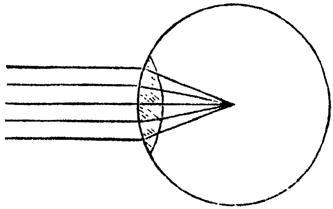


Fig. 101.

centre of a sphere of which the front surface of the lens forms a part (Fig. 101).

In the case of a plano-convex lens $\frac{1}{f} \approx \frac{1}{2} \cdot \frac{1}{r}$ or $f \approx 2r$; that is, if the curved surface is in front, the focus is on the circumference of the sphere of which the front surface is

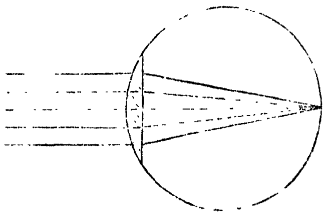


Fig. 102.

part (Fig. 102). Similar relations exist for double equi-concave and for plano-concave lenses.

84. General construction for images formed by lenses. Let A B (Figs. 103 and 104) represent an object placed on the principal axis of a lens. To determine the position of the image of the point A, it will be sufficient to determine the point of intersection, after refraction through the lens

of any two rays originally diverging from A. We have seen, in Art. 81, that the path of a ray is readily determined when it is incident parallel to the principal axis, or passes

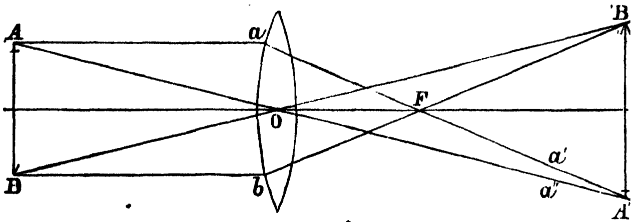


Fig 103.

through the centre of the lens. Let us consider, then, rays coming from A (Figs. 103 and 104) along both these paths. The ray A a, incident parallel to the principal axis, is refracted along $a\,a'$ in a direction passing through F, the principal focus of the lens. The ray A O passing through O, the centre of the lens, suffers no deviation, but continues its course along the straight line A O a''. The two refracted rays $a\,a'$ and O a'' actually intersect (Fig. 103), or appear to intersect (Fig. 104) at A′, which is, therefore,

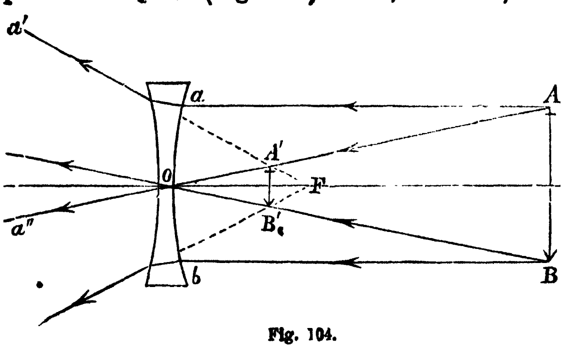


Fig. 104.

the image of A. Similarly, the image of B is formed at B′, and the images of all points between A and B being assumed to lie between A′ and B′, the complete image

A′ B′ is determined. When the rays really intersect, as in Fig. 103, the image formed is said to be **real**, but when they only apparently intersect, as in Fig. 104, the image is **virtual**. A real image is always formed on the side of the lens opposite to that on which the object is placed, and may be received on a screen, or seen by an eye* so placed as to receive *the rays involved in its formation.* A virtual image, having no real existence, cannot be said to be *formed* anywhere, but it is always seen on the side from which the light comes by an eye placed on the opposite side of the lens. A virtual image cannot be received on a screen.

85. Relative position of image and object. The formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

deduced for conjugate foci, evidently establishes a relation between the distances of the object and image from the centre of the lens; for an image is an assemblage of foci,

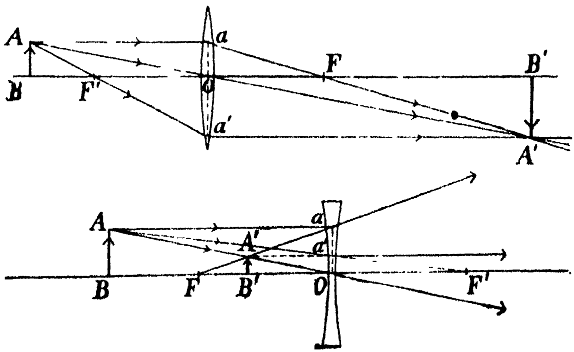


Fig. 105.

conjugate to corresponding points on the object. This relation may be proved geometrically in the following way. Let A B and A′ B′ (Fig. 105) represent respectively an

* The eye must be at not less than the distance of distinct vision from the image.

object and its image formed by the lens L. The construction for the image is identical with that explained above, with the addition of a ray from A through F′ which after refraction at the lens travels parallel to the principal axis.

From the triangles A′ B′ F and a O F we have—

$$\frac{A'B'}{aO} = \frac{B'F}{OF} \qquad (1) \quad (\text{Euc. vi. 4.})$$

Similarly, from the triangles A′ B′ O and A B O we have—

$$\frac{A'B'}{AB} = \frac{OB'}{OB}.$$

But—

$$AB = aO.$$

$$\therefore \frac{A'B'}{aO} = \frac{OB'}{OB}. \qquad (2)$$

Therefore, from (1) and (2) we have—

$$\frac{B'F}{OF} = \frac{OB'}{OB}.$$

If now, O B be denoted by u; O B′ by v; O F by f; and the usual sign convention be observed, we get—

$$\frac{f-v}{f} = \frac{v}{u}.$$

$$\therefore uf - uv = vf.$$

$$\therefore uf - vf = uv.$$

Therefore, dividing by ufv, we get—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

The variation of the position of the image with that of the object may be traced by the same method as that adopted in Art. 82 for conjugate foci.

The following cases should be noted:—

I.—Convex lenses.

1. A real object, at a distance from the lens greater than the focal length, has a real image, also at a distance greater than the focal length, and it is inverted. Fig. 103 illustrates this case; A B represents the object, and A′ B′

the image. The magnification is numerically greater than 1 if the distance of object from mirror < twice the focal length.*

2. If the object be real, and its distance be less than the focal length,* the image is on the same side of lens as

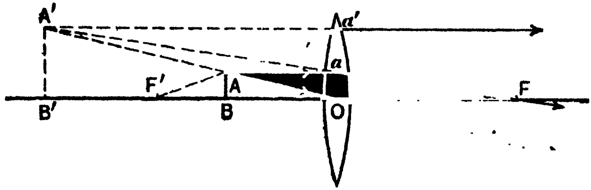


Fig. 106.

object, and is virtual, erect, and magnified. Fig. 106 shows this case.

3. If the object be virtual, the image lies between the lens and the object, and its distance from the lens is less than the focal length.* It is real, erect, and diminished.

II.—Concave lenses.

1. A real object always has a virtual erect image, nearer lens than the object and diminished, and always nearer the lens than its focus. (Fig. 104 illustrates this case.)

2. A virtual object, nearer the lens than the focal length, has a real, magnified, erect image, more distant than the object. (Fig. 104 shows this case if the direction and the rays be reversed.)

3. A virtual object further away than the focus on the negative side of the lens has a virtual inverted image further from the lens than the focus on the positive side.

I., 1, 2, 3 correspond respectively to II., 3, 2, 1. It should be noted that when the object is real, both lenses and mirrors form images which are erect when virtual, and inverted when real.

* In these cases the numerical value of the focal length is taken *i.e.*, the sign of f is not considered.

86. Relative size of image and object. The magnification produced by a lens is expressed by the ratio $\frac{\text{image}}{\text{object}}$. When the image is erect, the ratio is considered to be positive; when inverted, the ratio is taken as negative. In Fig 105, let A B represent the object and A′ B′ the image.

1. From the triangles A O B, A′ O B′ we have—

$$\frac{A'B'}{AB} = \frac{OB'}{OB} = \frac{v}{u}.$$

$$\therefore \frac{\text{Image}}{\text{Object}} = \frac{v}{u},$$

both sides of the equation being of negative sign in the upper figure.

2. From the triangles O F′ a', B F′ A we have—

$$\frac{a'O}{AB} = \frac{OF'}{BF'} = \frac{OF'}{OB - OF'}$$

but—

$$a'O = A'B',$$

$$\therefore \frac{A'B'}{AB} = \frac{OF'}{OB - OF'} = \frac{-f}{u+f}.$$

$$\therefore \frac{\text{Image}}{\text{Object}} = \frac{f}{u+f},$$

account being taken of the sign.

3. From the triangles F B′ A′, F O a, we obtain—

$$\frac{A'B'}{Oa} = \frac{FB'}{FO} = \frac{FO - B'O}{FO},$$

but—

$$Oa = AB,$$

$$\therefore \frac{A'B'}{AB} = \frac{FO - B'O}{FO} = \frac{f-v}{f};$$

$$\therefore \frac{\text{Image}}{\text{Object}} = -\frac{v-f}{f}.$$

Thus we have—

$$m = \frac{\textbf{Image}}{\textbf{Object}} = \frac{v}{u} = \frac{f}{u+f} = -\frac{v-f}{f}. \qquad (5)$$

These three relations may be very simply connected by the general relation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

by eliminating either u or v. To prove 2 from 1, v must be eliminated. Multiplying both sides of the equation by u we get—

$$\frac{u}{v} - 1 = \frac{u}{f}.$$

$$\therefore \frac{u}{v} = 1 + \frac{u}{f} = \frac{u+f}{f};$$

$$\therefore \frac{v}{u} = \frac{f}{u+f};$$

From this relation the relative sizes of image and object may be determined without finding the position of the image.

Considering the relation—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u}$$

the following results are readily deduced—*

1. If $v > u$, the image $>$ the object.
2. If $v = u$, the image $=$ the object.
3. If $v < u$, the image $<$ the object.

Applying those results to the general case I., 1, of Art. **85**, we get three particular cases, according as the image is *magnified equal to the object*, or *diminished*. In the second case, where the image is equal to the object, we have, for a *convex* lens, $v = -u$, and therefore in the formula—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we get—

$$-\frac{2}{u} = \frac{1}{f} \text{ or } u = -2f$$

That is, when the image formed by a convex lens is equal to the object, the distance of both image and object from the lens is equal to twice the focal length of the lens, and therefore the distance between the object and the image is equal to four times the focal length of the lens. It may also readily be proved that this distance is the minimum distance between object and image.*

It may also be noted from I., 1, that in order to produce a magnified image the object must be placed at a distance from the lens numerically greater than the focal length but less than twice the focal length of the lens.*

* Notice that in these paragraphs, as in many others where no confusion is likely to arise, on matters of sign, the sign convention of Art. 40 is not obeyed.

87. Exercise. To construct the rays formed by refraction in a lens by which an image is seen by the eye. If the object be near the principal axis the eye should be near the axis; it will then receive

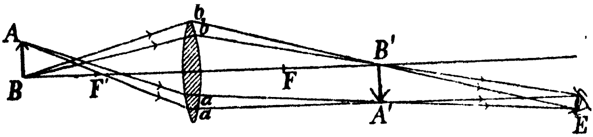


Fig. 107.

rays which have penetrated the lens. Fig. 107 illustrates the case of a convex lens yielding a real image. A B is the object, A′ B′ the image (found by Art. 84). To find the course of the rays by which A is seen, join A′ to the extremities of the pupil of the eye, then produce the lines so obtained backwards to cut the lens in *a a* and join *a a* to A. The rays by which A is seen are all included in the

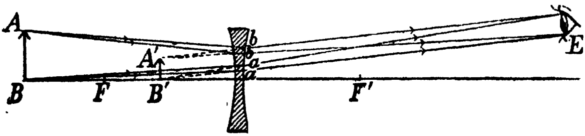


Fig. 108.

incident pencil A *a a*. Similarly for B. Fig. 106 illustrates the case of a virtual image formed by a convex lens, and Fig. 108 the case of a virtual image formed by a concave lens.

88. Combination of lenses in contact. Let two thin lenses of focal lengths f_1 and f_2 be placed in contact. The problem is to determine the focal length of a single lens which is optically equivalent to this combination (Fig. 109).

Imagine light from a point P, at a distance u from O, the centre of the combination,* to be incident first on the lens of focal length f_1. Then, *considering the action of this lens only*, the focus of the refracted pencil will be at a

* The thickness of the lenses is supposed to be so small, compared with the other distances involved, that the centre of the combination may be taken at any point in their combined thickness.

point, P′, at a distance v' from the lens, such that we have—

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1}. \qquad (1)$$

But this refracted pencil passes through the second lens, and after doing so is refracted through another point P″, at a distance v from the lens, such that—

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2}. \qquad (2)$$

The combined action of the lenses is thus to cause a pencil diverging from P, at a distance u from the centre, to be

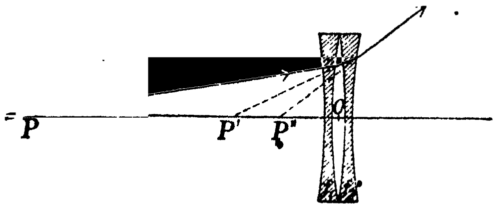


Fig. 109.

refracted through P″, at a distance v from the centre. Therefore, if F be the focal length of the combination, we have—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}. \qquad (3)$$

But by adding (1) and (2) we get—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}. \qquad (4)$$

Therefore, from (3) and (4) we get—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}.$$

$$\therefore F = \frac{f_1 f_2}{f_1 + f_2};$$

that is, a single lens of focal length $\frac{f_1 f_2}{f_1 + f_2}$ is optically equivalent to two thin lenses in contact, ~~and of~~ focal lengths f_1 and f_2. When the positions of equivalent lens and the combination are the same the image produced by the

equivalent lens is produced in the same place and is of the same size as that produced by the combination.

By extending the problem to a number of thin lenses in contact we obtain—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots \text{etc.} = \Sigma\frac{1}{f},$$

care being taken in actual work to give the proper signs to the numbers representing the several focal lengths.

89. Power of a lens. The reciprocal of the focal length of a lens expressed in metres is called the **power** of a lens. The unit of power is that possessed by a lens of one metre focal length and is called the dioptrie. The power of a converging lens is considered positive, and that of a diverging-lens negative, so that the sign of the power is the reverse of that of the focal length.* The theorem proved in the last article may thus be expressed: **The power of a combination of lenses in contact is equal to the sum of the powers of the constituent lenses.**

90. Spherical aberration by refraction. In discussing the formation of foci and images we have only considered

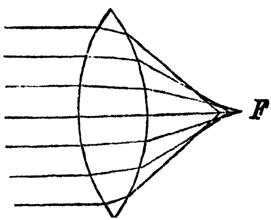


Fig. 110.

cases in which the curvature of the faces of the lens is small. If the faces be greatly curved, rays diverging from

* Sometimes—especially by opticians—a convex lens is taken as positive and a concave lens as negative, in defiance of Art. 80. Students must be on the lookout for this possible confusion; the context will sometimes reveal which kind of lens is meant.

any point in the object are not all brought together at one conjugate focus, but those rays which pass furthest from the centre of the lens have a focal distance shorter than that found by the rules we have learnt. This is shown in Fig. 110, where the marginal rays are seen to intersect before the central ones which cross at the principal focus F. This wandering of the intersections of the refracted rays from the focus is called *aberration*. (See also Art. 78.)

Exp. 16. To illustrate the different focal lengths of the central and marginal parts of a convex lens. Take a large convex lens of a short focus and, covering the centre of the lens with a large circle of black paper, use it to throw an image of a candle flame on a screen. Then, without altering the relative positions of candle, lens, and screen, replace the paper disc by a black paper ring which covers all the lens except a small central circle. The image of the flame will now be indistinct, and the screen will have to be moved further from the lens to make the image sharp.

Since spherical curves are always employed, all lenses are subject to this defect in some degree, but if the curvature be small, the defect is very slight. It may be reduced to any required degree by the use of diaphragms (Art. 46), which cover up more or less of the edge of the lens, exactly as in the case of curved mirrors, but obviously what is gained in definition by their use is lost in brilliancy.

The aberration may also be largely diminished by the use of a plano-convex lens, instead of one which is double convex. The curved surface must face the rays which are the more nearly parallel to the axis. Each ray is now very nearly equally deviated at the two surfaces, and higher mathematics proves that under these conditions the aberration is a minimum. Note how this fact is utilised in the construction of optical instruments (Figs. 185-188).

The aberration produced by a lens may also be greatly diminished by a suitable choice of the radii of curvature of the surfaces; for instance, in the case of crown glass, for which $\mu = 1 \cdot 5$, the aberration produced by a double-convex lens of given focal length is a minimum when the radius of the second surface is six times that of the first surface. Such a lens is called a *crossed lens*. Formulæ may be

obtained and used as a guide in this work, but in practice it is largely a question of repeated grinding and testing.

91. Caustics by refraction. An inspection of Fig. 110 will show that the intersection of the rays from different parts of the lens must give rise when the curvature is great to a luminous curved surface which is known as a *real caustic by refraction,* just as we found concave mirrors to produce real caustics by reflection.

The presence of a caustic surface affords the easiest method of illustrating spherical aberration to an audience. If the region to the right-hand side of the lens in Fig. 110 is filled with smoke the caustic surface is rendered very apparent.

CALCULATIONS.

92. THE following relations, obtained in the preceding chapter, may again be noticed:—

1. Prisms.

(1) $$\mu = \frac{\sin \frac{1}{2}(D + A)}{\sin \frac{1}{2} A}. \qquad \text{(Art. 74.)}$$

(2) $$D = \mu - 1) A. \qquad \text{(Art. 74.)}$$

Formula (2) is approximately true only when A is small, and is rigorously true only when A is infinitely small. It should, therefore, not be used in calculations except when A is small; for example, less than 10°.

2. Lenses.

(3) $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right). \qquad \text{(Art. 82.)}$$

(4) $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}. \qquad \text{(Art. 82.)}$$

(5) $$\frac{\text{Image}}{\text{Object}} = \frac{v}{u}. \qquad \text{(Art. 86.)}$$
$$= \frac{f}{u + f}.$$

In the above formulæ all distances are measured from the centre of the lens, and the usual sign convention is adopted; that is, distances measured from the centre of the lens, in a direction **opposed** to the incident **light** are considered **positive**, and distances measured in the **same direction** as the **incident light** are considered **negative**. In accordance with this convention the **focal length** (f) of a **convex lens** will be **negative**, and that of a **concave lens positive**.

In applying the formulæ the rules given in Art. 40 must be attended to. Of these (1) and (2) are so important, and their neglect so often leads to mistakes, that we shall again deal with them in their relation to the formulæ here considered.

(1) On substituting in any formula a numerical value for any of the symbols, the sign of the former must always be attached. For example, take the formula—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

If the image of an object, placed 20 cm. from a lens, be formed at a point 40 cm. *on the other side of the lens*, then, to find f, we have—

$$u = 20, \quad v = -40;$$

and
$$\therefore \frac{1}{-40} - \frac{1}{20} = \frac{1}{f};$$

and
$$f = -\frac{40}{3} = -13\tfrac{1}{3} \text{ cm.}$$

that is the lens is **convex**, and its focal length is $13\frac{1}{3}$ cm.

(2) In applying a formula to determine one of the involved distances, *no sign must be given to the unknown distance.* Thus, in the example given above, no sign is at first given to f, but the result, when worked out, shows it to be negative.

In applying formulæ (5*a*) and (5*b*), which express the relative size of image and object, the question of sign should be carefully attended to, for the interpretation of the result is simple and important. In these formulæ, a *positive* result indicates that the image is *virtual* and *erect*; for the image and object are then on the same side of the lens. Similarly, a *negative* result indicates that the image is *real* and *inverted*, the image and object being then on opposite sides of the lens. (See Art. 85.)

EXAMPLES V.

1. The refracting angle of a prism is 60°, and the minimum deviation produced in a pencil of monochromatic light is 40°. Find the refractive index of the prism, given that sin 50° = ·766.

Here, applying—

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2} A},$$

we get—
$$\mu = \frac{\sin \frac{1}{2}(60 + 40)}{\sin \frac{1}{2}(60)} = \frac{\sin 50}{\sin 30} = \frac{\cdot 766}{\frac{1}{2}} = 1{\cdot}53.$$

2. Find the focal length of a double-concave lens, the radii of curvature of its faces being respectively 25 cm. and 50 cm., and the refractive index of its material being 1·5.

Here, in formula (3)-

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right),$$

we have, supposing the light to be incident on the more concave face—

$$\mu = 1{\cdot}5, \quad r = 25 \text{ cm.}, \quad s = -50 \text{ cm.}$$

$$\therefore \frac{1}{f} = (1{\cdot}5 - 1)\left(\frac{1}{25} + \frac{1}{50}\right)$$

$$= \frac{1}{2} \times \frac{3}{50} = \frac{3}{100}$$

$$\therefore f = 33\tfrac{1}{3} \text{ cm.}$$

If we suppose the light to be incident on the other face we get the same result; thus, as before—

$$\mu = 1{\cdot}5, \quad r = 50 \text{ cm.}, \quad s = -25 \text{ cm.}$$

$$\therefore \frac{1}{f} = (1{\cdot}5 - 1)\left(\frac{1}{50} + \frac{1}{25}\right) = \frac{3}{100}.$$

3. An object is placed 12 inches from a convex lens of 8 inches focal length. Find the position and nature of the image.

Here, in formula (4),

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

we have u = 12 inches, f = − 8 inches (*convex* lens), and v is required—

$$\therefore \frac{1}{v} - \frac{1}{12} = \frac{1}{-8}.$$

$$\therefore \frac{1}{v} = -\frac{1}{8} + \frac{1}{12} = -\frac{1}{24}.$$

$$\therefore v = -24 \text{ inches}.$$

that is, the image is 24 inches on the other side of the lens.

Again, applying (5*a*), we have—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u} = \frac{-24}{12} = -2;$$

that is, the image is twice the size of the object, and is *real* and *inverted*.

4. An object, 3 cm. long, is placed 10 cm. from a concave lens of 20 cm. focal length. Find the size and nature of the image. Here, from (5*b*) we get—

$$\frac{\text{Image}}{\text{Object}} = \frac{f}{u+f} = \frac{20}{10+20} = \frac{2}{3}.$$

$$\therefore \frac{\text{Length of image}}{3 \text{ cm.}} = \frac{2}{3}.$$

$\therefore$ Length of image = 2 cm., and the image is *virtual* and *erect.*

A more usual, but less direct method of working this question is, first to determine v, and then to determine the size and nature of the image from formula (5*a*).

5. A concave lens whose focal length is 12 inches is placed on the axis of a concave mirror of 12 inches radius at a distance of 6 inches from the mirror. An object is so placed that light from it passes through the lens, is reflected from the mirror, again passes through the lens, and forms an inverted image coincident with the object itself. Where must the object be placed?

[In problems such as this, where by reflection and refraction the image of the object is made to coincide with the object itself, the solution is easy, if we remember that rays diverging from a point in the object, *on the principal axis*, return to the same point, and therefore travel to and fro by the same paths. But, if a ray, after reflection at a mirror, return along its incident path, it follows that it must be travelling *along a normal to the mirror.*]

In this case we know that, after the first refraction through the lens, the rays of the refracted pencil—originally diverging from a point in the object on the principal axis—are normal to the mirror, and therefore diverge from its centre of curvature. To find the position of the object we have therefore only to find a point on the principal axis such, that rays diverging from this point appear, after refraction through the lens, to diverge from the centre of curvature of the mirror.

Hence, in the formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we have—

$v = 6$ inches, $f = 12$ inches, and u is unknown.

$$\therefore \quad \frac{1}{6} - \frac{1}{u} = \frac{1}{12}.$$

$$\therefore \quad u = 12 \text{ inches.}$$

That is, the object must be placed 12 inches from the lens on the side remote from the mirror.

6. When a luminous point is placed on the principal axis of a convex lens (A) and at a distance a from it an image is formed 10 inches from the lens on the other side, if a second lens (B) is placed close to A, the image is 15 inches off. Determine the focal length of the lens B, and state whether it is concave or convex.

The action of the lens B is evidently to cause a pencil of rays, originally converging to a point P, 10 inches behind the lenses, to become less convergent, and to converge to a point P′, 15 inches behind the lenses. Thus P and P′ are conjugate foci with respect to the lens B, P′ being the image of P, and therefore, in the formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

we have $u = -10$ inches, $v = -15$ inches, and f is unknown.

$$\therefore \frac{1}{f} = -\frac{1}{15} - \frac{1}{-10} = -\frac{1}{15} + \frac{1}{10} = \frac{1}{30}.$$

$$\therefore f = 30;$$

that is, the lens is *concave*, and its focal length is 30 inches.

7. Show that if the angle of a prism be greater than twice the critical angle for the medium of which it is composed, no ray can pass through it.

8. The angle of a prism is 60°, and the refractive index of its material $\sqrt{2}$. Show that the minimum deviation is 30°.

9. A glass prism of refracting angle 5° is immersed in water; find the approximate deviation produced in a ray of light for which the absolute refractive indices of glass and water are respectively $\frac{3}{2}$ and $\frac{4}{3}$.

10. The minimum deviation produced by a hollow prism, filled with a certain liquid, is 30°; if the refracting angle of the prism is 60°, what is the index of refraction of the liquid?

11. Show that when a ray of light is refracted through a prism, in the position of minimum deviation, the course of the ray in the prism is perpendicular to the line bisecting the angle of the prism.

12. In order to determine the refractive index of a double convex lens, the radii of curvature of its surfaces were measured and found to be 30 cm. and 31 cm. respectively. Its focal length was also determined, and found to be 30·5 cm. Find the refractive index of the glass.

13. Find the focal length of a plano-convex lens, given that the radius of curvature of its convex surface is 50 cm., and that the refractive index of its material is 1·6.

14. Prove that the focal length of a plano-concave glass lens is equal to twice the radius of the concave surface. ($\mu = \frac{3}{2}$.)

15. A gas flame is at a distance of 6 feet from a wall. Where must a convex lens, of 1 ft. focal length, be placed in order to give a distinct image of the flame on the wall? Prove your result.

16. An object, 1 inch long, is placed at a distance of 1 foot from a convex lens of 10 inches focal length. Find the nature and size of the image.

17. If an object, 10 cm. from a convex lens, has its image magnified four times, what is the focal length of the lens?

18. An object is at a distance of 3 inches from a convex lens of 10 inches focal length. Find the nature and position of the image.

19. An object is placed 6 inches from a lens, and an image, three times as large, is seen on the same side of the lens as the object. Find the focal length of the lens.

20. A convex lens of 10 inches focal length is combined with a concave lens of 6 inches focal length. Find the focal length of the combination.

21. Find the focal length of a lens which is equivalent to two thin convex lenses of focal lengths 20 cm. and 30 cm. placed in contact.

22. A convex lens of focal length 12 cm. is placed in contact with a concave lens, and the focal length of the combination is found to be -24 cm. Calculate the focal length of the concave lens.

23. A convex lens of 3 inches focal length is used to read the graduations of a scale, and is placed so as to magnify them three times. Show how to find at what distance from the scale it is held, the eye being close up to the lens.

24. The image formed by a convex lens is n times the size of the object: Show that the distance of the object from the lens is $-\frac{n+1}{n}f$.

25. A candle flame is placed 6 inches from a plane mirror, and a convex lens, of 3 inches focal length, is placed between the candle and the mirror, and 2 inches from the latter. Find the position of the image.

26. A candle flame is placed 20 cm. from a plane mirror. Find where a convex lens of 5 cm. focal length must be placed in order that the image of the flame may coincide with the flame itself.

27. On a sheet of paper placed vertically is written a capital L. If an observer stand 3 feet in front of the paper and hold a double-convex lens of 6 inches focal length halfway between his eye and the lens he will see an image of the letter. Draw a picture of the image as seen, and state whether it is larger or smaller than the object.

28. A convex lens is focussed on a mark on a sheet of paper; a thick plate of glass is then put between the paper and the lens, and it is found that the mark can no longer be distinctly seen. Explain this, and illustrate by a diagram the path of the ray in the two cases.

29. A convex lens of 6 inches focal length is employed to read the graduations of a scale, and is held so as to magnify them three times. Find how far it is held from the scale.

30. Show by accurate measured drawings the paths of rays of light, incident at 30° from the normal, on (a) a thick plate of glass with parallel sides, (b) a glass prism at 60°.

31. A ray of light falls normally on the middle of one face of a glass prism whose section is an equilateral triangle. Show by a measured drawing the whole path of the ray.

32. An object 3 inches high is placed successively at distances of 45, 20, 18, and 8 inches from a convex lens of 10 in. focal length. Calculate in each case the position and height of its image, state whether it is real or virtual, erect or inverted, and give rough sketches showing the paths of the rays in each case.

This method is only applicable, and indeed it need only be used, for convex lenses when the focal length is large and E can be placed

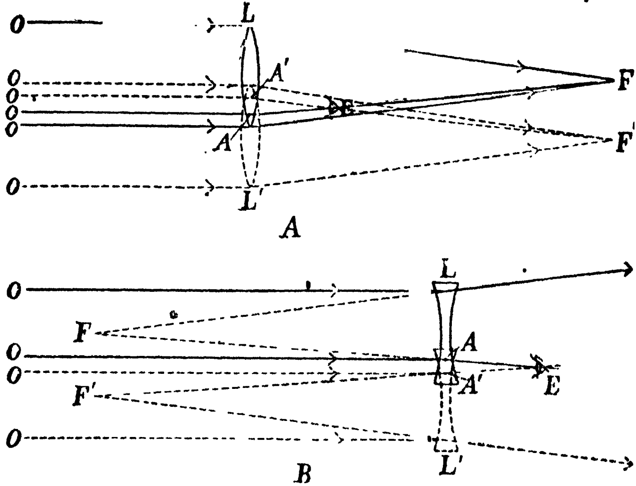


Fig 111.

between the lens and its principal focus. If the real image of an object be viewed through a convex lens, it will be found that lens and image move in the same direction.

94. Determination of the focal length of a lens. The experimental determination of focal length is of great importance. The methods adopted depend upon the nature of the lens, and upon the degree of accuracy required. We shall consider a few of the simpler approximate methods for each class of lenses.

I.—Convex lenses.

1. The simplest method of determining the focal length of a convex lens is to allow a beam of parallel light to be incident on the lens, in a direction parallel to the principal axis, and then to measure the distance of the focus of the refracted pencil from the lens.

Exp. 19. For this purpose mount the lens in a suitable stand or clip, with its axis parallel to a graduated wooden rod, along which

the stand slides. At one end of this bar, and at right angles to its length, fix a white cardboard screen with its centre approximately on the same level as the principal axis of the lens. Point this arrangement towards the sun or some other well-defined distant object, and adjust the position of the lens by the method of oscillations (Exp. 1) until a clearly defined image of the sun, or other object chosen, is formed on the screen. The distance between the lens and the screen, as indicated by the graduations on the rod, gives the required focal length.

2. This method is based on the fact that if a beam of parallel rays leaves the lens, the source of the rays must be at the principal focus.

Exp. 20. Focus a telescope T (Fig. 112) on a distant object. Fix the lens L to the telescope, in front of, and coaxial with, the object glass. Now place a sheet, P, of printed matter in front of the lens,

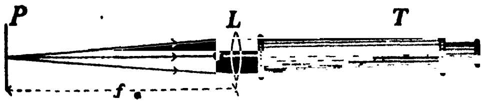


Fig. 112.

and move it to and fro along the line of sight until the print is clearly seen by an eye looking through the telescope. Since the telescope is set for parallel rays, P must be at the principal focus of L.

3. This method is a combination of methods (1) and (2). A source is placed at the principal focus of a lens. The parallel beam resulting on refraction is reflected by means of a plane mirror, and re-traversing the lens is brought to a focus alongside the object.

Exp. 21. Take a simple form of optical bench, lamp, and object as described in Art. 47 and Exp. 22; place the lens L (Fig. 113) in a clip, and close behind it place a plane mirror M. Move the lens and mirror about until the rays returned by reflection at the mirror form an image of the gauze on the screen by the side of the gauze itself.* Employ the method of oscillations (Exp. 1) in order to get the best position. Measure O L; it is the focal length, for, since

* An image may be produced by reflection from the lens surfaces (Art. 95, II., 2). In the case under consideration the right image disappears if the mirror is taken away.

the rays return along their old paths, they must have been *directly* incident on M; and have therefore left L as a parallel beam. It will be necessary to slightly tilt M to one side in order that the image

Fig. 113.

may be observed apart from the object. This method is perhaps the simplest method, and is especially useful in the case of long-focus "convex" lenses. This and nearly all the following experiments should be carried out in a darkened room.

4. This method is an application of the formula—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

Exp. 22. Mount the lens in one of the uprights of the optical bench, with its axis parallel to the length of the bench. In two other uprights, placed one on each side of the lens, fix a lighted candle and a screen, the flame and the centre of the screen being adjusted to the level of the principal axis of the lens. By properly adjusting the positions of these two uprights, relative to that carrying the lens, focus a sharply defined image* of the flame on the screen. Determine the distances v and u by noting on the scale of the bench the distances between the screen and the lens and between the candle flame and the lens. Substitute these values in the above equation, taking care that v is represented by a negative number. These measurements should be made for several different values of u, and the mean of the results taken as the mean value of f.

Instead of the candle flame it is better to employ a small, sharply defined object brightly illuminated by a suitably placed light. A piece of glass with a scale etched on it, two fine wires stretched across a hole in a piece of cardboard, or a piece of wire gauze answer the purpose extremely well. Fig. 114 shows a simple form of bench fitted up for this experiment. A is the box (Art. 47) containing the lamp—a small electric incandescent lamp is the most convenient,—

* The image should be as little coloured as possible. (See Art. 108.)

the gauze is mounted on the cardboard, B, by stamp paper. C is the lens carried by an adjustable clip kept tight by an elastic band, D is the screen of white cardboard upon which the image is received; in

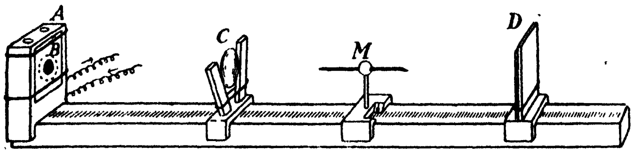


Fig. 114.

some experiments a plane mirror is affixed to D by means of an elastic band.

M is a metal measuring rod of known length, provided with pointed ends. When measuring distances its ends are brought into contact with the different surfaces, and readings are taken from a fiducial mark cut upon its wooden base.

If, in the general formula above, we replace u by d_1, v by $-d_2$, and f by $-f_1$, where d_1, d_2, f_1 are simply the numerical values of u, v and f, we obtain the formula

$$\frac{1}{d_1}+\frac{1}{d_2}=\frac{1}{f_1},$$

which is simpler for purposes of calculation; the arithmetic is still further simplified if tables of reciprocals are employed. Plot the results of the experiment on squared paper as in Art. 48 (4). Take

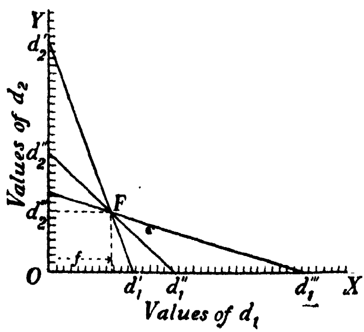


Fig. 115.

values of d_1 along OX and values of d_2 along OY, (Fig. 115). Join up corresponding points. All these lines should intersect one another at the point whose co-ordinates are equal to the focal length.

5. The displacement method. Let A and B (Fig. 116) represent respectively the positions of a bright object and a screen. Then, if a *magnified* image of the object A be formed on the screen at B by a lens placed at C, a *diminished* image can also be obtained on the screen by placing

Fig. 116.

the lens at a point C′ such that B C′ = A C, for, if A C and B C are conjugate focal distances, then the *equal* distances A C′ and B C′ are also conjugate. Let A B be denoted by l and C C′ by a, then if A C and C′ B be each denoted by d_1, and C B and A C′ be each denoted by d_2, we have—

$$A\,B = A\,C + C\,B,$$

$$\therefore \quad l = d_1 + d_2; \qquad (1)$$

and—

$$C\,C' = A\,C' - A\,C,$$

$$\therefore \quad a = d_2 - d_1. \qquad (2)$$

Therefore from (1) and (2) we get—

$$d_1 = \frac{l-a}{2}, \text{ and } d_2 = \frac{l+a}{2};$$

and therefore by the formula of Page 182,

$$\frac{2}{l-a} + \frac{2}{l+a} = \frac{1}{f_1}.$$

$$\therefore f_1 = \frac{l^2 - a^2}{4\,l}. \qquad (3)$$

Hence, by measuring l and a, f may, by application of this formula, be readily determined. This method does not involve any error due to inexact knowledge of the position of the centre of the lens.

Exp. 23. Using an optical bench find the focal length of a convex lens by this method. Measure with a pair of dividers and a fine scale, the dimensions of the object (either the diameter of the circle in B (Fig. 114) or the width of a convenient number of wires of the gauze) and image in each case. Note that the image

is as much magnified for one position of the lens as it is minified for the other position. The proof of this is simple. Denote corresponding dimensions of object and the two images by $0, I_1, I_2$. Then when lens is at C, $\frac{I_1}{0} = \frac{d_2}{d_1}$, and when lens is at C′, $\frac{I_2}{0} = \frac{d_1}{d_2}$. Therefore,

$$\frac{I_1 I_2}{0^2} = \frac{d_2}{d_1} \cdot \frac{d_1}{d_2} = 1; \textit{i.e.}, 0 = \sqrt{I_1 . I_2}.$$

Hence if I_1 and I_2 are measured, 0 can be calculated. This method proves very useful if the object cannot be directly measured.

A particular case of this method is applied in Silbermann's

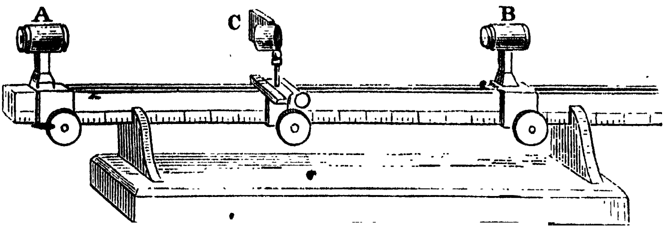


Fig. 117.

Focometer (Fig. 117). If a, in formula (3) above, becomes zero, we have, neglecting sign—

$$f = \frac{l^2}{4l} = \frac{l}{4}.$$

When this is the case the points C and C′ in Fig. 116 are evidently coincident, and we have A C = C B. That is, image and object are equidistant from the lens, and are therefore equal in size (Art. 86). This is the fact made use of in this instrument, which consists of a fixed scale carrying three slides, A, B, and C. Mounted in tubes on A and B are two glass scales photographed from the same negative, the graduations being uncovered and facing the lens. The slide C carries the lens whose focal length is to be determined. The positions of these slides must be adjusted until the image of the scale in A is seen to coincide exactly with that in B. It will then be found that A and B are equidistant from C, and the distance A B read off on the scale, gives l, from which f (= $l/4$) is easily calculated.

6. The magnification method. This method is applicable to thick lenses and combinations of lenses (*e.g.*, the photographic lens), as well as to thin lenses, hence its importance. Let a lens, focal length f, at a distance u_1 from the object produce an image, magnification m_1, at a distance v_1. Also let the same lens and object at a distance u_2 produce an image of magnification m_2 at a distance v_2.

Then $$\frac{1}{f}=\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{v_2}-\frac{1}{u_2},\ m_1=\frac{v_1}{u_1},\ m_2=\frac{v_2}{u_2},$$

$$\therefore\quad \frac{v_1}{f}=1-\frac{v_1}{u_1}=1-m_1;\ i.e.,\ v_1=(1-m_1)f.$$

Similarly $$v_2=(1-m_2)f,$$

$$\therefore\quad v_1-v_2=f(m_2-m_1);\ i.e.,\ f=\frac{v_1-v_2}{m_2-m_1}.$$

The apparatus of Fig. 117 is very useful for this method.

Exp. 24. Find the focal length of a thick convex lens. Use as an object some compass points, the legs being opened so that the points are some exact distance apart, say 1 cm. or ½-inch. Focus a magnified image of the points on a finely divided scale, and note the number (n_1) of scale divisions bridged by the tips. Now move the scale towards the lens through a measured distance (d, say 1 inch for a short-focus to 6 or 9 inches for a long-focus lens. Adjust the position of the compass points (the lens must not be moved) until an image of them again rests on the scale Note the number (n_2) of scale divisions bridged by them now. Then find, by direct application, the number (n) of divisions of the scale bridged by the compass points themselves. In the first position the magnification $m_1=n_1/n$, in the second $m_2=n_2/n$. Focal length $=d/(m_2-m_1)$.

II.—Concave lenses.

We have already seen that with a concave lens the image of a real object is virtual, and so cannot be received upon a screen. This renders it difficult to determine the focal length of a concave lens,* but the following methods may be adopted with fairly accurate results:—

1. One face of the concave lens is covered with a circular piece of black paper, through which two large pinholes have been made, on a diameter of the circle, at points equidistant from the centre. A beam of parallel light is then directed on the lens in a direction parallel to the principal axis. All the incident rays except those passing through

the pinholes are stopped by the black paper, and if a screen be placed behind the lens, two bright spots will be formed upon it at the point where the rays passing through the pinholes meet its surface.

Let a and b (Fig. 118) represent the position of the pinholes, then, the incident light being parallel, the rays

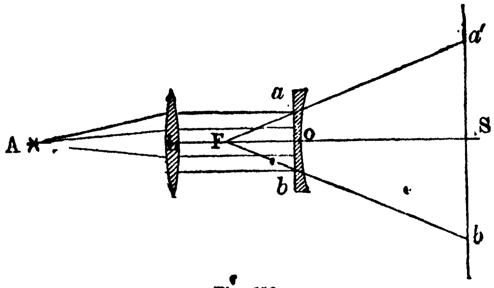


Fig. 118.

refracted through at a and b diverge from the focus F, and bright spots of light are formed, at a' and b', on a screen placed at any point S, behind the lens.

From the figure we have—

$$\frac{a\,b}{a'b'} = \frac{\mathrm{F\,O}}{\mathrm{F\,S}} = \frac{\mathrm{F\,O}}{\mathrm{F\,O}+\mathrm{O\,S}}.$$

Therefore, if the focal length of the lens be denoted by f, we get—

$$\frac{a\,b}{a'b'} = \frac{f}{f+\mathrm{O\,S}}.$$

From this relation f can be determined when $a\,b$, $a'\,b'$, and O S are known.

Exp. 25. Place the source of light, A, at the focus of the convex lens, and then place the concave lens and screen in position. Measure $a\,b$ and $a'\,b'$ with a pair of compasses and a fine scale and read off the distance O S from the bench. Only very rough results can be obtained by this method.

2. Let P (Fig. 119) denote the position of the object, and P′ the position of the image formed on the screen at S by the convex lens L. If the *concave* lens be placed at L′ in

such a position that L′ P′ is less than its focal length, then the rays converging to P′ become less convergent and thus meet at a more distant point P″ (Art. 85, II., 2). If the screen be placed at S′ an image of the object at P is formed on it; this image may be considered as the image

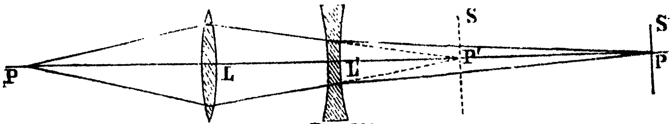


Fig. 119.

of a virtual object P′, and if L′ P′ and L′ P″ be measured the focal length of the concave lens may be calculated from the relation—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

Exp. 26. Mount the lens L on the optical bench and locate P′ by means of a screen S. Note its position. Mount L′ on a stand so as to be the same level as L and place it in position. The image is now thrown back to P″. Locate it by the screen as before. If L′ P″ is very long, there will be a considerable range for which the image is approximately in focus. To overcome this difficulty place L′ nearer P″; P″ is now considerably nearer P′.

3. If L′ (Fig. 119) is so placed that L′ P′ is equal to its focal length, the rays leaving L′ will be parallel to one another, and may be made to retrace their paths by means of a plane mirror placed at right angles to them.

Exp. 27. Place a plane mirror M (Fig. 120) behind L′ and move the lens along the bench until an image of the gauze at P is formed

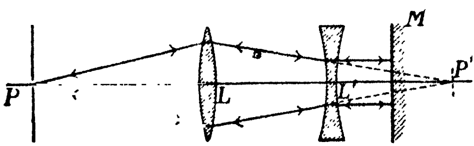


Fig. 120.

alongside P. Note the position of L′ and then remove it and M The rays now come to a focus at P′; P′ L′ is the focal length of the concave lens.

4. It has been shown in Art. 88 that if two thin lenses of focal lengths f_1 and f_2 be placed in contact so as to act as one compound lens, then the focal length, F, of the combination is given by the relation—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}.$$

If a concave lens, of focal length f_1, be combined in this way with a convex lens of *shorter* focal length, f_2, the combination is evidently equivalent to a convex lens, and its focal length, F, may be determined by either of the methods given above. Similarly f_2 may be determined, and the required focal length f_1 may then be calculated from the relation just given. Care must be taken over the signs.

95. Radius of curvature of a mirror or surface of a lens.

I. **Concave** surfaces.

The concave surface of a lens may be regarded as a concave mirror, and its radius determined as in Art. 48.

II. **Convex** surfaces.

Art. 48 gives two methods; the following are also in general use—

1. If a convex surface is so placed in a converging beam that the focus of the beam is the centre of curvature of the surface, the rays are reflected back upon themselves and retrace their former paths.

Exp. 28. (1) Place the surface to be measured (M, Fig. 121) behind a convex lens, L, of short focal length, and adjust their positions until a sharp image of the gauze (Art. 47) is formed on the screen by

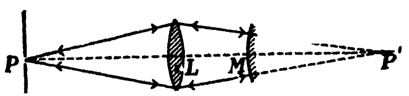


Fig. 121.

rays returned after reflection at the convex surface. Note the position of M, then remove it; the rays now travel on and an image of P is formed at P′. Since the rays were incident directly on M they were converging to its centre of curvature, hence M P′ is the radius of curvature. This method may also be used to find the radius of the convex surface of a lens; the second surface will not interfere.

2. This method is only applicable to the convex surfaces of lenses, but for them it is the simplest and best method, and is easily performed at the same time as the determination of the focal length (by Art. 94, I., 3).

Exp. 29. Mount the lens L (Fig. 122) in the clip, with the surface under experiment turned away from the illuminated gauze. Some light is reflected from the surfaces of the lens. Starting with the lens up close to the gauze gradually withdraw it, until one of these

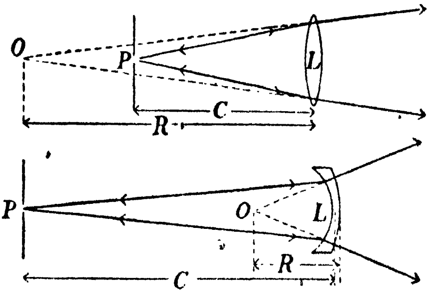


Fig. 122.

reflected beams throws an image of the gauze back on the gauze itself. If necessary, tilt the lens slightly to one side, in order to throw the image on the screen and thus allow of its being clearly seen. Find the exact position of the lens by the method of oscillations (Exp. 1). Measure the distance L P from lens to screen, and denote it by c.

Since the rays of light originally emergent from P return along their original paths, it is clear that they must be incident normally on the back surface of L, and hence those rays which penetrate L diverge from O, the centre of curvature of this surface. Therefore P and O are conjugate foci; and denoting the focal length of the lens by f, and the radius of curvature of the back surface by R, we have—

$$\frac{1}{R} - \frac{1}{c} = \frac{1}{f}.$$

$$\therefore\ R = \frac{cf}{c+f}.$$

R and c are of course positive, but f is positive or negative according as the lens is concave or convex.

The method is applicable to the surfaces of double-convex lenses, concavo-convex lenses,* and to convexo-concave lenses,† in which the radius of curvature of the convex surface is numerically smaller than the focal length of the lens.

96. Determination of the refractive index of the glass composing a lens. The focal length and radii of the two surfaces being found by the experiments described above, μ can be found from the formula—

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

in which r is the radius of the surface upon which the light first falls. Great care must be taken over the signs.

97. Experimental illustration of the deviation produced by a prism. In Arts. 75, 76, the phenomenon of deviation of light by a prism was illustrated by the displacement of the *virtual* image of a slit seen through the prism. We can, however, illustrate deviation by noticing the displacement of a *real* image, obtained by means of a lens and prism. The apparatus described below may be employed for this purpose, and will also serve to give rough measurements of the deviation produced.

Exp. 30. A small truly circular table, about 30 cm. in diameter, is fitted round its edge with a strip of thin tough paper, in such a way that the upper edge of the strip projects about 5 cm. above the face of the table. On this projecting edge, about half-way up, a scale, showing degrees, is marked all round the strip. At the 180th

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* In performing Exp. 29 with a concavo-convex lens, the first focussed image of the gauze on the screen B (Fig. 114), as the lens is gradually withdrawn from the illuminated gauze, is due to reflection at the back surface of the lens and gives c. As the lens is still further withdrawn another image is focussed on the screen. This image is formed by reflection at the concave surface of the lens, and the distance of the lens from the screen gives the radius of that surface. This is Exp. 8.

† In performing Exp. 29 with a convexo-concave lens, the distance of the first image—as the lens is gradually withdrawn—gives the radius of the concave surface, the distance of the second image gives c.

division on the scale, a narrow vertical slit, S (Fig. 123), about 2 cm. long and ·5 mm. wide, is so cut in the paper that one edge accurately coincides with this division. Take the apparatus into a dark room and illuminate the slit by a properly shaded sodium flame. Fix a mounted convex lens, L (f_1 = 5 cm., about), on the table, between the slit and the centre, in such a position that a clearly defined image of the slit, having one edge coincident with the zero on the scale, is obtained on the paper strip at S′.* At the centre of the table fix a small stand T T, which can be rotated round a central vertical axis. On this stand place a prism so that its edge is vertical, and the plane bisecting its refracting angle passes through the axis of rotation of

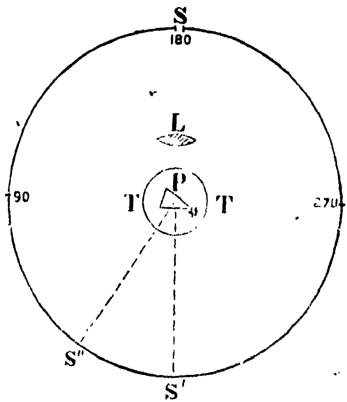

Fig. 123.

the stand. Rotate the stand until the rays of light coming from the slit, through the lens, are refracted through the prism; observe that the position of the image of the slit is changed, and that the change of position indicates that the rays are deviated by the prism in a direction away from its refracting edge. Note also that as the rotation continues the position of the image changes, indicating that the magnitude of the deviation produced depends upon the position of the prism relative to the incident light. If for any position of the prism, the image is formed at S″, then the magnitude of the deviation is measured by the angle S′ P S″, which may be at once read off on the scale. Now rotate the prism so as to cause the deviation to diminish. As the prism is rotated, always in the same

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* If the lens be not employed the light leaves P in diverging pencils and an indefinite image is formed. With the lens in the correct position the pencils converge as in Figs. 124, 127, and form a real image on the circular scale.

direction, the image will travel at a gradually decreasing rate towards S′, and at a certain point will become stationary, and then turn back in the opposite direction. The deviation at the instant at which the image is stationary is the *minimum deviation*, which can thus be determined by noting, on the scale, the division at which the image ceases to advance towards S′ and begins to turn back. The image obtained on the scale, after the interposition of the prism, is not clearly defined except at, and near, the position of minimum deviation, and consequently measurements made near this position will be more correct than for other positions. Accurate measurements of deviation are made by means of the *spectrometer*. (Art. 165.)

98. Exp. 31. Determination of the angle of a prism. Take the simple apparatus of the last Article, and having adjusted L in position, place P on the stand so that the refracting edge is over the axis of the rotation of the stand, and the faces O R, O T are

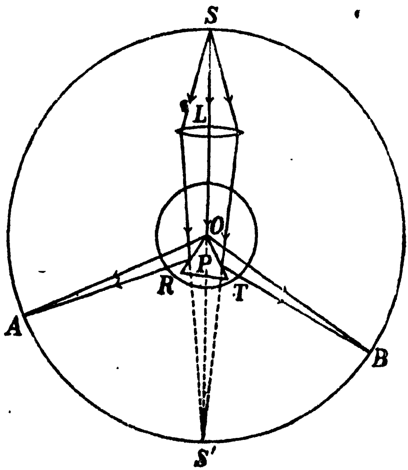

Fig. 124.

nearly symmetrical about S O (Fig. 124). The converging beam of light falling on P is now reflected in two portions, and the reflected beams converge to points A and B on the scale. Read off on the scale the magnitude of the angle A O B. It is twice that of the angle of the prism, for by Art. 30 the

$$\angle AOS' = 2 \angle ROS'$$
$$\angle BOS' = 2 \angle TOS';$$

Hence $$\angle AOB = 2 \angle ROT.$$

99. Determination of the refractive index of the substance of a prism. Having now determined the refracting angle of the prism and the minimum angle of deviation, we have all the data necessary for the calculation of the refractive index of the material of the prism for sodium light. For if A is the refracting angle, and D the minimum angle of deviation,

$$\mu = \frac{\sin \frac{D + A}{2}}{\sin \frac{A}{2}} \qquad \text{(Art. 74).}$$

The refractive index of liquids can be measured by enclosing them in hollow prisms whose walls are made of thin parallel plates of glass suitably cemented together at their edges.

The refractive index of metals has been measured by using very acute-angled prisms made on glass by the electric discharge or by chemical decomposition.

EXAMPLES VI.

1. In an experiment made to determine the focal length of a convex lens by the method of Art. **94**, I. 4, the following corresponding values of d_1, d_2 were observed:

d_1	cm. 52·5	cm. 38·1	cm. 32·5	cm. 28
d_2	30·5	41·4	51·9	69·5

Find by a graphical method the focal length of the lens.

2. An object and screen were fixed on an optical bench at a distance apart of 94·1 cm. Between them a convex lens was moved about, and in two positions, 71·3 cm. apart, images were formed on the screen. Find the focal length of the lens.

3. A contact-combination of a convex and a concave lens had a focal length of −19·3 cm. The focal length of the convex lens being −10·02 cm., find that of the concave lens.

4. A convex lens was used to form a real image of an object. Between this image and the lens, and at a distance of 29·25 cm. in front of the image, a concave lens was placed, and it was found that the image was thrown back 12·25 cm. Find the focal length of the concave lens.

5. A convex lens yielded a real image of a piece of illuminated gauze at a distance of 150 cm. from itself. At 50 cm. behind the lens was placed a concave lens, and behind this a plane mirror, and it was found that an image of the gauze was thrown back on the gauze itself. Find the focal length of the concave lens.

6. A convex lens yielded a real image of a piece of illuminated gauze at a distance of 40·38 cm. from itself. An equi-convex lens was placed behind this lens, and it was found that an image of the gauze was found alongside the gauze itself when the second lens was 29·68 cm. behind the first. Draw the diagram, and find the radii of curvature of the surfaces of the second lens.

Given that the focal length of this lens is $-10 \cdot 1$ cm., find the refractive index of the glass of which it is composed.

7. A long-focus equi-convex lens was fixed in a clip on an optical bench. A plane mirror was also mounted on a stand, as in Fig. 113. The base-pieces of the two stands could be moved together. At the start they were placed close up to the illuminated gauze, with the mirror facing the gauze and the lens between the gauze and the mirror. They were then gradually withdrawn together. At distances from gauze to lens of 57·2 and 112·1 cm. images of the gauze were thrown on the screen alongside the gauze itself. At the latter distance, when the mirror was withdrawn, the image disappeared. Find from these data the focal length of the lens, the radii of curvature of its surfaces, and the refractive index of the glass.

8. The same experiment was repeated with a convex meniscus. The concave surface was placed facing the gauze, and at distances from gauze to lens equal to 8·9, 35·2, and 35·3 cm. images were thrown back on the screen. At the last distance the image disappeared when the mirror was withdrawn. From the above data find the focal length of the lens, the radii of curvature of the two surfaces, and the refractive index of the glass.

9. A hollow glass prism of refracting angle 39° 33′ was filled with water and set on the prism table of a simple spectroscope (Art. 97). The angle of minimum deviation for sodium light was found to be 13° 57′. Find the refractive index of water.

10. A similar experiment was performed with carbon bisulphide. Find its refractive index from the following data: Refracting angle of prism, 40° 24′; angle of minimum deviation, 28°.

EXAMINATION QUESTIONS.

Questions Set at Various University Examinations.

1. What is meant by the Focal length of a Convex Lens? Show how to find it (1) by aid of the sun, (2) by an artificial flame.

2. What is the focal length of a lens, and how would you determine it experimentally? In the case of a convex lens, if the object be as near as possible to the image, where must the lens be?

3. Show how to determine by a geometrical construction the diameter of the sun's image formed by a convex lens of 6 feet focal length, assuming that the sun's diameter subtends, as seen from the earth, an angle of $\frac{1}{2}^{\circ}$. How does the brightness of the image depend upon the size of the lens and its focal length?

4. How would you determine the focal length of a convex lens if sunlight were not available?

5. What is the focal length of a lens? How may the focal length of a concave lens be determined?

6. A candle is placed at a fixed distance opposite a wall. A convex lens, held between the candle and the wall, throws on the wall a well-defined magnified image of the candle flame when it is 1 foot from the candle, and a well-defined diminished image when it is 11 feet from the candle. Find the focal length of the lens.

7. How is the focal length of a convex lens best determined without the aid of sunlight?

An object is placed 8 inches from the centre of a convex lens, and its image is found 24 inches from the centre of the other side of the lens. If the object were placed 4 inches from the centre of the lens, where would the image be?

8. A small object is placed close to a thick plate-glass mirror. An eye near the mirror observes not one image of the object only, but a great number. Explain the formation of these. Which of them is the brightest, and why?

9. A small gas flame is placed on the axis of a lens distant from it 120 cm. By means of a ground-glass screen an inverted image of the gas flame is found on the farther side of the lens 200 cm. from it. What is the nature of the lens, and what is its focal length?

10. Explain how to determine the focal length of a convex lens. If an object at a distance of 3 inches from a lens has its image magnified three times, find the focal length of the lens.

11. Two thin lenses have each a focal length of 1 inch. Draw as carefully as you can to scale, the path of a beam of light from a distant object (1) when the lenses are in contact, (2) when they are separated by $1\frac{1}{2}$ inches, (3) when they are separated by 3 inches.

12. A bright point is situated on one wall of a room 9 feet wide. A convex lens, 1 ft. focal length and 2 inches in diameter, is placed 6 feet from the wall in the normal from the point. What is the width of the circle of light thrown by the lens on to the opposite wall?

13. Draw a curve showing the relation between the distance of an object and that of its image as measured from a lens, as the distance of the object is progressively varied. Take, for simplicity, the case of a convex lens of negligible thickness.

14. Give a careful sketch of the arrangement of a lamp, slit, lens, and scale, by means of which the image of the slit formed by the lens and reflected by a plane mirror may be thrown on to the scale. Trace in your sketch the course of a pencil of the rays.

15. Draw a curve showing in the case of a convex lens the connection between the distance of the object from one principal focus and the distance of the image from the other.

16. How can convex lenses be distinguished from concave lenses by the appearance of objects seen through them.

17. An object is 20 feet from a screen. Given two convex lenses respectively of 9 inches and 18 inches focal length, explain how you will obtain (1) an erect and magnified, (2) an inverted and magnified image of the object on the screen.

18. How would you determine experimentally the focal length of a convex lens in such a way as to avoid an error arising from the thickness of the lens, your source of light being a paraffin lamp?

19. Describe fully the method you would adopt in order to determine the index of refraction of a transparent prism for sodium light.

CHAPTER IX.

DISPERSION.

100. Homogeneous and compound light. We have seen that there is reason to believe that the physical cause of light is a species of transverse vibratory motion in the ether. When this motion is made up of a series of waves, all of the same wave-length, then the light is said to be *homogeneous* or *monochromatic*. It is, however, more generally the case that the wave motion is made up of an infinite number of waves of different wave-length. The light is then said to be *non-homogeneous* or *compound*.

Monochromatic light is of a definite colour, corresponding to its wave-length, and difference in wave-length is always indicated by a difference in colour. Compound light may also be of all shades of colour, or may be *white* or colourless, but its colour is no indication of its composition; two lights of almost identical colour may be made up of very different constituents, and may even exactly match the colour of any of the monochromatic lights. Thus the colour of light is not a reliable indication of its composition. In the case of white light, however, we can always state that it is compound, for all monochromatic lights are coloured; but, without experiment, we cannot state what may be the constituents of any given source of white light. Solar light and the other white lights with which we are most familiar—for example, gaslight, lamplight, electric light—are very similar in composition, and include all possible shades of monochromatic light. The reason of this is evident when we remember that white light of this nature always results from incandescence, and that an incandescent or white-hot body has passed through all the phases of change of colour attendant on rise of temperature. It is, therefore, giving out light of all wave-lengths, from the dark red which first appeared when it began to get red-hot, to the violet which was added when it first appeared to be perfectly white.

101. Newton's Experiment. The light coming from the sun was first shown by Newton (1676) to be of a composite character.* The experiment by which this fact was demonstrated is known as Newton's Experiment, and is worthy of special notice, both on account of the historical interest attached to it, and because of the great importance of the fact which it illustrates.

In its simplest form Newton's Experiment may be performed in the following way. A beam of sunlight is

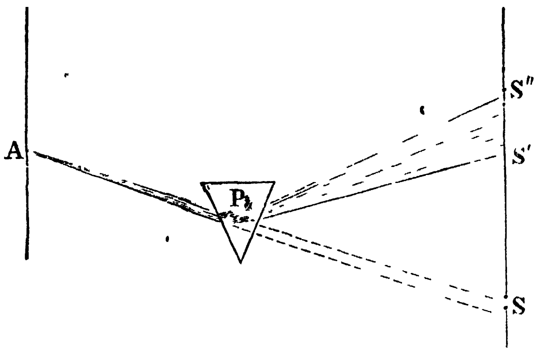

Fig. 125.

admitted into a dark room through a small circular aperture, A, in a shutter or blind. The beam will be seen in the room as a small pencil of light diverging from A (Fig. 125), and, if allowed to fall on a vertical screen at

* The prevailing opinion before Newton's time was that the prism actually made the coloured lights out of the white light. Newton showed, or he thought he did, that white light really contained the coloured constituents, and that his prism merely sorted them out. More recent research, however, inclines to the theory that white light does not consist of a number of regular trains of waves, but is made up of irregular disturbances or pulses. The prism resolves these into simple components upon which it impresses different periodicities, which components therefore advance in different directions determined by their periodicities and wave-lengths. Newton's theory is therefore not as correct as the older one, if that could be called a theory. **Since, however, the new theory has not yet been decisively established Newton's theory will be adhered to in this book.**

S, it forms a small elliptical bright spot, which is a rough image of the sun. If now a prism, P, with its edge horizontal be placed, edge downwards, in the path of the beam, the latter will be deviated from its original course, and deflected upwards so as to form an image at S S″. This image differs from that first formed at S in several important particulars; the vertical diameter is much longer, and, instead of appearing as a bright patch on the screen, it is made up of several coloured bands, arranged horizontally. In fact, the image is made up of several overlapping images, similar in shape to that originally seen at S, but each of a different colour. This shows that the beam of *white light* incident on the prism is, on refraction through the prism, separated into its different coloured constituents, each of which forms its own image on the screen, and thus the many-coloured compound image at S′ S″ is formed. Such compound images are called *spectra*. When a *spectrum* is formed by decomposition of the solar light, as in the case we have just considered, it is called the *solar spectrum*, and on a first analysis may be taken as made up of the six colours—red, orange, yellow, green, blue, and violet.* Of these the red rays are the least deviated, and therefore appear at S′, the bottom of the image S′ S″. The violet rays are the most deviated, and are therefore found at S′, the top of the same image. The intermediate rays are arranged in the order given above, from below upwards, between S′ and S″.

The student must beware of thinking that the spectrum can be divided into six distinct blocks of different colours, and that solar light is made up of only six different constituents, corresponding to the six colours. This is not the case; the number of constituents making up solar light are *infinite*, but, considered with reference to their action on the eye, they may be divided into six sets, each of which corresponds to a definite *colour sensation*, and comprises an infinite number of rays, each corresponding to a certain *shade* of the colour which characterises the set to which it belongs.

* Newton described *seven* principal colours, including indigo between blue and violet, and others have followed him; but no such colour is to be seen by normal eyes in a pure spectrum, and if it were, indigo is only a kind of blue, and there is no more reason to subdivide the blue than the green or any other colour.

102. Refrangibility. When the rays of a compound beam of light are refracted through a prism, as in Newton's Experiment, each constituent suffers refraction to a different extent. This is sometimes expressed by saying that the constituents of the compound beam are of different *refrangibilities*; the most *refrangible* rays are those which undergo the greatest deviation, while the least *refrangible* are those which suffer least deviation. In the *solar spectrum* the red rays are the least refrangible, while the violet rays are the most refrangible. The intermediate rays increase in refrangibility as we pass from red through orange, yellow, green, and blue to violet.

From what has been said above, it will be seen that difference in refrangibility corresponds to difference in wave-length. Light of high refrangibility is of short wave-length, and the corresponding index of refraction for any given medium is relatively high, while light of low refrangibility is of long wave-length, and the corresponding index of refraction for any given medium is relatively low.

This different refrangibility is due to the different frequency of the various coloured rays. Colour, in fact, in light, corresponds with pitch in sound. The violet waves are the shortest, the red the longest. A violet light, therefore, corresponds to a high note, a red light to a low note.

103. Pure spectrum. The spectrum obtained by the method of Newton's Experiment is indistinct and badly defined because of the overlapping of the images of which it is composed. Such a spectrum is said to be *impure*. To obtain a *pure* spectrum a very narrow slit must take the place of the aperture in the shutter, and some means must be adopted to obtain a spectrum made up of a series of adjacent but not overlapping images of this slit. In Fig. 126 let *s* denote the position of the slit. The pencil of rays, diverging from *s*, forms a broad band *a b* on the screen S S'. If now the prism P be interposed *in the position of minimum deviation*, with its edge parallel to the length of the slit, the rays of the pencil are deviated and dispersed in such a way

that the red light appears to come from r', a virtual image of the slit s, and forms a band $r\,r$ on the screen.* Similarly the violet light seems to come from v' and gives the violet band $v\,v$; and so on, for each colour of the spectrum. It is

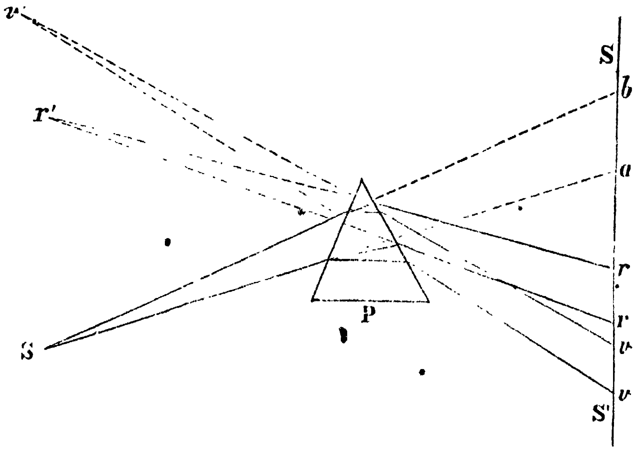

Fig. 126.

evident that the spectrum thus contained on the screen is composed of a series of overlapping bands, and is therefore not *pure.*

If, however, a suitable lens, L, be placed, as shown in Fig. 127, so as to give, when P is removed, a distinct image of the slit at a, and the prism, P, be then interposed, in the position of minimum deviation for the mean rays, the pencil of rays converging to a, will, after refraction through the prism, be dispersed and give rise to a series of pencils converging to the points r, o, y, g, b, v. Real images of the

* In general, oblique refraction does not produce a point image of a point source (Arts. 62 and 75), but in the special case when the prism is symmetrically placed with respect to the incident and emergent rays, the emergent pencil does diverge from a point, and hence we can consider r' and v' as definite images.

slit are thus formed at these points of light of each colour, and, as each image is narrow and distinct, like that at *a*, there is no overlapping and a *pure* spectrum is obtained. If the slit itself is not sufficiently narrow, the images may

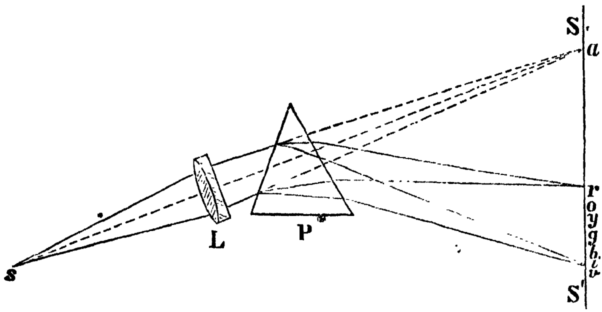

Fig. 127.

be broad enough to overlap and thus give an impure spectrum. Instead of placing the lens at L (Fig. 127), it may be placed on the other side of the prism in such a position (Fig. 128) that real images of the virtual foci lying between r' and v' (Fig. 126) are formed on the screen.

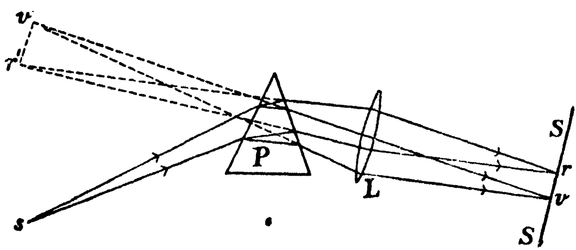

Fig. 128.

A third method of obtaining a real, pure spectrum is described in Chap. XII. under the spectroscope (Art. 164).

It is thus evident that to obtain a real, pure spectrum we must have—

1. A very narrow slit.

2. The prism in the position of minimum deviation for the mean rays and therefore approximately for all rays.

3. A lens, so placed as to form a *clearly defined* spectrum on the screen.

The second condition is of great importance, for it is only when the prism is in the position of minimum deviation that clearly defined images can be obtained. In practice it will be found convenient to illuminate the slit by means of a lamp, and instead of the large screen shown in the figure, to employ a smaller one, placed first at a to receive the direct image of s, and then at r . . v to receive the spectrum. The prism is placed in the position of minimum deviation by rotating it until the position of the spectrum is as near as possible to a (Art. 97).

It should here be noticed that the above arrangement of apparatus is necessary to obtain a *real* pure spectrum, which can be received upon a screen. A *virtual*, pure spectrum can be seen by merely looking through a prism at a narrow slit. An eye placed near the prism, so as to receive the emergent pencil, sees a small but very bright and pure spectrum at the virtual foci of the different constituents of the pencil entering the eye. The violet end of this spectrum is seen nearest the refracting edge, because the violet rays are most refracted. This is evident from Fig. 128, the lens and screen there shown being replaced by the lens of the eye and the retina. The red end of the spectrum is seen at r' and the violet end at v'.

Exp. 32. 1. Cut a narrow slit (about one or two millimetres in width) in a piece of cardboard or metal foil, and place it vertically in front of a bright white light, such as the sun, a bright cloud, an electric arc, a good lamp flame, or an incandescent mantle. Take a glass prism (a glass lustre from a chandelier will do) and stand it in a vertical position in the path of the light issuing through the slit. At some distance behind place a white screen. Note the spectrum formed, its deviation, and the fact that the violet end is deviated more than the red end, and that the centre is very nearly white. Set the prism in the position of minimum deviation.

2. Close behind the prism stand another similar prism also in the position of minimum deviation. Note that the deviation of the spectrum is increased and that it is further drawn out.

3. Take this second prism and now place it horizontally with its

refracting edge downwards, close behind the first prism. Note now that the spectrum is elevated, and the violet end more so than the red end so that the spectrum is now on the slant. This is Newton's celebrated *crossed-prisms* experiment.

4. Repeat (1) but instead of using the screen place the eye to receive the emergent beam; the spectrum seen is nearly pure.

5. Take now a lens of about a foot in focal length and set up the prism and lens as in Figs. 127 or 128. A pure real spectrum is now formed on the screen. Note the purity of the colours compared with those produced in (1).

104. Dispersion. We have seen that when a beam of compound light is refracted through a prism, each constituent of the beam suffers deviation to a different degree. The light of shortest wave-length is deviated most, and that of longest wave-length least, and thus the different constituents of the incident beams are, as it were, separated, each travelling in a definite direction determined by the deviation it has experienced. This separation of the different constituents of a compound beam of light by refraction through a prism is called *dispersion*, and is measured, for any two rays of the refracted pencil, by the angle between these rays. Thus, in Fig. 129, if A B represent a ray incident on a prism at B, and split up by refraction through the prism into a pencil of rays bounded by B C D and B E F, then the angle D O F measures the dispersion for the extreme rays of the emergent pencil.

105. Dispersive power. The dispersive power of a medium is determined by the ratio of the *extreme* dispersion produced by a prism of small refracting angle of that medium to the *mean* deviation produced by the same prism, when a beam of white light is refracted through it, in the position of minimum deviation.

Thus, in Fig. 129, the dispersion is measured by the angle D O F and the *mean* deviation by A′ O′ H; therefore, in accordance with our definition,* we have—

$$\text{Dispersive power} = \frac{\text{D O F}}{\text{A}' \text{O}' \text{H}}.$$

* In Fig. 129 the angle of the prism is intentionally made large for the sake of clearness.

Now, the angle D O F is the difference between the deviations of the extreme rays A B C D and A B E F; therefore, if μ_v denote the index of refraction for A B E F, and μ_r the

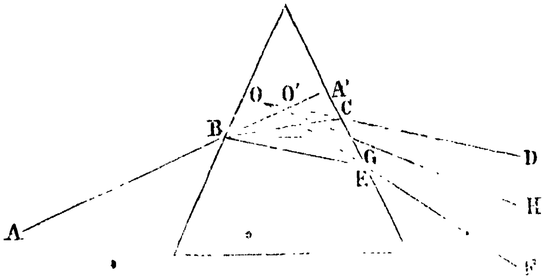

Fig. 129.

index of refraction for A B C D, the deviation for A B E F is approximately given for a prism of small angle, A, by—

$$D_v = (\mu_v - 1) A,$$

and for A B C D the deviation is given by—

$$D_r = (\mu_r - 1) A$$

Similarly, if μ denote the index of refraction for A B G H, the mean ray of the refracted pencil, then the mean deviation is given by—

$$D = (\mu - 1) A.$$

Thus we have—

$$D O F = D_v - D_r = (\mu_v - 1) A - (\mu_r - 1) A = (\mu_v - \mu_r) A,$$

and—

$$A' O' H = D = (\mu - 1) A,$$

and therefore—

$$\text{Dispersive power} = \frac{(\mu_v - \mu_r) A}{(\mu - 1) A} = \frac{\mu_v - \mu_r}{\mu - 1}.$$

Dispersive power is usually denoted by ω, so that the above relation is generally written—

$$\omega = \frac{\mu_v - \mu_r}{\mu - 1}.$$

This is regarded as the strict definition of ω. The definition first given is only approximate.

Experiment shows that in general the dispersive powers of different media are different. Water has little dispersive power, crown glass has about the same, flint glass has nearly twice as much as crown glass, and carbon bisulphide has still more, and therefore hollow prisms filled with this liquid are usually employed in spectrum work for lecture illustration.

In the following table the refractive indices of these substances are given for the definite red, greenish yellow, and violet lights corresponding respectively to the A, D, and H Fraunhofer Lines (Art. 111). The dispersive power is obtained from the formula—

$$\omega = \frac{\mu_H - \mu_A}{\mu_D - 1}$$

	μ_H	μ_D	μ_A	ω
Water (20°)	1·344	1·333	1·329	·045
Carbon Bisulphide (20°) ...	1·700	1·628	1·609	·143
Crown Glass (Heavy) ...	1·551	1·534	1·528	·043
Flint Glass (Heavy) ...	1·653	1·619	1·609	·073
Rock Salt (24°)	1·569	1·544	1·537	·059

106. Achromatic prisms. To realise the full significance of dispersion let us consider the following experiments:—

(1) Let two exactly similar prisms P and P′ (Fig. 130),

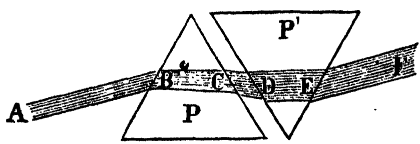

Fig. 130.

of the same material, be placed as shown in the figure, with their refracting edges turned in opposite directions and their adjacent faces parallel, and let a beam of solar light A B

be incident on P. On refraction through P, this beam is dispersed, and the refracted pencil, C D, after emergence from P, is incident on P′. Now P′, being exactly similar to P, but having its edge turned in opposite direction, will produce an equal and opposite effect to that produced by P; that is, the pencil, E F, after emergence from P′ will be parallel to A B, and the dispersion produced by P will be destroyed by P′, so that the beam E F will be, in all respects, exactly similar to A B. In fact, the action of the combined prisms is the same as a *plate* of the same medium. (Art. **53.**)

This experiment, which may be employed to illustrate the recomposition of white light, shows that whenever, by the action of two similar prisms *of the same material*, dispersion is destroyed, the deviation is also destroyed. Sir Isaac Newton, after some research in this direction, came to the conclusion that this result was true generally, whether the prisms were of the same material or not, and that it was impossible to obtain deviation without dispersion, or dispersion without deviation. This conclusion we now know to be wrong, for let us see what it leads to. Let d and D denote respectively the extreme dispersion and mean deviation produced by P, and d' and D′ the dispersion and deviation produced by P′, then if, on refraction through the combined prisms in the way explained above, the dispersion and deviation are simultaneously destroyed, we have—

$$d = d' \text{ and } D = D',$$

and therefore—

$$\frac{d}{D} = \frac{d'}{D'}, \quad i.e., \ \omega = \omega.$$

If, then, Newton's conclusion were true generally, it would mean that all media have the same dispersive power. Experiment has shown that this is not the case (Art. **105**).

(2). Let the prisms C, F, (Fig. 131), made of media of different dispersive powers, but of equal refracting angles, a, yield spectra $r\ v$, $r_1\ v_1$, and let D, D_1, be the middle points of these spectra. If F have the greater dispersive power, $r\ v$ is not as long nor as deviated as $r_1 v_1$. It is possible now to cut down the angle of F till its spectrum is of the same length as that of C. Let F_1 be the new

prism yielding the spectrum $r_2\,v_2$, equal in length to $r\,v$. Let $O_2\,D_2$ be now less than O D, hence, on combining C and F, as in (4), we get a combination which deviates, but does not disperse. The deviation

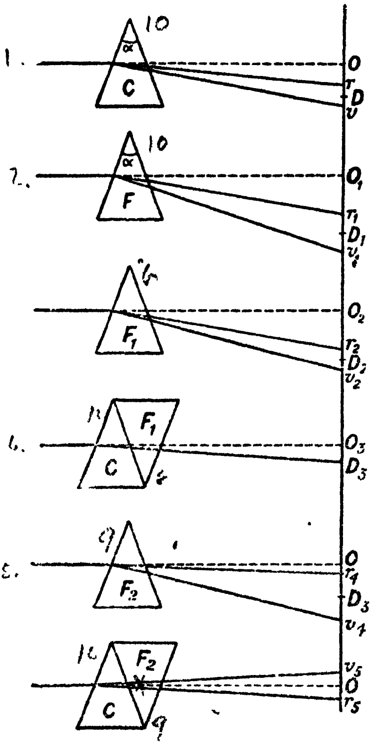

Fig. 181.

$$O_3\,D_3 = O\,D - O_2\,D_2.$$

This fact was discovered by Hall in 1730, and first used by Dollond, a London optician, in 1757.

Example. Suppose that C and F are made of crown and flint glass respectively, and a is small. The dispersion produced by C $= (\mu_H - \mu_A)\,a = \cdot 023\,a$ (See table, Art. 105). If β is the value of the angle of F_1, the dispersion which it produces $= (\mu_H' - A_A')\,\beta = \cdot 044\beta$. Since these dispersions are equal we have—

$$\beta = \frac{\cdot 023}{\cdot 044} = 0\cdot 52\,a.$$

Therefore as combined in (4) the deviation produced

$$= (\mu_D - 1)\,a - (\mu_D' - 1)\,\beta$$
$$= 0\cdot 534\,a - 0\cdot 619\,\beta$$
$$= 0\cdot 534\,a - 0\cdot 322\,a = 0\cdot 21\,a.$$

(3) Instead of cutting down the angle of F, as described above, we may cut it down until the mean deviation which it produces is equal to that produced by C. This gives us F_2, where $O_3\,D_3 = O\,D$; $r_4\,v_4$ is still greater than $r\,v$, and hence on combining F_2 with C we get (6), which disperses but does not deviate. The dispersion $r_5\,v_5 = r_4\,v_4 - rv$. That is, the beam of light after emergence from the combined prisms will continue approximately in its original

direction, but the beam itself will be dispersed, and, if allowed to fall on a screen, will show a spectrum. This is the principle of the *direct-vision spectroscope* (Art. **169**).

Example. Using the data supplied above, we have, if γ be the angle of F_2,

$$(\mu_D - 1)\,\alpha = (\mu_D' - 1)\,\gamma$$
$$0{\cdot}534\,\alpha = 0{\cdot}619\,\gamma$$
$$\therefore \gamma = \frac{0{\cdot}534}{0{\cdot}619} = {\cdot}86\,\alpha.$$

Therefore as combined in (5) the dispersion produced

$$= {\cdot}023\,\alpha - {\cdot}044\,\gamma$$
$$= {\cdot}023\,\alpha - {\cdot}038\,\alpha$$
$$= -{\cdot}015\,\alpha.$$

The negative sign shows that the dispersion is in an opposite direction to that produced by C.

107. Irrationality of dispersion. If the dispersive powers of two materials be calculated for several pairs of selected rays it will be found that the ratio of the dispersive powers varies with the rays selected. This *irrationality* is sometimes very apparent, some media compressing the red end of the spectrum and extending the violet end, others doing the reverse, and a few others giving spectra whose colours are not in the usual order. This last phenomenon is called *anomalous dispersion.* Hence in general if the spectra of two prisms be so arranged that the extreme rays are equally distant from each other, the intermediate rays of one spectrum will not exactly correspond in position with the intermediate rays of the other. Hence also when the dispersion is destroyed in a prism or lens combination for a given pair of rays, there is still left a residual dispersion of some colours; the coloured images which are formed by these residual rays are known as *secondary spectra.*

108. Dispersion in a lens. When a pencil of compound light is refracted through a lens, it suffers dispersion just as in refraction through a prism. Thus, if a diverging pencil of solar light, P a b (Fig. 132), be incident on the convex lens L, then the red rays, being the least refrangible, are brought to a focus at R, while the violet rays converge to a focus V nearer the lens. The orange, yellow, green, and blue rays converge to points intermediate between R and V, and thus, instead of the refracted rays all meeting in one focus, the rays of each colour converge to their own focus,

and the image formed on a screen placed anywhere near V or R will be coloured at its edges. If the screen be placed anywhere near V, between the line *v r* and the lens, then the outer edge of the image will be red, but, if placed beyond *v r*, then the outer edge shows violet. This fact is taken advantage of in focussing an image on a screen. The

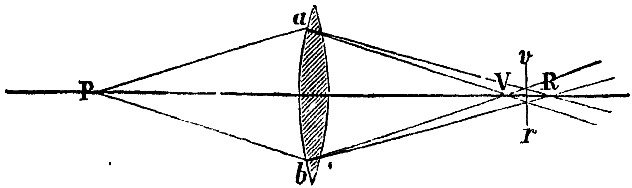

Fig. 182.

points V and R are very close together, and the best *definition* of the image is obtained when the screen is at *v r*. This adjustment is readily made by gradually changing the position of the screen until the colour showing at the outer edge of the image changes from red to violet. When this change of colour takes place the screen is in the position indicated by the line *v r*.

This effect of the dispersion of light when refracted through a lens is called **chromatic aberration** and was a great source of trouble in the construction of optical instruments, until it was shown that it was possible to obtain deviation without dispersion; that is, that it was possible to make the rays converge to a focus without obtaining a coloured image. This result, as in the case of prisms, is achieved by combining two lenses of different dispersive powers, and such that the chromatic aberrations which they singly produce are equal and opposite. For example, if a convex lens of crown glass (focal length, 20 cm.) be combined with a concave lens of flint glass (focal length, 34 cm.), the combination is equivalent to a convex lens of about 49 cm. focal length, and the image produced by it is almost entirely free from all colour defects. Such a combination is said to be *achromatic*. This subject will be dealt with more fully in a later chapter (Art. 160).

109. The prismatic spectrum. The *prismatic* spectrum is the spectrum obtained by the decomposition of white light on refraction through a prism.

All radiant waves are capable of refraction and dispersion, and thus, when a beam of white light is refracted through a prism, the emergent beam is made up of a series of rays, separated and arranged in order of *continuously* increasing refrangibility. Beginning at the less refrangible end of the spectrum determined by this emergent beam, and travelling in the direction of increasing refrangibility, we pass a group of rays known as the *dark heat rays* which do not excite the sensation of sight Then we come to another group, the rays of the *visible spectrum* ranging through the colours red, orange, yellow, green, blue, and violet. This group of

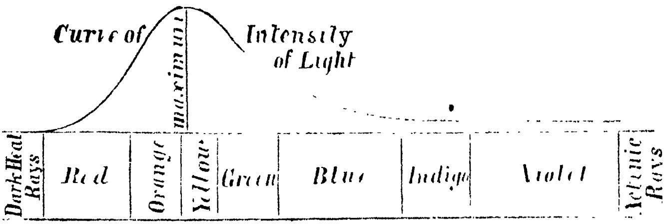

Fig. 133.

rays, in addition to possessing heating properties, has the peculiar property of exciting the optic nerve, and thus producing the sensation of sight. In this visible spectrum the intensity of the light is different in different parts, being a maximum in the yellow and gradually diminishing on both sides towards the red and violet, as shown by the ordinates of the *intensity curve* of Fig. 133. Beyond the visible spectrum we come to the *dark chemical rays* or *actinic rays*. These extend for a considerable distance beyond the violet, and are characterised by their power of producing chemical action in a certain class of substances.

The complete spectrum may thus, at first, be regarded as made up of *dark heat, luminous*, and *actinic rays*. The only essential difference between these is that of wave-length, which *continuously* decreases from the first to the last.

110. Spectra of gases or vapours. If a small quantity of a substance such as strontium chloride or cupric chloride be brought into the non-luminous flame of a Bunsen burner, the salt will, owing to the high temperature, be vaporised and the flame coloured, crimson in the case of the strontium chloride and green in the case of the cupric chloride. If now this light be examined by a prism as in Art. **103** * it will be found that the image on the screen does not show every variety of colour, nor does it show merely the red or green parts respectively. It is indeed quite distinct from the spectrum already described, and consists of a number of lines (images of the slit through which the light is admitted) whose positions are for the same substance invariable. The positions of the chief lines for strontium

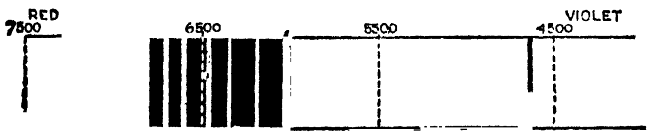

Fig. 134.

chloride are given in Fig. 134. The numbers supply the scale of wave-length in terms of "tenth metres," *i.e.*, 10^{-10} metre, or in ten millionths of a millimetre.

It is also found that each element when vaporised in the flame gives one or more (often a large number) of such lines, each and all characteristic for the given element, so that no two lines occupy the same position. In Fig. 135 the position and relative strength of the lines for the metals sodium, lithium, potassium, calcium, and thallium are given, as observed when a volatile salt of these metals is introduced in the Bunsen flame. For most other metals higher temperatures are required, and in these cases induction sparks are caused to pass between the poles of the metal in question.

* For accurate work a spectrometer (Art. 165) should be employed.

To obtain the spectrum of a gas the gas is introduced into a dumbbell-shaped tube (Fig. 196), which is then sealed up and partially exhausted. The induction discharge * is then passed between two terminals sealed into the ends of the tube, when the gas becomes incandescent. The light in the central portion is examined as before. If the pressure is large, the spectrum is continuous. As the pressure is

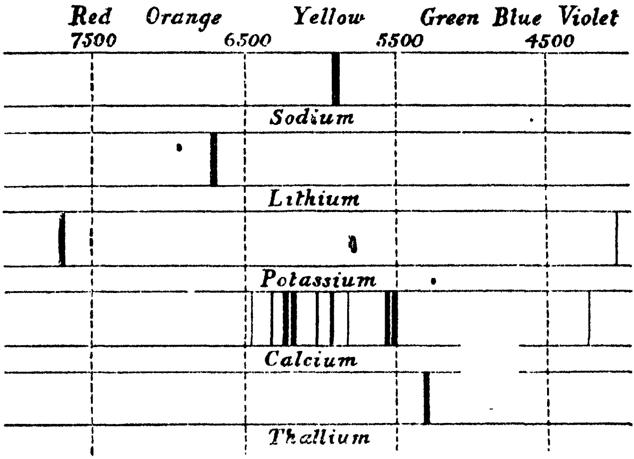

Fig. 185.

decreased the spectrum becomes discontinuous, and finally narrows down to a series of bright bands or lines, whose position is characteristic for each gas.

It is often possible to recognise the presence of a substance by means of the colour which it imparts to the flame, though when several substances are present together one may mask the other. Sodium salts, for instance, give a bright yellow colour to the flame, which will completely mask the violet tinge given by potassium salts. But if the light from a mixture of substances be passed through a

* See "Higher Textbook of Magnetism and Electricity," Arts. 58 and 59.

prism, the lines of each appear in their proper places, and it is possible to recognise each and all of them by measuring the position of the lines which are visible.

Exp. 33. Place a little calcium chloride in a watch-glass, and moisten it with hydrochloric acid. Dip a clean platinum wire into the pasty mass, and then hold the wire in the hot part of the Bunsen flame. Note the red coloration imparted to the flame. Repeat with the following metals: sodium (yellow), lithium (rose), potassium (violet), barium (apple green), strontium (crimson), copper (bluish or emerald green), thallium (green). Art. 166 deals further with this subject.

111. The Solar Spectrum. If the slit be illuminated by sunlight a bright and apparently continuous spectrum will be thrown upon the screen, but with a sufficiently narrow slit it will be found to be crossed by a number of dark * lines, some well defined and easily seen, others extremely thin, and only visible after careful focussing. These lines were first observed by Wollaston, but Fraunhofer in 1804 was the first to accurately map their positions in the spectrum, hence they are usually called **Fraunhofer Lines.** In position, and in distinctness also, these lines correspond in almost every case to the bright lines already spoken of as obtained from one or other of the elementary bodies.

To fully understand the formation of these dark lines consider the following experiment:—

An intensely white-hot substance is obtained and its spectrum thrown upon the screen. Between it and the prism is now placed a Bunsen flame into which a piece of common salt is inserted. The sodium flame by itself emits light of a greenish-yellow colour whose wave-length is approximately 5893 tenth-metres. When, however, both sources are in action the continuous spectrum due to the white-hot surface is found to be crossed in the yellow by a well-defined dark line at the same place as had previously existed the bright yellow sodium line. The inference is that the amount of light proceeding from the sodium vapour is relatively so small that it may be neglected, and the only effect we have to consider is that which the presence of the vapour may have upon the rays proceeding from the white-hot body. It follows from thermodynamic considerations that, if the white

* The light is probably not absolutely wanting, but so faint as to appear dark by contrast.

source be hotter than the vapour in the Bunsen flame, the ether waves proceeding from the white-hot body will pass readily through the sodium vapour, *except those vibrations whose wave-length corresponds to those of the screen of vapour.* These are in a large measure absorbed or quenched in the screen and the dark line results. In the same way if the layer of vapour contains also lithium there will be an absorption at wave-length 6705 corresponding to the position of the lithium line, and so on for each substance whose vapour is present.

In the case of the sun the white-hot (3500° C.) radiating surface is the body of the sun itself, the *photosphere*, and the absorbing layer is an envelope or atmosphere of the cooler vapours emitted from the body of the sun, termed the *chromosphere.* We shall under such circumstances have the spectrum of white light interspersed with dark lines corresponding to all the substances so present in this layer of vapour. This is the explanation, due to Kirchhoff (1859), of the existence of dark lines in the sun's spectrum, and the coincidence of these lines in position with those given by various terrestrial elements convinces us of the existence in the sun of a large number of the elements identical with those which form part of the earth's crust.

Fig. 136 shows the positions of the best-defined Fraunhofer lines. There are three well-marked lines A, B,

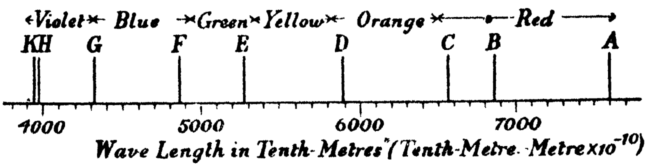

Fig. 136.

and C, in the red; one, D, in the orange; another, E, in the green; another, F, in the blue; another, G, on the borderland of the blue and violet, and two lines, H and K, in the extreme violet.

Of these A and B are due to absorption by oxygen in our own atmosphere, the rest are caused by absorption in the chromosphere as follows: C and F by hydrogen; D by sodium vapour; E, H, and K by calcium vapour; and G by the vapour of iron.

112. The invisible parts of the spectrum. (1) *The actinic or ultra-violet rays.*—If an ordinary photographic plate be exposed in the prismatic spectrum it will be found on development that the red light has had little effect upon the plate, which is, however, strongly affected by the blue and violet light * and also to a large extent by the rays beyond the violet, these *ultra-violet* rays being particularly able to decompose the sensitive silver salt (silver bromide), though not able to excite the sensation of vision. Glass prisms and lenses absorb ultra-violet light to a great extent, but if quartz prisms and lenses be used it will be found that the ultra-violet portion of the spectrum of an incandescent body extends to an enormous extent beyond the violet end of the visible spectrum, and when solar light is employed it will be found to be crossed by dark lines, just as the visible spectrum is crossed by the Fraunhofer lines. Its extent may also, to some degree, be measured by means of a fluorescent body (Art. 115).

Ultra-violet light has intense chemical and electrical effects. The ultra-violet constituents of the light from an electric arc or spark falling on a negatively charged zinc plate causes it to rapidly lose its charge, and the radiation from a mercury arc-lamp, which is very strongly ultra-violet, rapidly charges the air through which it passes with ozone. Note also the Finsen Cure for Lupus. It is, however, of interest to note that the decomposition of the atmospheric carbon dioxide which takes place in the leaf cells of plants with the liberation of oxygen is mainly effected by yellowish-green light.

(2) *The dark heat or infra-red rays.*—In 1810 Herschel found that as he moved a small thermometer through the solar spectrum from violet to red, it showed only a little rise of temperature in the blue end of the spectrum, a little more in the green, a large rise in the red end, and even for some distance below the red end the thermometer was very sensibly heated. Since then Langley has done much work

* In ordinary photographic work the inability of the ordinary plate to record colours in their correct luminosities is a serious drawback in all work other than that of mere black-and-white. Special plates called orthochromatic have, however, been manufactured which, in conjunction with a specially prepared coloured screen, nearly reproduce the correct luminosities.

on the infra-red portion of the spectrum. As glass absorbs these dark heat rays, he used prisms and lenses made of rock salt or fluorspar instead, and replaced the thermometer by a lamp-blacked linear thermopile * or bolometer.† Lampblack absorbs all the radiation which falls on it, hence the energy measured at any point is the *total* energy sent to that portion of the spectrum.

If solar light be used the position of maximum energy is found within the visible spectrum, but if the electric arc or incandescent lamp be used the maximum is found some distance down in the infra-red, the distance being greater the lower the temperature of the source.‡ This is well shown by Fig. 137, due to Langley, where the

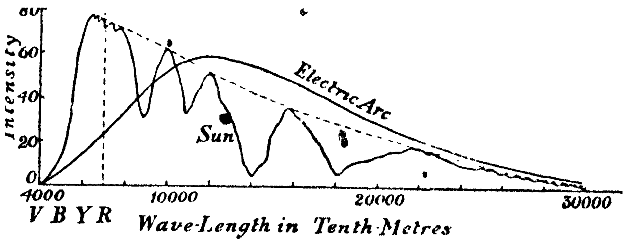

Fig. 137.

energy spectrum of solar light is given by the irregular curve and that of the light emitted from the electric arc-lamp by the smooth curve. The extent of the visible spectrum is V B Y R. The big depressions in the former curve are due to absorption by the atmospheres of the sun and earth. If no absorption had occurred the solar spectrum would probably have followed the dotted line.

Exp. 34. Send an electric current of gradually increasing strength through a platinum wire in a dark room. The wire soon becomes just perceptibly warm to the touch, then too hot for the hand to bear, and soon so hot that the heat radiated from it may be felt at a distance of several inches. But it is still invisible. All the rays yet emitted are obscure rays. After a time, as the temperature increases,

* See "Higher Textbook of Heat," § 97; "Higher Textbook of Magnetism and Electricity," § 218.

† See "Higher Textbook of Heat," § 109.

‡ An exception to this occurs in the case of the spectrum of the electric sparks obtained by sparking between metals. In this case the position of maximum energy is in the *ultra-violet*

the wire becomes faintly visible, first as a peculiar flickering "grey glow," followed as the temperature rises by an emission of the extreme red rays of the spectrum. With further rise of temperature the yellow and orange rays are added to the red and obscure rays, then follow the green, blue, and violet rays. The wire being then white hot and very near its melting-point the experiment must come to an end. This, considered in the light of the experiment with the thermometer in the various spectrum rays, shows that the varying thermal and luminous effects depend upon the frequency or wave-length of the emitted rays, and that the greatest heating effects are due to waves of low frequency and comparatively great length.

It is thus possible to estimate the temperature of an incandescent body, the light of which is due to heat, by carefully noting its colour. As a body is heated the first colour, a dull red, appears at 525° C. This turns to cherry colour at 800° C., and to a bright cherry at 1000° C. Bright orange appears at 1200° C., white at 1300° C., and dazzling white at 1500° C. and above. Intermediate temperatures may be read by means of a new instrument based on the above scheme, and called *The Optical Pyrometer.*

Recent research shows that the visible spectrum forms only a small part of the whole spectrum. The variation of wave-length in the visible spectrum is from about 4000 tenth-metres in the violet to 7500 tenth-metres in the red. Ultra-violet rays have, however, been found of wave-length less than 1000 tenth-metres, while infra-red rays have been found of lengths varying from 7500 tenth-metres up to 600,000 tenth-metres (·006 cm.) The wave-length of the shortest electric waves hitherto obtained is about 5 mm.

113. Transmission and absorption of radiation. Experiments show that when radiation of a definite kind is transmitted through a substance the amount transmitted decreases in geometrical progression as the thickness increases in arithmetical progression—that is, each layer of the substance, of a given thickness, transmits the same proportion of the radiation which enters it. Thus, if I_0 denote the quantity of radiation entering the substance, the amount which emerges after traversing unit thickness is I_0a, where a is a constant. Similarly, on transmission through another layer of unit thickness it is reduced to I_0a^2

and therefore, after transmission through a layer of thickness n the quantity of transmitted radiation is given by—

$$I = I_0 a^n.$$

The constant a has been called the *coefficient of transmission*. It is independent of the intensity of the incident beam and depends only on the nature of the substance and the wave-length of the radiation employed. When radiation of a compound nature is transmitted by any substance, its various constituents are absorbed in different degrees, and thus the nature of the transmitted radiation is subject to continuous change. The character of this change is, however, such that the nature of the transmitted radiation tends to become constant and capable of further transmission without absorption. For this reason radiation which has passed through a thick plate of any substance passes readily with little loss through another plate of the same substance.

Absorption, then, is the prime factor in the production of colour. If white light falls upon a plate which absorbs unequally the rays of different wave-length, the emergent light will be coloured. For considerable thicknesses the colour remains the same for different thicknesses, the shade becoming darker, but with thin enough layers the colour gradually alters. Thus thin plates of cobalt glass transmit chiefly blue light, while thick plates transmit a preponderance of red.

The light reflected from a body is also very often coloured. This is often due to the reflection not being wholly superficial. A portion of the incident light penetrates the body for some distance, suffers internal reflection and returns to the front surface, from which it emerges in line with the reflected light. This portion has undergone absorption and hence is usually coloured.

Exp. 35. Admit a beam of white light into a dark room. Reflect it by a coloured surface and catch the reflected beam on a white screen. Observe that this screen now appears of the same colour as the reflecting surface. Note also the reflections of coloured shop-signs.

A large crystal of sulphate of copper is transparent and deep blue, because it absorbs all but the blue components

of white light before the light has travelled very far through it. But when the crystal is crushed to a fine powder, not only does it become opaque, but its deep colour is reduced to a pale Cambridge blue. The light cannot then penetrate far enough into the crushed mass for any great absorption to take place. So it is with other coloured crystals. When crushed they assume a much paler tint.

114. Absorption spectra. There are many solid bodies which show a characteristic colour by light reflected without penetration from their surfaces, whereas thin films or plates viewed by transmitted light appear of a different colour. Such bodies are said to possess *surface colour* and seem to have a preference for reflecting certain rays and transmitting others. Thus gold when burnished is yellow by reflected light, but if a thin film of gold (gold leaf) be examined by transmitted light it is of a dull green colour. Many of the aniline dyes exhibit the same phenomena, and if an alcoholic solution of fuchsine (commonly called magenta dye) be allowed to evaporate to dryness on a glass plate it will be found that it transmits red light but reflects green.

Solutions of salts in many cases appear coloured by transmitted light; a solution of copper sulphate is blue, one of potassium permanganate is a rose purple, potassium chromate is yellow. Such phenomena are in all cases due either to selective reflection or to *selective absorption.* A body whose surface indiscriminately reflects all the rays appears white; the yellow colour of gold is due to the fact that most of the red, green, blue, and violet rays are transmitted and gradually absorbed, the predominant reflected rays being yellow. Similarly copper sulphate is of a blue colour because the rays of other colours contained in white light are absorbed in the solution and only the blue rays transmitted.

Selective absorption can be readily illustrated by arranging a bright white light, prism, and lenses, as in Art. 103, to throw a pure spectrum upon a white screen and then interposing various substances in the path of the rays. The spectrum will be found in some cases to be crossed by definite dark bands or lines, but in some cases whole portions of the spectrum will be blotted out.

Another method is to throw an ordinary spectrum on the screen, and then insert in the path of the light between the prism and the screen a prism of the material under examination, the refracting edge of this prism being placed perpendicular to that of the former as in Exps. 32-3. The spectrum now obtained will be blotted out in places, and strongly curved on each side of these spaces. A little observation will show that for rays on the red side of the absorption

band the refractive index is abnormally increased, whilst for rays on the blue side the refractive index is abnormally decreased. This is Kundt's Law. See also Art. 107.

Exp. 36. Place the following bodies in the path of the light and observe the results enumerated below.

Ruby glass	Red light only transmitted.
Cobalt blue glass . . .	Red and blue rays only transmitted.
Bichromate of potash * . .	Red and orange rays only transmitted.
Ammonio-sulphate of copper*	Blue and violet rays only transmitted.
Permanganate of potash * .	Spectrum crossed by several dark bands in the yellow, green, and blue region of the spectrum (Fig. 138).
Blood (dilute)* . . .	Two dark bands in orange and yellow, violet end of spectrum blotted out.

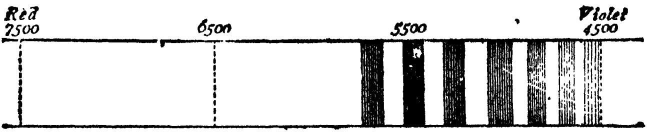

Fig. 138.

Many other similar examples are presented by dyes and by fluids derived from organisms.

The position of these bands is just as definite and characteristic as the lines are in flame spectra, hence the spectroscope (Art. 163) may be used for the purposes of recognising such bodies in solution.

Many gases give absorption spectra. Thus vapour of iodine gives a large number of narrow dark bands and water vapour gives such a characteristic absorption spectrum that its appearance in the spectrum of the sky is looked upon by meteorologists as an almost certain forecast of rain.

115. Fluorescence.† If a test tube containing a solution of sulphate of quinine be moved along through a spectrum which is cast on a screen, it will be observed that in the red, yellow, and green it appears red, yellow, and green respectively. In the blue and violet a change appears, the solution glows with a bluish light, and this bluish fluorescence exists even when the test tube is some distance into the ultra-violet. If the same solution be examined in sunlight, it

* Dilute solutions of these substances should be placed in little parallel-sided glass cells.

† The name is derived from *fluorspar*, the natural occurring form of calcium fluoride.

will be found to exhibit this fluorescence at its edges. This phenomenon was investigated by Sir G. Stokes, who showed that the alteration was caused by the quinine absorbing light-energy of one wave-length and emitting a part of it as light of longer wave-length. The quinine above absorbs ultra-violet light and renders it violet. Similarly, chlorophyll will appear red in the blue part of the spectrum and uranium glass, yellow in the green portion.

The fluorescence is invariably confined to the surface layers, the reason being that all the light which the substance is able to attack is disposed of in the region near the surface, and that which passes on is therefore rendered inactive.

If a spectrum be thrown up on a screen painted with sulphate of quinine, it will be found to be much extended at the violet end, and, if solar light be used, dark absorption bands will be found at various places, just as the Fraunhofer lines are found in the ordinary visible spectrum. Thus the ultra-violet region may be mapped.

Many common substances afford examples of fluorescence. Among them we may especially note ordinary paraffin oil (blue), eosin (red ink, red), fluorspar (blue), and an infusion made from fresh horse-chestnut bark (blue). The yellow salt barium platinocyanide is largely used in X-ray work, since it fluoresces brilliantly in these rays; and thus, if a dense object be interposed between a point-source of these rays and a prepared screen, a shadow of the object is thrown upon the screen, and, if the object vary in thickness, corresponding portions of the shadow will vary in intensity.

Exp. 37. Chip some fresh horse-chestnut bark into a beaker of warm water. Note the blue colour of the solution. Take it out into the sunlight and concentrate light on it by means of a large convex lens. Note the blue shimmer where the cone of light enters the solution.

116. Phosphorescence. In the case of bodies just considered, the fluorescence ceases almost as soon as the bodies are withdrawn from the light; but in the case of some substances—notably the sulphides of calcium, barium, and strontium—the emission of light will continue for some

hours after the exciting light has been cut off. Balmain's *luminous paint* consists of a mixture of the above sulphides, and if a card coated with this substance is exposed to a bright white light, or even ultra-violet light, and then taken into a dark room, it will emit a peculiar violet-coloured light, the rate of output of which is intensified by heating. The name phosphorescence is rather misleading. because the glow of slowly oxidising phosphorus is entirely a chemical change, while the phenomenon here dealt with is purely physical. The glowing of pure phosphorescing bodies is due entirely to the same cause as the glow of fluorescing bodies, fluorescence being only a phosphorescence which dies away more rapidly. To study the duration of the period during which they are luminous, Becquerel invented a *phosphoroscope*, which consists of sectors revolving at the ends of a cylindrical darkened chamber. By means of this instrument a substance can be exposed to the light; then the light is cut off and the substance is viewed after any required interval. By this apparatus he showed that all substances are more or less phosphorescent; and more recently Professor Dewar has shown that such bodies as feathers, egg-shells, etc., phosphoresce brilliantly when cooled to the temperature of liquid air.

117. The Emission of Light and the causes which produce it.

I. The commonest method of causing a body to emit light is to raise it to a high temperature (cf. Art. 100 and Exp. 34). In a flame the high temperature is due to the result of chemical action, in an electric incandescent lamp it is caused by passage of the current through a high resistance.

The phenomenon of **Calorescence** is a variety of this method of some historical interest. Professor Tyndall found that if he passed the radiation from a hot body through a solution of iodine in carbon-bisulphide, and focussed the waves of long wave-length (which are the only ones to penetrate this solution) upon a piece of thin blackened platinum foil, the latter was heated to redness. The invisible infra-red radiations are thus converted in part, at least, into luminous radiations of much shorter wave-lengths. This was thought to be the converse of fluorescence, hence the name calorescence; but it is obvious that the effect is a true temperature effect, for the infra-red radiations are poured into the foil faster than it can radiate them at low temperatures and a balance is only

obtained when the temperature of the foil has risen so high that it has become incandescent.

II. The cases of the production of light other than that of high temperature are collected under the general term **Luminescence**. The different kinds of luminescence may be summarised as follows :—

(1) *Photo-luminescence.* This is caused by the action of light. Fluorescence and Phosphorescence come under this head. The emission of light by a Welsbach mantle is supposed to be partly a true heat radiation, and partly phosphorescence.

(2) *Tribo-luminescence.* This is due to mechanical effects such as friction, percussion, and cleavage. Simple instances occur when quartz-crystals are rubbed together, a lump of sugar is crushed, and mica is cleaved.

(3) *Electro-luminescence.* This occurs in a vacuum discharge tube (see Art. 110 and Fig. 196). The glow in the body of the tube is probably produced by the impact of negative corpuscles (see Footnote, p. 248) against the gaseous molecules. At lower pressures the parts of the walls of the tube struck by the corpuscles also glow. In the same way many naturally occurring crystals glow when exposed to the radiations from radium and other radioactive bodies.

(4) *Chemi-luminescence.* This is usually due to oxidation, the most noteworthy cases being that of phosphorus, and decaying animal and vegetable matter.

(5) *Thermo-luminescence.* This occurs when certain bodies are slightly warmed, the temperature being far too low to produce a red-heat. This effect is noticeable with diamonds and with fluorspar (particularly with the variety called chlorophane).

(6) *Anima-luminescence.* This is observed chiefly with the glow-worm, marine infusoria, and the firefly. The glow is probably due to the action of an oxidising ferment. (See p. 26.)

118. Colours of Bodies. If a piece of red cloth or a red poppy be held in the red part of the spectrum, it appears red. Held in the green or blue part of the spectrum it appears black. So, a green leaf is distinctly green in the green portion of the spectrum, but is black elsewhere. Similarly a piece of cloth exhibits the colour which it has in the sunlight only at that part of the spectrum which is coloured like itself.

These experiments show that a body which is red in daylight is able to reflect red rays only. As it appears dark in the green or blue light of the spectrum, it reflects no green or blue rays. The same reasoning applies to other colours; a green surface reflects only green, blue only blue rays, and so on.

We now understand, with the help of Art. 113, the meaning of the colours which bodies are seen to exhibit in white light. White light is made up of many colours. When it falls upon a red surface, only that part of it which is red is reflected. The other spectrum colours are absorbed by the surface. When a body appears yellow, we are to understand that all the colours except yellow are absorbed.

So far we have assumed that the colours of bodies are simple, and not made up of a mixture of two or more different colours. It is difficult to obtain a pure green or a pure yellow or blue, and hence it often happens that when a coloured cloth is held in other parts of the spectrum than that which matches its colour, it appears coloured. A piece of green baize appears bluish in a blue light and yellowish in a yellow light, because most greens contain some blue and yellow in their composition.

If a piece of cloth of different tints be looked at in a light which is deficient in one or more of the colours of the spectrum, or in a light where one of the colours predominates, it does not always *look* the same as when viewed in daylight. If the light lacks one of the tints which the cloth exhibits in daylight, then that particular tint cannot be reflected by the cloth. In gas light yellows are brightest, because a gas-flame is chiefly a yellow light. But since a white body seen by yellow light appears yellow, the difference between white and yellow by gas light is much less distinct than by the white light of day.

We have now no difficulty in explaining the colours of transparent plates. A plate of red glass lets only red rays pass through, a green plate transmits the green rays, and so forth. A blue plate does not allow red rays to pass through it. Hence, if a beam of sunlight be made to fall upon a piece of red glass, and if a piece of blue glass be held in the course of the red light, it follows that as none of the red light can pass through the blue glass, the two plates together cut off all the light, and the source of light, if it is visible at all, appears black. This is found to be very nearly the case.

Exp. 38. Look at a gas flame through two plates of different colours, say blue and red, red and green, yellow and blue. The flame is invisible or nearly so, if the tints are sufficiently deep. It is difficult to get glasses of pure colour, so that ordinary red glass allows some other rays than red to pass through. Some of these may be able to pass through the second piece of glass.

Observe that a white cloth appears red in a red light, because it reflects red, blue, green, or any other colour. In fact, it appears white in daylight because it reflects all the colours.

119. Primary and complementary colours. A *primary* colour is defined as that which cannot be imitated to the eye by the mixture of any other colours. Maxwell showed that there are three primary colours—red, green, and violet—and that any other colour can be formed by mixing suitable proportions of these.

Colour mixtures may be effected in the following way: A spectrum is thrown upon a screen which is provided with adjustable slits, which can be opened to various extents and also shifted to occupy different places in the spectrum. In this way beams of different colours are let through, and these can be combined by a judicious arrangement of mirrors and lenses. **Newton's disc** (Art. 121) affords another method.

Any two colours which produce the sensation of white when they are mixed together are said to be *complementary*.

If we open a slit in the yellow and move another slit up and down the spectrum, we shall find that when it is in the blue the mixture of the two beams produces white. Thus yellow and blue are complementary, so also are red and greenish-blue, and green and purple. If, however, we mix yellow and blue pigments, the result is not white, but green. This is due to the fact that neither blue nor yellow pigment can be obtained of the same purity as in the spectrum. There is a little green in the composition of both of them, and while the yellow is mostly united with the blue to form white, there is some green left over, which accordingly is the resultant colour.

Exp. 39. Throw a continuous spectrum upon a screen. Cut away part of the screen, so that all the colours except red pass through. By means of a reversed prism (Art. **106**, 1) and lens combine all the colours that pass through the opening. Then the combined colours form a colour—greenish-blue—exactly complementary to the red.

120. Theories of colour vision. Colour blindness. The commonly accepted theory of colour vision is that due to Young and Helmholtz, and it postulates that, just as there are three primary colours, so we have three sets of nerves by which colours are appreciated. The three colours to which these nerves respond are the red, green, and blue. It is to be noted that these are not the three primary colours.

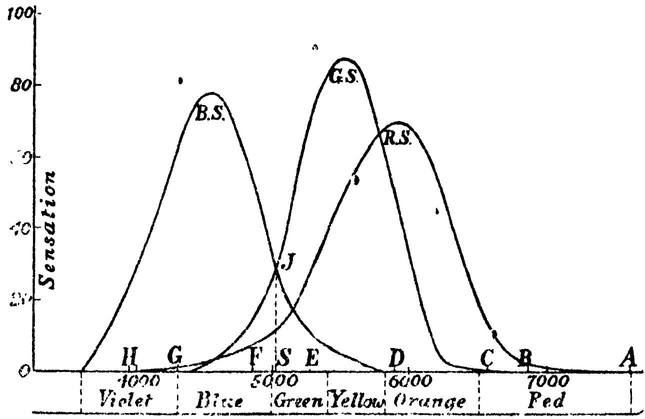

Fig. 139.

Red, green, and blue are therefore the three fundamental sensations, and the colour of a body simply depends upon the proportion in which the three sets of nerves are excited. Fig. 139—due to Captain Abney—shows that each region of the spectrum exercises all the sensations to varying degrees, and if we find a number of points in the spectrum such that the sum of the red-sensation ordinates is equal to the sum of the green-sensation ordinates, and also equal to the sum of the blue-sensation ordinates, the combination of the colours at these points would result in white light.

If one set of nerves is absent or inactive, the person is said to be colour-blind. Suppose, for example, a man is red-blind (this is the commonest form of colour-blindness), then he has no sensation corresponding to the curve R S (Fig. 139), and therefore the red constituent of all colours is unappreciated. To such an observer

the point S in the spectrum appears white, for the ordinate J M is the same both for blue and green sensations, and, according to him, the spectrum is composed of two colours, yellow and blue, and other colours which are more or less shades of these. Red to him is only dark yellow, and very often the spectrum is considerably shortened at the red end. Green is rather muddy yellow, and the middle of the spectrum (around S) is white or grey, while the violet he prefers to call dark blue. Of course colour-blind persons learn by experience to give many colours their proper names, but in some occupations, such as engine-driving and signalling, colour-blindness is such a great defect that candidates for such posts are carefully weeded out by tests both with the spectrum and with coloured skeins of wool.

121. Recomposition of white light. The experiments we considered in Art. **103** were analytical. We have determined the composition of white light by decomposing it into its constituent coloured rays. But there are many ways by which we can reverse the process, that is, we may start with the separate colours, and recombine them into a beam of white light, thus effecting a synthesis:

1. By the use of a second prism exactly like the first, but with its refracting edge turned in the opposite direction, as in Fig. 130.

2. By receiving the spectrum on a line of plane mirrors so that a separate colour falls on each, and then inclining the mirrors so that all the coloured rays are reflected to the same spot on a screen.

3. By interposing in the course of the spectrum rays an achromatic lens. A cylindrical lens with its geometric axis parallel with the slit gives the best results.

4. By **Newton's disc**. This is a circular disc of cardboard divided into sectors painted with the colours of the spectrum and attached to a whirling table. When the disc is rapidly rotated it appears nearly white, not because there is any real mixture of colours, but because of the fact that luminous impressions on the retina of the eye persist for a small fraction (about $\frac{1}{10}$) of a second, so that before the impression due to any one colour has died away it is succeeded by all the other colours. Thus there is a physiological though not a physical blending of the colours.

5. By combining complementary colours as in Art. **119**.

122. The rainbow. When the sun is shining upon a cloud of rain or on the spray from a waterfall or fountain, an observer standing with his back to the sun, and facing the rain or spray, often sees a circular arc of colour apparently in the midst of the water drops. Such an arc is called a rainbow, and is due to the reflection and refraction of the light falling upon the drops of water.

Let us now consider what happens when a parallel beam of light coming from a distant point (Fig. 140) falls upon a spherical drop of water whose centre is O. The ray S*a* incident along the normal will be reflected back along its

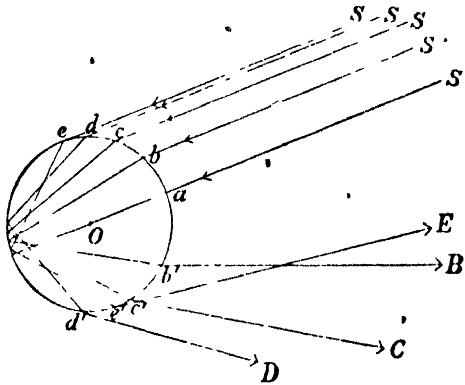

Fig. 140.

former path. Rays S*b*, S*c*, S*d*, S*e* . . . will suffer refraction on entering the drop, internal reflection at the back, and refraction on leaving, and will emerge in varying directions *b*′ B, *c*′ C, *d*′ D, *e*′ E. . . . The deviation of the ray S*a* is two right angles. Considering the other rays, it can be shown by accurate construction or by mathematics that the deviation of the rays decreases as we move out through *a b c d e* until a minimum is reached; after that it again increases. Now the deviations of the rays on each side of the ray of minimum deviation are all very nearly equal to the minimum deviation, so that an approximately parallel beam emerges in the direction *c*′ D, *d*′ D, and if an observer

is situated along these lines he will receive a large quantity of light.

Let $S\,d\,R\,d'D$ (Fig. 141) be the ray suffering minimum deviation ϕ.

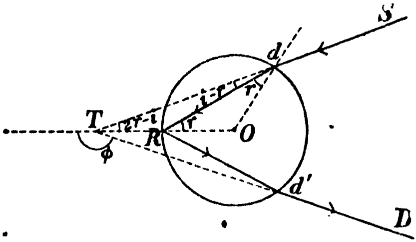

Fig. 141.

$$\phi = 180 - 2(\angle\, RTd) = 180 - 2(2r - i) = 180 + 2i - 4r.$$

Plot a curve between ϕ and i. The curve is convex towards the axis of i, the minimum value of ϕ being about 138°.

All other drops which yield this minimum deviation

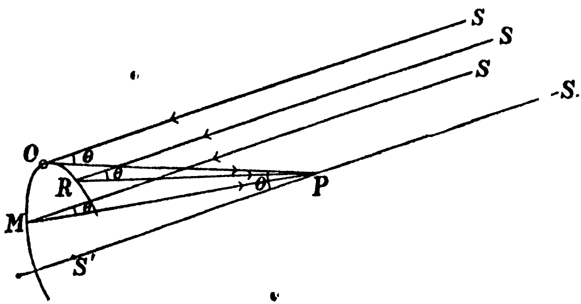

Fig. 142.

will lie on a circle M O R (Fig. 142) which forms the cross section of a cone whose semivertical angle is θ where $\theta + \phi = 180°$. Thus a circular arc of light will be seen, the centre S_1 of which is the point in the heavens exactly opposite to the sun. A complete semicircle is therefore only seen by an observer on the earth when the sun is on the

horizon, but from a balloon a complete circle may often be observed. So far we have considered only monochromatic light. Since, however, refrangibility depends on wave-length, the angles of minimum deviation will be different for different colours, so that a spectrum-coloured circular arc is seen. Violet light is more deviated than red, hence $\phi_v > \phi_r$, and therefore $\theta_r > \theta_v$, so that the radius of the red arc is greater than that of the violet. Calculation gives the values of θ_r, θ_v as 43° and 41° respectively, which are substantiated by actual measurement.

123. The Secondary Bow. Light may also reach the eye after two reflections inside the drop as shown in Fig. 143.

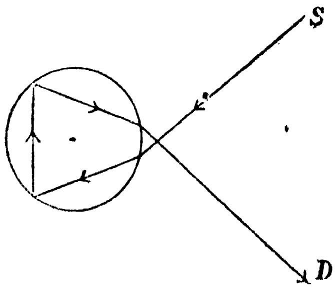

Fig. 143.

The deviation in this case is greater than 120°, and as before a position of minimum deviation occurs, making the light a maximum along the direction which it determines. This yields a bow concentric with the primary bow. The violet deviation being as before greater than the red, the angular radius of the violet arc is greater than that of the red, the numerical values being 54° and 51° respectively.

The space between the two bows is darker than the spaces within the primary bow and without the secondary bow.

In the same way as the above we may have bows arising from three, four, five, etc., internal reflections within the drop, but they are very faint, and are rarely or never seen. Since the sun is not a point source of light, the rainbow is

not a pure spectrum, and if it should happen that the sun is shining through a thin cloud—which has the effect of increasing the diameter of the source of light—the rainbow is nearly white.

Exp. 40. By means of a tank of water and a piece of glass tubing of about 1 mm. bore, obtain a vertical downward-flowing smooth jet of water. Place a flame—a candle will do—about 4 feet from the jet. and place the eye close to the jet with the back of the head towards the candle, taking care not to block the light. The primary and secondary rainbows will be seen together with a large number of spurious bows. By means of pins and paper map their directions and confirm the values of the angular radii given above.

124. Lunar rainbows are also occasionally seen. Owing to their faintness they usually appear destitute of colour.

The large circular rings or *Halos* which are sometimes seen around the moon, and more rarely around the sun, are due to refraction through tiny hexagonal ice crystals. The smaller brightly coloured circles or *Coronas* seen close round the moon, especially when it is full, are due to diffraction effects produced by very small water drops in high clouds; their formation lies beyond the scope of the present book (see Art. 134).

The ice crystals being in the shape of hexagonal prisms, and the refractive index of ice being 1·31, light which enters at one prismatic face cannot emerge at the next, but it may at the next but one, and of course at the one opposite and parallel to it. Considering two alternate faces it is evident that the crystal will act like a 60° prism; and applying the formula—

$$\mu = \frac{\sin \frac{D + A}{2}}{\sin \frac{A}{2}}$$

for light passing through at minimum deviation, D is found to be 22°.

If therefore a thin cloud of such crystals exists, the axes of many of the crystals being perpendicular to the line joining an observer to the moon, he will receive scarcely any light refracted through these prisms except at a point about 22° from the moon; and hence a bright circle of radius 22° will be seen. The theory would make the circle of light coloured as in the case of the rainbow, but as a rule the only colour seen is a red tinge on the inside of the circle.

Exp. 41. Measure the angular diameter of a Halo. Be on the look-out for halos around the moon, and when a good one appears take a foot-rule and a set-square of between 4 and 7 inches side. Place one end of the scale close to the eye, and sight the edge straight at the moon. Place the set square so that one of the sides enclosing the right angle may slide along the edge of the scale, and move the square forwards and backwards until the angular point not in contact with the scale just reaches out to the halo. Make sure that the scale is still pointing straight at the moon, and then take the reading of the point of the square at the right angle. If this is y, and the length of the side of the square standing out from the scale is x, the angular radius of the halo is $\tan^{-1}\frac{x}{y}$. Therefore the angular diameter $= 2\tan^{-1}\frac{x}{y}$. If x is about 5 inches, y will be about 12 inches.

125. The scattering of light by very fine particles.* It can be proved experimentally in the case of sound waves and mathematically for all wave motion that the smaller the wave-length of any radiation the more perfectly will an obstacle stop the waves and scatter them. Red light has a wave-length double that of violet light; hence when white light is travelling through a medium containing very fine particles in suspension we should expect the scattered light to have a blue tint and the transmitted light a red tint. This is borne out by the facts that street lamps look very red in a fog, and the setting sun appears redder as it approaches the horizon. Also the smoke from a lighted cigarette or wood fire, a distant haze, and a reservoir of water containing very fine particles in suspension look blue. The **colour of the sky** may be explained in like manner, the scattering in this case being due either to fine salt particles, to fine metallic dust from meteorites, or to the gaseous molecules themselves.

A pretty experiment to illustrate the colour of the setting sun can be performed by illuminating a screen by a parallel beam of white light from an optical lantern, and then inserting in the path of the beam a glass cell containing a freshly-made dilute solution of sodium thiosulphate—the "hypo" of the photographers—to which a little dilute hydrochloric acid has been added. Precipitation of sulphur gradually occurs, and the image on the screen gradually changes colour to orange and to red, and finally is obscured.

* See also Art. 163.

EXAMPLES VII.

1. Draw the section of a prism. Draw also the section of a beam of sunlight passing through the prism, and show by your sketch how this light is acted on by the prism.

2. A spectrum cast upon a white screen is looked at through a purple glass. What appearance does it present, and what is the cause of this appearance?

3. How would you disprove experimentally the assertion that white light passing through a piece of coloured glass acquires colour from the glass; What is it that really happens?

4. If you hold one piece of glass up to the sun it appears dark red; if you hold another up to the sun it appears dark blue. If you put the two glasses together you cannot see the sun through them at all. How is this?

5. A lamp-frame, looked at through a glass prism, appears to be coloured blue on one side and red on the other. Draw a picture tracing the rays from the lamp to the eye, and showing which side of the coloured image is red and which side is blue.

6. Given a powerful source of light, such as a lime-light or an electric-light, explain how you could obtain a spectrum of it on a screen.

7. Describe an experiment proving that white light is compound. How can it be shown that the constituents into which it is resolved are not likewise compound?

8. Describe how you would proceed to examine the spectrum of a salt. What variations in your method would be necessary if you wished to examine the spectra of (i) gases, (ii) metals.

9. If a shower of very small glass equilateral prisms ($\mu = 1{\cdot}65$) fell between you and the sun, what would be the general effect? Calculate the angular radius of the halo seen.

EXAMINATION QUESTIONS

QUESTIONS SET AT VARIOUS UNIVERSITY EXAMINATIONS.

1. Find the dispersion produced by a thin prism of angle 15° having a refractive index for red light of 1·5 and for violet light of 1·6.

2. Define the term Spectrum. What is a *pure* and what is an impure spectrum? Describe a method of procuring a pure spectrum and explain why your arrangement attains this object.

3. Give a drawing showing how white light is dispersed by a prism.

The image of a very small source of white light is thrown on to a screen by a convex lens. If the screen is brought nearer the lens, the spot of light is enlarged to a round patch, and its edge is coloured. How do you explain this?

4. Define "dispersive power." Explain how to combine prisms so as to produce (*a*) deviation without dispersion, (*b*) dispersion without deviation of a given mean ray.

5. Show that it is possible to make an achromatic prism with two sorts of glass.

6. By what experiments would you show that the radiation from an arc lamp extends beyond red at one end, and beyond violet at other end of the spectrum? In what respect do these invisible radiations differ from the visible radiations?

7. What evidence is there that radiant heat is physically of the same nature as light?

8. To what are the colours of objects as ordinarily seen due—for instance, red glass, blood, red cloth, red copper?

9. What is meant by phosphorescence, and by what means did Becquerel observe it?

10. A round stick one inch in diameter is placed vertically with its centre 6 in. in front of a vertical screen. At a distance of 1 foot from the screen two sources of light, one blue and the other yellow, are so adjusted that the two shadows of the rod due to them overlap to the extent of ½ inch. Draw a diagram approximately to scale of the shadow on the screen, and describe the colour effects visible on it.

11. How would you arrange an experiment to determine the percentage of light that is transmitted through a neutral-tinted glass plate?

If a plate of such glass allowed 40 per cent. of the light incident upon it to pass through, how much light would be transmitted by a plate of the same glass of four times the thickness, assuming that no light is lost by reflection at the surfaces in either case?

12. Define the coefficient of transmission of a substance for radiation. If a plate of blue glass one millimetre thick transmit half the radiation from a gaseous sodium flame which falls on it what is the coefficient of transmission? Does it follow that the plate would transmit half the radiation from a lamp?

13 (*a*). A snow-covered landscape when viewed through a piece of red glass appears red. How does the glass produce such a result? How would you verify your explanation by experiment?

(*b*) Red light is found to penetrate a fog more effectively than light of any other colour. This being so, would the penetrating power of an arc lamp be increased by passing its rays through red glass?

CHAPTER X.

VELOCITY OF LIGHT.

126. THE velocity with which light travels through any medium is inconceivably great, but varies with the nature of the medium. The velocity *in vacuo* is taken as *the* velocity of light, and the velocity in any other medium may then be determined, as explained in Art. **53**, from the absolute refractive index of that medium. For, if V denote the velocity of light in *vacuo*, and V_m its velocity in any given medium, then—

$$\frac{V}{V_m} = \mu,$$

where μ denotes the absolute refractive index of the medium.

Hence, if we can determine the velocity of light in any medium, such as air, we can calculate its velocity in any other medium, or *in vacuo*.

The velocity of light has been determined in three general ways:—

(1) From observations of celestial phenomena.

(2) By direct terrestrial experiments.

(3) By indirect electrical methods, the consideration of which is outside the scope of this book (cf. Stewart, "Magnetism and Electricity," Art. 194).

The first and last of these give, approximately, the velocity *in vacuo*, the second the velocity in air.

127. Determination of the velocity of light from observations of celestial phenomena. The first computation of the velocity of light by this method was made, in 1675, by **Roemer,** a Danish astronomer. He deduced his result from observations of the eclipses of Jupiter's first satellite, Io. This satellite is eclipsed to us once during each revolution when it passes behind the planet into the shadow cast by the sun. This occurs at intervals of about 42 hours. The instant at which the eclipse should take place can be accurately calculated from dynamical considerations based upon the mean of a large number of observations. Roemer timed the eclipse when the earth was in that part of its orbit nearest to Jupiter, and from this time calculated the times of the eclipses which would occur throughout the year. During the succeeding months he set himself to observe these eclipses and he found that the observed times were always later than the calculated times, and also that the difference between these two times varied with the relative position of the earth and Jupiter. From a careful analysis of the observations it was found that the difference between the observed and calculated times, increased as the earth moved away from Jupiter, reached a maximum about sixth-elevenths of a year afterwards when the distance between these two bodies had attained its greatest value and gradually decreased again to zero in another six-elevenths of a year when the earth was again in a position nearest to Jupiter. From this it is evident that the interval between the actual occurrence of the eclipse and the instant of its observation on the earth is equal to the time taken by light in travelling from Jupiter, or rather from Jupiter's satellite to the earth.

Let S (Fig. 144) represent the sun, E_1, E_2, E_3 the orbit of the earth, and J_1, J_2, J_3 that of Jupiter. Both earth and Jupiter move round the sun in the same direction; the times of revolution being one and twelve years respectively. Starting with the planets in conjunction at E_1, J_1, they will be in opposition at E_2, J_2 six-elevenths of a year later and again in conjunction at E_3, J_3 twelve-elevenths of a year after the previous conjunction, and it is now evident that as the earth moves to E_2 and Jupiter to J_2 the observed times of the eclipse lag behind the calculated times, the lag being a maximum at E_2, J_2. As the motion still ensues the lag decreases and by the time the earth and Jupiter have reached to the positions E_3, J_3 the

observed and calculated times once more agree. It is also evident that the maximum difference between the calculated and observed

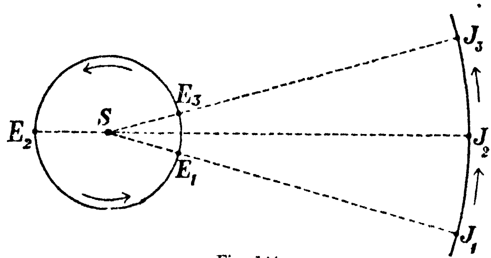

Fig. 144.

times is equal to the difference in the times taken by light in travelling the distances $J_1 E_1$ and $J_2 E_2$, *i.e.*, equal to the time taken in travelling a distance equal to the diameter of the earth's orbit.

The diameter of the earth's orbit is about 185,600,000 miles. The eclipse at E_2 is always about 16·5 minutes later than the time calculated from observations at E_1. Hence we have—

$$\text{Velocity of light} = \frac{185{,}600{,}000}{16{\cdot}5 \times 60} \text{ miles per second.}$$

This gives a velocity of about 187,000 miles per second.

About fifty years after the time of Roemer, Bradley, the English astronomer, gave an explanation of the phenomenon of *astronomical aberration*, based on the fact that light travels through space with a definite velocity. This phenomenon is due to the fact that both the earth and light travel through space with definite velocities, and hence the direction in which light from a star reaches the earth will be in the direction of the velocity of the light *relative to the earth.* Thus, if A (Fig. 145) represent the position of the earth when light from a star, S,* starts from S′, and if the velocities of the earth and of light be such that the former travels from A to B while the latter

* Many of the stars are at such great distances from the earth that, neglecting aberration, they are apparently seen in the same direction whatever the position of the earth in its orbit. The distance of the nearest fixed star is greater than 200,000 times the distance of the sun.

travels from S′ to B, then the direction in which the star is seen from the earth is parallel to A S′, and not to the true direction, A S″.

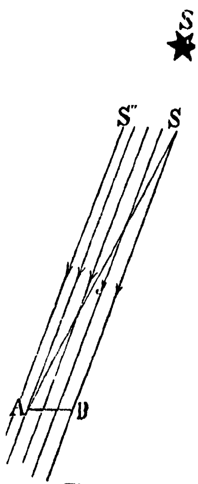

Fig. 145.

From our construction it is evident that—

$$\frac{A\,B}{B\,S'} = \frac{\text{Velocity of the earth}}{\text{Velocity of light}}\,,$$

and, when the angle S″ A B is a right angle, that is, when the true direction of the star is at right angles to that in which the earth is moving in its orbit, we have—

$$\frac{A\,B}{B\,S'} = \tan B\,S'\,A = \tan S'\,A\,S'',$$

and the angle S′ A S″ is called the *aberration* of the star.

Hence, if V denote the velocity of light, and v the velocity of the earth in its orbit, we get—

$$\frac{v}{V} = \tan\theta,$$

where θ denotes the aberration of the star.

Of the quantities involved in this relation v and θ can be determined by astronomical observation, and V can then be calculated.

Aberration is perhaps more clearly understood if we consider a man running through a shower of rain falling vertically. The drops will strike him in the face (or he will strike his face against the drops), and the rain will therefore appear to him to come from a point not straight above him but somewhat in front. The effect depends entirely on the velocity with which the rain is falling *when it reaches him*, not on how long it has been falling. So the displacement of a star by aberration is the same for all stars, and quite independent of their distances. As the star is never seen in its true position the angle cannot be measured direct. The angle measured is the angle between the apparent positions when the earth moves in opposite directions. This of course is double the aberration, and has been found to be 20·44 seconds.

The value obtained for V by this method is about 185,000 miles per second.

128. Direct determination of the velocity of light by terrestrial experiment. Two distinct methods have been devised to determine the velocity of light by direct experiment.

I. Fizeau's method. The principle of this method is simple. Let S (Fig. 146) represent a source of light and M a plane mirror. Now if a ray of light, S M, be incident normally on the mirror M, it will be reflected back along M S, and an observer behind S will see an image of S in the mirror. But, if a toothed wheel, having the teeth

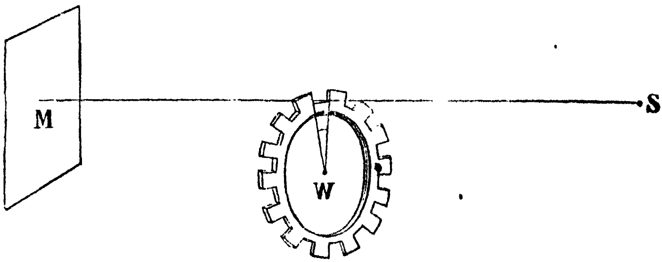

Fig 146.

and spaces of equal width, be interposed at W in the position indicated in the figure, it may be rotated at such a rate that the light incident through any space will, after reflexion, be received on the back of the next tooth, and thus no image of S will be seen in the mirror. When this is the case it is evident that during the time taken by the wheel to rotate through the angular width of one of the spaces, light travels from W to M and back again. Hence, to determine the velocity of light, from this experiment we have that—

$$V = \frac{2\,W\,M}{t},$$

where t denotes the time in which the wheel rotates through the angle subtended by one of the spaces, at the centre of the wheel.

It is further evident that if the wheel be rotated at twice the above rate the reflected ray will pass through

the next space, and the image will again become visible, and if rotated at treble the rate extinction again takes place, and so on.

In the application of this principle Fizeau employed somewhat complicated apparatus, the essential parts of which are shown in Fig. 147. A source of light, S, was placed so as to send a beam of light down the side tube *t* on the mirror *m*, made of unsilvered glass. This mirror is inclined at an angle of 45° to the axis of *t*, and reflects the light along the main tube T on to the lens L. The position of this lens is so adjusted that the rays emerge parallel, and after traversing a distance of about 5½ miles, fall on the lens L′, which causes them to converge through the

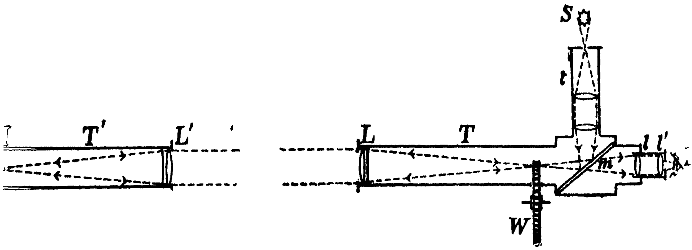

Fig. 147.

tube T′ on to the mirror M, from which they are reflected back along the path by which they came. On reaching the mirror *m* the light is partially reflected to S, but a portion passes through and reaches the observer's eye at E, after passing through the lenses *l* and *l′*, which are adjusted to give distinct vision of the image.

The wheel, placed at W, is driven by clockwork, and by adjusting its rate of rotation the image can be made to disappear and reappear successively several times. The wheel employed by Fizeau had 720 teeth and 720 spaces, the width of the latter being equal to that of the former. The distance W M was about 8,663 metres, and the first eclipse of the image took place when the wheel revolved 12·6 times per second. Hence, the time taken by the wheel to rotate through the angle subtended by one of the

spaces is $\frac{1}{2 \times 720 \times 12 \cdot 6}$ of one second. In this time light travels from W to M and back again, a distance of $2 \times 8{,}663 = 17{,}326$ metres. Therefore, for the velocity of light we have—

$$V = 17{,}326 \times 2 \times 720 \times 12 \cdot 6 = 314{,}000{,}000 \text{ metres per second.}$$

This result, obtained in 1849, is about 195,000 miles per second, and is somewhat in excess of the result obtained by more recent experiments.

This method has one great defect, arising from the fact that it is impossible to determine the exact rate of rotation of the wheel at which extinction of the image takes place. The rate can be appreciably varied without allowing the image to become visible, because the quantity of light reaching the eye, when the rate of rotation is approximately equal to that producing exact extinction, is too small to affect the retina. In some recent experiments by Messrs. Forbes and Young this defect was removed by arranging the apparatus so that two images, formed by mirrors at different distances, could be seen. The rate of rotation of the wheel was then adjusted until the two images appeared of the same intensity. This method was found more practicable, and gave more trustworthy results. The method of reducing the observations is rather difficult, and need not here be considered. The mean value obtained for V was 301,400,000 metres per second.

In 1876 M. Cornu carried out a careful determination by Fizeau's original method, but on a larger scale, the distance W M (Fig. 147) being 15 miles, and found the value of V to be 300,330,000 metres per second in air, corresponding to 300,400,000 metres per second in *vacuo*. This result is also too high, more recent work having shown it to be less than 300,000,000 metres per second.

II. Foucault's method. This method is somewhat more complicated both in theory and in practice. Adopted in 1850 it utilises the principle of the rotating mirror as first employed in 1834 by Wheatstone, to determine the duration of the electric spark. The principle of the method is as

follows:—Solar light is transmitted through a narrow rectangular aperture *s* (Fig. 148), down the middle of which extends a vertical wire. The light proceeds through the achromatic lens L, falls obliquely upon a plane mirror *m*, and then comes to a focus at M. At M is placed a concave mirror whose centre of curvature is at *c*, the middle point of *m*. For a certain position of *m*, a pencil of light, *s c*, starting from *s* is reflected from *m* to M (the central ray being incident along the normal *c* M), and then reflected back along the same path to *s*. For convenience of observation a thin parallel plate of glass is inserted between L and *s* at an angle of 45° to the axis of the central ray, so that the

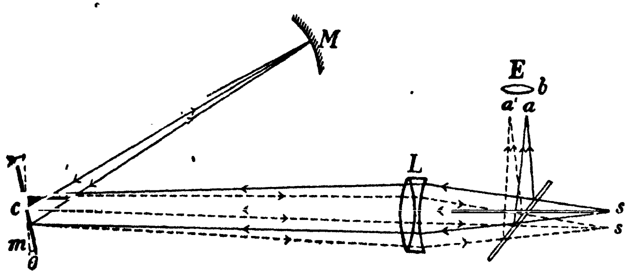

Fig. 148.

reflected beam is in part reflected and comes to a focus at *a*, which can be observed through an eyepiece, *b*.

If now the mirror *m* be made to revolve it will pass through the position just considered once in each revolution, and therefore an image of *s* will be seen, for an instant, once in each revolution. When the revolutions become sufficiently rapid (about 30 per second) these quickly succeeding images persist on the retina, and blend into one permanent image, still seen at *a*. When, however, the speed of rotation is greatly increased, the mirror, *m*, turns through an appreciable angle while the light is travelling from *c* to M and back again. For example, if the mirror turn through the angle θ while light travels from *c* to M and back to *c*, then the ray M *c* will not be reflected along *s*, but along *c s'*, and the eye at E sees the image of *s* at *a'*.

Hence, if we can determine the angle $s c s'$ and the distance c M, we can calculate the velocity of light. For, by Art. 31, the angle $s c s' = 2\theta$, and light travels a distance $2 c$M during the time that the mirror revolves through an angle θ. If the mirror makes n revolutions per second, then the angular velocity per second is $2\pi n$, and the time in which the angle θ is described is given by—

$$t = \frac{\theta}{2\pi n} \text{ seconds.}$$

Therefore, if cM be denoted by l, the velocity of light is given by—

$$V = \frac{2l}{t} = \frac{4\pi n l}{\theta}.$$

Of the quantities involved in this relation, n and l are readily determined, and θ is equal to $\frac{1}{2}(s c s')$. In practice it would be very difficult to measure $s c s'$ with any accuracy, but no difficulty is incurred in an accurate measurement of $a a'$ which equals $s s'$, and θ is then evaluated in terms of the several distances involved. These distances are easily measured, and thus V can be calculated.

In the actual experiment the distance cM was 20 metres, the mirror was a piece of silvered glass, and was rotated by means of a small air turbine. The deflection $a a'$ only amounted to 0·7 mm., but by means of the micrometer eyepiece this could be read to an accuracy of 1 in 150. The result finally obtained by Foucault was 298,000,000 metres per second.

In 1880 **Professor A. A. Michelson,** of the United States Navy, introduced great improvements in Foucault's method, the chief being the transference of the lens L from its position in Fig. 148 to a position between c and M. In this way the distance cM could be greatly increased (a distance of 2,000 feet was attained) without any diminution in the brightness of the image. A deflection of 133 mm. was obtained, the plate of glass and micrometer eyepiece could thus be discarded and $s s'$ directly measured. The rotating mirror was driven by an air turbine under perfect control, and its speed was measured by an electrically

driven vibrating tuning-fork. His final result is 299,882,000 ± 60,000 metres per second.

Professor Newcomb (1882) further modified the method by using a cubical mirror so that the brightness of the image was increased fourfold, and the distance *c*M was further increased to 12,000 feet. His value of V is 299,810,000 ± 60,000 metres per second.

Taking the mean of the best determinations we get the velocity of light *in air* to be 299,890,000 ± 60,000 metres per second; this corresponds to a velocity *in vacuo* of 299,970,000 ± 60,000 metres per second. In English units the velocity in air is 186,350 ± 40 miles per second.

129. Connection between the velocity of light and the refractive index of the medium. If a long tube containing water or other transparent medium be placed between *c* and M (Fig. 148), the displacement, from *a* to *a'*, will be greater or less, according as the velocity of light in the given medium is less or greater than the velocity in air. Experiment shows that light travels more slowly through a dense than through a rare medium, that is, the greater the refractive index of the medium, the less is the velocity of light through it.

Foucault, who was the first to carry out the above experiment, did not succeed in measuring the ratio of the velocities. This was left for Michelson; he used a tube of water 3 metres long and found the ratio of the velocity of light in air to that in water to be 1·330. Experiments on other transparent bodies have yielded similar results.

The question has also arisen whether the velocity of light of various colours is the same in air (or in vacuo)? Evidence shows that the velocities are identical, for if not at an eclipse of a star or at its reappearance it would appear coloured, and the image in Foucault's experiment would be dragged out into a spectrum. In other media the velocity is greater for red rays than for violet rays; thus Michelson found that red light travelled about 5 per cent. faster than blue light in carbon bisulphide.

130. Theories of the nature of light. Many of the effects of light and many crude optical instruments were known to the ancients, but of the theory of optics they were wholly ignorant. Pythagoras (B.C. 540-510) and Plato maintained that vision was a threefold phenomenon. Their idea was that the eye sent out a stream of potency or divine fire which combined first with the light of the sun and then with the emanation from the third body, the second combination completing the action of vision. Aristotle, in B.C. 350, struck the right chord by maintaining that light was not a material emission from a source, but a mere quality or potentiality of a medium existing between our eyes and the body seen.

131. The emission or corpusculary theory of light. In more recent times Sir Isaac Newton (1642-1727) upheld the corpuscular or emission theory of light. Light was supposed to be a swarm of corpuscles ejected from a luminous body at a great speed, and these corpuscles on entering the eye excited the sensation of vision. In open space their motion is rectilinear, but near material surfaces the motion proceeds along curves, this being due to either a repulsion or attraction of the corpuscle by the matter in question, a property which only extends to a short distance from the surface.

Let us now consider what happens to a corpuscle when it approaches a polished surface A B (Fig. 149). Suppose the conditions are favourable for reflection. The path is straight up to *a*, at which

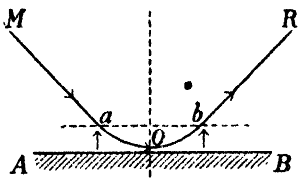

Fig. 149.

it first experiences repulsion. By the time it reaches O (a point still some distance from the surface) the velocity normal to A B has been neutralised, and when *b* has been reached the force of repulsion has

endowed it with a velocity normal to A B, and equal in magnitude to that which it previously possessed. Since the velocity parallel to A B has not been altered its final velocity along O R is equal to its initial velocity along M O, and therefore the angles of incidence and reflection are equal.

If the condition is favourable for refraction, suppose the corpuscle is attracted by the surface as soon as it reaches *a* (Fig. 150), and its

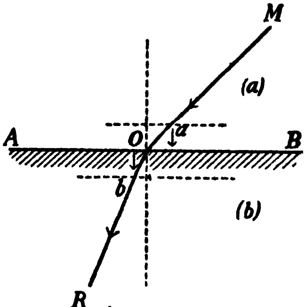

Fig. 150.

velocity, normal to the surface, is gradually increased until it reaches *b*, after leaving which it continues to move in a straight line. In this case the velocity normal to the surface has been altered, and if, as in the figure, it has been increased, the ray is bent towards the normal. As before, the velocity parallel to A B is unchanged, so that the final velocity along O R is greater than the initial velocity along M O. If the corpuscle had been subjected to a continuous repulsion throughout *a b*, not strong enough, however, to prevent it entering the second medium, the final velocity would have been less than the initial velocity, and the ray would have been bent away from the normal.

If v_a, v_b be the velocities of light in the upper and lower media, i and r the angles of incidence and reflection, then since the component of the velocity parallel to the surface remains constant,

$$v_a \sin i = v_b \sin r.$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{v_b}{v_a},$$

a constant for the same two media.

To explain away the seemingly haphazard processes of attraction and repulsion, Newton endowed his corpuscles with periodic "fits" which made them alternately more liable to be reflected or refracted. This device is, however, very artificial, and Newton in his old age modified his theory until the corpuscles became almost a superfluity, and there was little difference between this theory and the undulatory or wave theory of Light which we now propose to describe.

132. The wave theory of light. This theory may be said to have begun with Aristotle, but it was not till Huyghens took it up in 1678 that it was enunciated in a scientific manner. Since then Fresnel, Young, and Stokes have finally established it.

It postulates (Art. 3) that there is spread throughout space an all-pervading medium or ether, and that transverse oscillations in this ether constitute thermal, luminous, and electrical radiations.

Many have been the properties with which the ether has been endowed. As we can neither hear, taste, see, smell, nor feel it, we can never be directly cognisant of its properties; in fact, we are only aware of its existence because the sciences of radiant heat, light, and electricity demand such a medium for the transmission of their effects.

A luminous body is supposed to set up transverse vibrations in the ether in its neighbourhood; this disturbance then travels out through the ether, and on entering the eye excites the sensation of vision. Since the propagation is attended with transverse wave motion the ether must possess properties akin to rigidity and density, and at different times it has been considered to be an incompressible fluid and an elastic solid.

In the dynamical conception of an ether many difficulties arise, but these are avoided if we concentrate attention rather on the phenomena exhibited than on the mechanism that produces them. States of strain and of motion in the ether produce what we are more familiar with as electric and magnetic field. Vibrations involve both motion and strain; thus Maxwell's **Electromagnetic theory of light** * escapes the question of the fundamental nature of the luminiferous ether, by explaining light waves as an electromagnetic phenomenon in which the electrical and magnetic fields in vacuo, or in any medium considered, undergo periodic changes. For most purposes it is therefore unnecessary to consider the properties of the ether; light phenomena as a rule can be explained by simply supposing

* See Stewart's "Textbook of Magnetism and Electricity," Ch. XXX.

either transverse waves of displacement or electromagnetic waves to be travelling through the ether. The properties of wave motion are usually treated fully in Textbooks of Sound,* so that in this place we shall not dwell on the mechanics of the problem.

We shall now describe the method by which Huyghens explained the reflection and refraction of light.

To explain reflection, let A A′ (Fig. 151), be a wave front of a plane wave incident on the polished surface A B. A now becomes a centre of disturbance and from it spreads outwards a series of

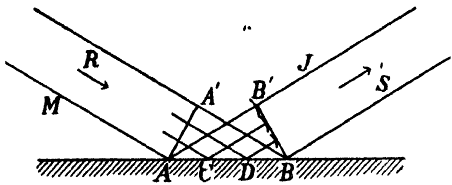

Fig. 151.

spherical waves. Similarly other points, C, D, etc., in turn become active centres. By the time A′ has reached B the largest wave from A extends a distance A B′ from A where A B′ = B A′, and if B B′ be drawn tangential to this sphere it will touch all the extreme spheres of disturbances emanating from points between A and B. Therefore B B′ is the new wave front and the light now proceeds in a direction perpendicular to it. The figure is symmetrical about a normal and thus the results are in agreement with the laws of reflection (Art. 23).

A simple illustration of the spreading and reflection of spherical waves is obtained by letting a tap gently drip into water placed in an elliptical dish. Spherical waves diverge from the point where the drops enter; and if this point is situated at one of the foci of the ellipse, the waves are reflected at the walls of the dish, and shrink in upon the other focus. If the experiment is performed in a good light, the presence of the waves is made more evident by the bands of light on the bottom of the dish.

For the case of refraction consider as before a wave front, A A′ (Fig. 152), of an advancing plane wave. Each point of A B becomes in time a centre of disturbance and by the time the whole wave

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* Catchpool's "Textbook of Sound."

front A A′ has crashed into A B the refracted wave front will have advanced to B B′, where

$$A B' = \frac{v_b}{v_a} \cdot A' B.$$

Thus if $v_a > v_b$ (as in figure), A′ B > A B′, and the light is bent towards the normal. Also—

$$\frac{\sin i}{\sin r} = \frac{\sin A' A B}{\sin A B B'} = \frac{A' B / A B}{A B' / A B}$$

$$= \frac{A' B}{A B'} = \frac{v_a}{v_b},$$

a constant for the same two media.

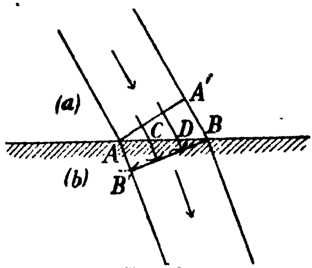

Fig. 152.

133. Crucial test between the emission and undulatory theories. Let μ be the index of refraction between two media a and b; and v_a. v_b the velocities of light in these media. The relation between the three quantities is given by

$$\mu = \frac{v_b}{v_a}, \quad \text{Emission theory.}$$

$$\mu = \frac{v_a}{v_b}, \quad \text{Undulatory theory.}$$

Thus in a medium, such as water, for which $\mu > 1$, the emission theory states that the velocity is greater than the velocity in air, while the undulatory theory postulates the reverse. Foucault decided this point (see Art. 128) in favour of the undulatory theory, and although this does not prove the undulatory theory to be right, it certainly proves that the emission theory, as at present enunciated, is wrong.*

These results were assumed in Art. 53.

* It is worthy of note that in recent electrical theory corpuscles have once more been called into existence. The apparent mass of a corpuscle is about the 1800th part of that of a hydrogen atom, and it carries a negative charge. They act, as it were, as anchors for the tubes of electrical force. The Newtonian corpuscular reflection can be imitated very well by an experiment on the reflection of the electric negative corpuscles. A small negatively charged piece of lime is heated to incandescence in a very high vacuum. Above a certain temperature it gives off a narrow pencil or stream of negative corpuscles, whose path is rendered luminous by molecular collisions. If across the path of this pencil a negatively charged plate is placed, the stream of corpuscles is reflected almost exactly according to Fig. 149.

134. The rectilinear propagation of light. This was a strong point with the supporters of the emission theory, for, granted that the corpuscles moved in straight lines until acted on by the surfaces of media, the theorem that light travels in straight lines is self-evident.

On the undulatory theory it proved at first a great stumbling block, for the opponents argued that as sound waves easily bent around corners so also would light waves. In the case of sound waves it is easily proved by experiment that the shorter the waves the more sharply defined are the sound shadows.* Now the wave-length of the sound waves caused by a tuning-fork of frequency 512 per second is about two feet, while the wave-length of the yellow or mean light of the spectrum is only $\frac{1}{44000}$ inch, hence we should expect, even on the wave theory, the shadows to be very distinct, but never *absolutely* sharp.

This latter result is borne out by the fact that the best shadows are always bordered by alternate light and dark fringes called *diffraction* bands. The propagation of light is thus only approximately rectilinear.

Exp. 42. Cut a very narrow slit in a postcard, and place it in front of a white light. Cut another slit in another postcard, and hold this up at some distance in front of the eye, so that the flame, slits, and eye are in alignment. Diffraction bands will now be clearly seen bordering the direct view of the flame.

These diffraction bands are due to the superposition of light waves which have traversed slightly different paths. Other instances occur when a candle is observed through a silk handkerchief, a street lamp through an umbrella, and the sun through half-closed eyelashes.

The colours obtained when oil is poured on water, or a piece of steel is heated in the flame, are also due to the same cause. So also are the colours of soap-bubbles, mother-of-pearl, and cracks in crystals and blocks of ice.

Recent work on the infra-red rays or waves is greatly in favour of the electro-magnetic theory. These waves have been traced to such a degree that not a very considerable gap now remains between the longest infra-red waves and the shortest electric waves (see page 218). Also the properties of the infra-red waves approximate as the wave-length increases to the properties of the electrical waves, tending to show that the difference between electric waves and light waves is essentially only one of wave-length.

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* See Catchpool's "Textbook of Sound," Ch. IX., Arts. 118, 119.

EXAMINATION QUESTIONS.

QUESTIONS SET AT VARIOUS UNIVERSITY EXAMINATIONS.

1. How has the velocity of light been determined from observations of the eclipses of Jupiter's first satellite?

Assuming that Jupiter's first satellite revolves round the planet in a constant period of forty hours, that the velocity of the Earth in its orbit is 18 miles per second, and that of light is 187,000 miles per second, find the greatest and the least apparent intervals between successive eclipses.

2. Describe the method by which Fizeau investigated the velocity of light.

3. Describe Foucault's method of measuring the velocity of light by means of a rotating mirror. What is the effect of introducing a tube of water (with glass ends) between the rotating and fixed mirrors, and what relation is there between the velocity of light in a medium and its refractive index?

4. Explain carefully some method of measuring the velocity of light. How has it been shown that lights of different colours travel through air at very nearly the same rate?

5. Describe a method of measuring the velocity of light based upon the use of a revolving toothed wheel. The distance between the two stations being 9·3 miles and the number of teeth being a hundred, the rotation is started and gradually increased in speed. Find the number of rotations of the wheel in a second when the light reflected from the distant station has disappeared and reappeared ten times. The velocity of light may be taken as 186,000 miles per second.

6. Point out the difference between a wave of sound, a wave of light, and a wave traversing a stretched string.

CHAPTER XI

SIMPLE OPTICAL INSTRUMENTS.

135. Artificial horizon. The altitude* of a star is frequently determined by a method based on the laws of reflection. The accuracy of the results obtained by this method furnish an indirect but rigorous proof of the truth of these laws. A vertical divided circle, adjusted in a vertical plane, carries a telescope T T (Fig. 153) which can be rotated round an axis passing through the centre of the circle. In making an observation the telescope is first pointed to a particular star, and the reading on the circular scale, for this position of the telescope, is accurately noted. The telescope is then turned into the position T′ T′, so as to view the image of the star, formed by reflection from the *horizontal surface* of mercury contained in the vessel M. The reading of the scale corresponding to this position is again noted, and the difference between the two readings—that is, the angle

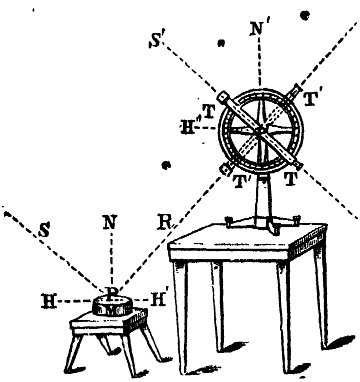

Fig. 153.

* The altitude of a star is the angle between the direction of the star and its horizontal projection.

S′ P′ R—gives twice the altitude of the star. For, *assuming the laws of reflection to be true*, we have—

	S P N = N P R,	
and therefore—	S P H = R P H′.	
But	R P′ H″ = R P H′	(Euc. i. 29);
therefore	R P′ H″ = S P H,	
also—	S′ P′ H″ = S P H.*	

Therefore we have—

$$S' P' R = S' P' H'' + H'' P' R = 2 \, S P H.$$

But S P H is the altitude of the star; therefore S′ P′ R is twice the altitude of the star.

The accuracy of the results obtained by this method conclusively proves the truth of the laws of reflection.

136. Hadley's sextant. The sextant is an instrument employed for measuring the angle between two distant objects, as seen from the position occupied by the observer. The principle of its action has already been explained in Art. 33. The essential parts of the instrument are shown in Fig. 154. The frame is made up of the circular arc, S S′, and the two arms, S C and S′ C. These two arms, which are radii of the circle of which S S′ is an arc, intersect at C the centre of the circle, and C I is an index arm which can be rotated about an axis passing through C. Two plane mirrors, A and B, are attached to this arrangement; A is fixed on the arm S′ C, and B is attached, at C, on the index arm C I; both mirrors being perpendicular to the plane of the paper. The mirror

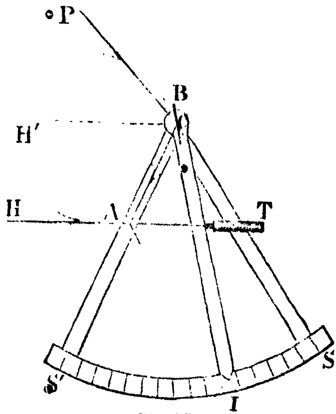

Fig 154.

* S P and S′ P′ are parallel. Compare footnote to Page 239.

A is unsilvered, or only partially silvered, so that an observer looking through the telescope, T, which is directed towards A, can see objects in the direction T H. When I is at S, the planes of the mirrors, A and B, are parallel, so that any ray H′ C, incident on B parallel to H T, is reflected along C A to A, and thence, along A T, to T.

The observer looking through T thus sees objects in the direction T H (or C H′), both directly through A, and by successive reflection from B and A respectively.

On moving the index arm C I towards S′, other objects, in addition to those seen directly through A, are brought into view, and if, when any particular object, in the direction C P, is brought into the field of view, the arm C I has been turned through an angle θ, then, by the principle of Art. **31**, the angle P C H′ is equal to 2θ. That is, the angle between an object seen in the direction C H′ and another object in the direction C P, is equal to twice the angle S C I.

Hence, in determining the angle between any two given distant objects, the instrument is first adjusted until one of the objects is seen directly through A, and also by reflections from B and A. The index I will then be at the zero of the scale on S S′. The arm C I is then moved until the other object, seen by reflections from B and A, appears to coincide with the first object still seen directly through the unsilvered part of A. The required angle is then obtained by doubling the angle S C I, which is given by the reading of the scale. Usually the scale is graduated on the principle of marking half-degrees as whole ones, so that the direct reading gives the required angle.

137. The heliograph and heliostat. A heliograph is simply a plane mirror suitably mounted, so that by its means sunlight can be reflected from one station to another, say, several miles away. It is used for the transmission of messages; the mirror is alternately tilted away from, and back to, its correct position according to a given code, and the observer at the distant stations notes the duration and regularity of the flashes, and from this constructs the message.

The heliostat is simply a heliograph mirror in which by suitable means the reflected beam is sent in the same direction all day long. This is done by mounting the mirror on a frame drawn by clockwork, the mirror being moved so that its normal always bisects the angle between the direction of the sun and the direction in which the light is to be sent.

138. Paraboloidal Mirrors. In Art. 46 we saw that the aberration produced by spherical mirrors could be largely decreased by the use of stops.

A real *remedy* may be applied by the substitution of a *parabola* instead of a circle as the generating curve of the mirror. A parabola is a curve formed by a section of a cone parallel to its side, and a paraboloidal surface is generated by the revolution of this curve around its axis. Portions of two parabolas are shown in Figs. 155 and 156, the former of low, the latter of high angle. No matter how

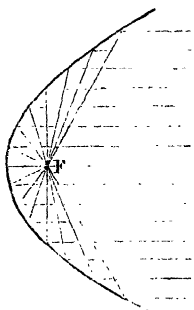

Fig. 155.

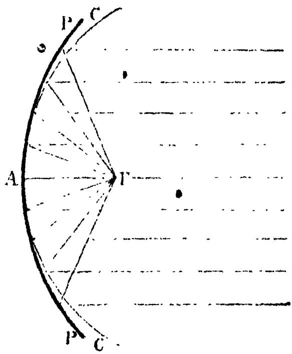

Fig. 156.

great the angle, parallel rays falling on a paraboloidal mirror parallel to the axis are converged accurately to a certain point called the focus, and, conversely, rays diverging from that point are accurately parallelised. When, as in lightships and railway signal lamps, it is necessary to parallelise rays as perfectly as possible, Fig. 155 shows the form of mirror employed. In the specula of large telescopes, which have to converge parallel rays as accurately as possible, an attempt is made to give such a figure as that shown (but with great exaggeration of the aperture actually used) in Fig. 156. In this figure the circle is shown for comparison of the curves.

139. Ellipsoidal mirrors. These afford the most accurate method for concentrating by a single reflection the light proceeding from one point upon another.

If F and F′ (Fig. 157) be the geometrical foci of the ellipse J R M, and R any point on it, the angles which F R, F′ R make with the normal R N are equal, hence if the luminous source be placed at F all the reflected rays will go through F′. Revolve the ellipse about the line F F′, and any portion of the surface generated will constitute an ellipsoidal mirror.

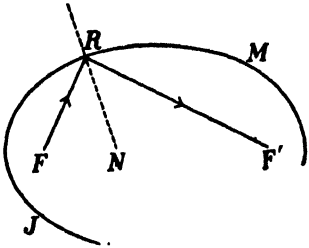

Fig. 157.

Cylindrical mirrors and lenses. These mirrors behave like plane mirrors for dimensions of objects parallel to their lengths and as concave or convex mirrors for dimensions perpendicular to their axes. They are used largely for shop-window illumination.

The lenses act similarly. In the laboratory they are often employed to produce images of illuminated slits; in the outside world they are used largely by spectacle-makers as a corrective to astigmatism (Art. 149).

140. Total reflection prisms. Let A B C (Fig. 158) represent the section of a prism with angles 90°, 45°, 45°. If an incident beam of parallel light, P Q, falls normally on the face A B, it suffers no refraction and very little loss by reflection, and meets the hypotenuse A C at an angle of 45°, which is greater than the critical angle (41°). Consequently it is *totally* reflected in the direction Q R, and reaching the face B C normally, it emerges without refraction and with very little loss by reflection from the surface B C. The beam is thus deviated through a right angle with very little loss of light.

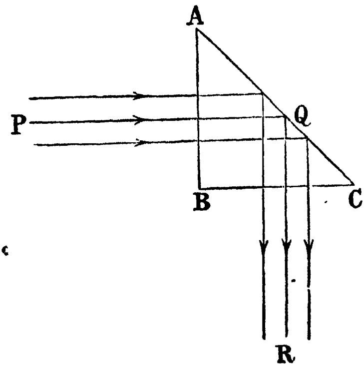

Fig. 158.

If a plane mirror of glass, silvered at the back, be employed to deviate a beam through a right angle, confusion is often caused by the succession of images which are formed (Art. 65). This error can be eliminated by silvering the front surface, but when we remember that even the most highly polished silver surface reflects regularly considerably less than the whole of the light incident upon it, and that there is difficulty in keeping a silver surface in good condition, we can easily see why such reflecting prisms are very frequently used in optical instruments. But such prisms can only be used to reflect light at an angle of incidence greater than the critical angle. Since the rays are intended to enter and leave normally, the prism should be isosceles with its angle B equal to the angle of deviation required. If used for other angles, there is more loss of light by reflection at B C or B A. The issuing beam is not identical with the original beam, but undergoes the same kind of reversal, right and left interchanging, as with reflection from a plane mirror.

Total reflection prisms are largely used in lighthouses. Right in front of the light is placed a large compound plano-convex lens, and around it arranged in rings are fixed a number of total reflection prisms of varying angle, such that all the light is sent out in a parallel beam. As important applications of total reflection prisms in optical instruments, see the Newtonian telescope, Art. **157**, Fig. 180, and the comparison spectroscope, Art. **164**, p. 313.

Wollaston's prism, sometimes called the *camera lucida*, is a totally reflecting prism with four angles, generally employed as an aid to sketching. A section of the prism is shown in Fig. 159; the angle A B C is a right angle, A D C is 135°, and the other two angles each $67\frac{1}{2}$°. Light incident normally on B C, in the direction P Q, is totally reflected from the face D C to the face D A, whence it is totally reflected along R S normally to the face A B. To an eye looking along S R, objects in the direction of Q P are seen in the direction S R P′, and the image thus seen may be traced on a sheet of paper placed at P′, vertically below S. The sheet of paper is seen past the edge A of the prism, while

the image is seen by reflection from the face A D. It is im-

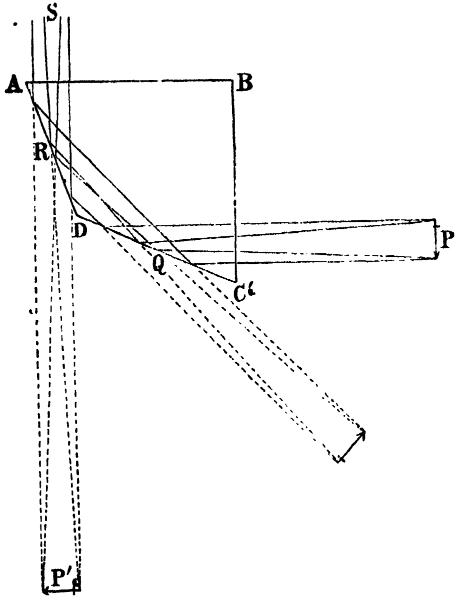

Fig. 159.

portant that the image should be in the plane of the paper, for then paper, pencil, and image are seen with the same focussing of the eye. For this reason a concave lens of short focal length is placed in front of the face B C when the object to be sketched is very distant. By adjusting the height of the prism in its stand (Fig. 160), the image can then be made to coincide with the plane of the paper. The two reflections at the faces C D and D A are necessary to give an erect image.

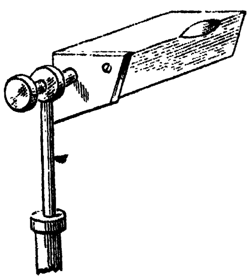

Fig. 160.

141. The camera obscura. The principle of this arrangement is indicated in Fig. 161. At the top of a small tent or wooden structure is a small cylindrical or cubical box,

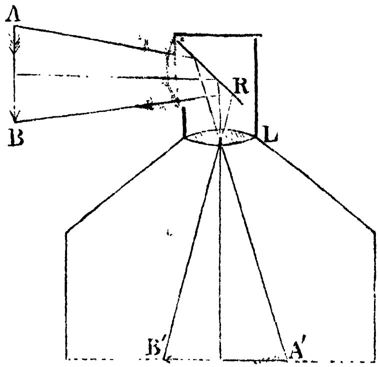

Fig. 161

which contains a mirror, R, and a lens, L, arranged as shown in the figure.

The mirror is inclined at 45° to the horizontal, and reflects the rays coming from any external object A B on to the lens L, which forms an image A′B′ on a white table or screen placed vertically below it. The room is perfectly dark and the inside carefully blackened, so that the image thus cast upon the screen or table may be clearly seen. The box containing the mirror and lens can be rotated, and thus images of all objects surrounding the tent are in turn cast upon the table. Instead of the mirror and lens it is better to employ a totally reflecting prism with the faces, which are turned towards A B and A′B′ respectively, convex and concave. The curvature of the concave surface is less than that of the convex, and the arrangement thus acts as a convex lens and a mirror combined. Since the objects under observation are relatively far away, all the images on the table are in focus at the same time. Their sizes are the same as if a simple aperture were used instead of the lens and mirror.

142. Photographic camera. If we disregard all unessential details, this instrument may be described as a box (Fig. 162) with a convex lens* in front, and at the back a ground glass screen, which can be replaced by a slide carrying a sensitised plate. When this instrument with the lens uncovered is towards any object that is to be

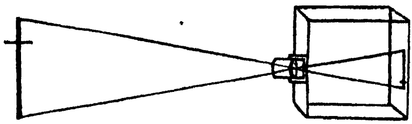

Fig. 162.

photographed, an inverted image of that object will be formed; and by sliding the lens or the back of the box in or out, the image may be accurately focussed on the ground glass screen.

If a suitably prepared photographic plate † be then substituted for the screen, the action of the light on this plate is such that when subjected to proper chemical treatment a *negative* is obtained, from which the ordinary photographs can be printed on sensitised paper.

The pinhole camera (Art. 8) is very useful for some kinds of photographic work, especially that of taking buildings, as it produces no distortion, whereas an ordinary lens gives images with strongly curved lines; and in fact many of the cheap cameras on the market are simply pinhole cameras. A disadvantage of such cameras is that the light can only enter through a small hole and thus long exposures are required, but by using extra rapid plates this difficulty can be surmounted.

Exp. 43. Make a pin-hole camera. Cut a strip of brown paper 8 ins. by 30 ins. Paste one side and roll it, with the pasted side inwards, on a roller about 3 ins. in diameter, and not less than 9 ins. in length. In this way a serviceable cardboard tube can be made. Over this roll a single layer of dry paper. Over this again roll

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* Usually an achromatic combination.

† The first photographs were taken by Daguerre in 1839.

a pasted sheet of brown paper as before. Thus two cardboard tubes about 8 ins. long and 3 ins. in diameter will be made so that one may slide stiffly in the other. Close one end of the wider tube by a thin piece of card, in the centre of which make a small pinhole. Close one end of the smaller tube with a piece of tracing paper or ground glass. Push this end of the smaller tube into the larger one, and looking into its open end, direct the pinhole to any brightly illuminated object. A small inverted image of the object will be seen on the tracing paper. Push the smaller tube farther in and note the reduced size but increased brightness of the image. Pull it farther out and note the increased size but diminished brightness of the image. Enlarge the pinhole and note the increased brightness but less distinctness of the image, its size being unaltered.

143. Why optical instruments are blacked inside. Cameras, telescopes, microscopes, and other optical instruments are always painted dead black inside in order to prevent internal reflections. In the camera, for instance, when the light reaches the sensitised plate a considerable proportion is scattered in all directions, and falling on the sides, top, bottom, and front of the box, would, if these were light in colour, be in great part reflected back to the plate and fog it. In other instruments such internal reflections would similarly confuse the effect produced by the direct rays.

144. The optical lantern. In attending any course of experimental lectures this instrument will be seen in constant requisition for a great variety of purposes, and the student should have some idea of its construction and mode of action. It is now surely quite time to drop the old-fashioned name *magic* lantern, except for the toys used for exhibiting pictures only.

Essentially it is a light-tight box (Fig. 163) enclosing a powerful light or radiant, S. For scientific experiments the radiant should be a powerful one, but *it is much more important that it should be very small*, the smaller the better. The limelight and electric arc are satisfactory in both these respects. In front of the box is the *condenser*, L, usually a pair of plano-convex lenses about 4 ins. in diameter. The radiant is so mounted that it can be slid backwards and

forwards in the optic axis of the condenser. When it is in the principal focus of the condenser a powerful parallel beam emerges. When slid further back the beam can be made to converge to any required point, and after the crossing of the rays to diverge from that point.

When used for projecting images of various pieces of apparatus or of pictures, an achromatic lens combination, of about 6 in. focus called the *objective*, O, is mounted in front of the condenser, and the radiant is placed so that

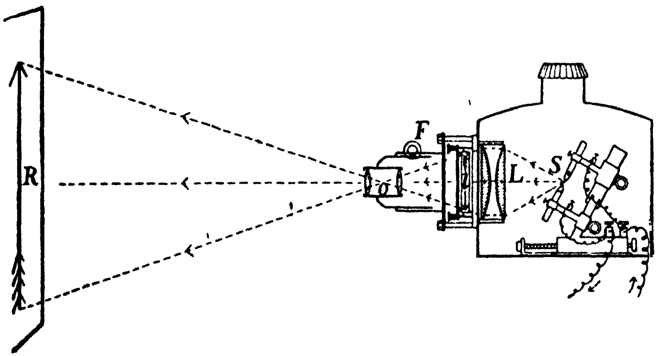

Fig. 163.

the whole of the convergent beam issuing from the condenser may be taken up by the objective.

The lantern slide, *l*, which is a transparent photograph, or other suitably prepared representation of the object to be shown is placed between O and L, as close as possible to the latter, and the tube carrying the former is then screwed backwards or forwards until a clearly defined image is focussed on the screen R. The image is inverted and magnified in the ratio R O : O *l*. Since the intensity of illumination of the image varies inversely as the square of R O, the best magnification that can be obtained depends ultimately on the illuminating power of S. The image is inverted, hence to get an erect image the lantern slide is put in upside down. In the projection of a piece of

apparatus on the screen inversion is often undesirable, and in such cases the beam is re-inverted by means of a total reflection prism.

The condenser is used simply to concentrate (or condense) the light on the slide. It plays no part in the focussing and hence need not be corrected for either kind of aberration.

145. The eye. The human eye is essentially an optical instrument, similar in principle to the photographic camera described in Art. **142**. Fig. 164 shows, diagrammatically, a vertical section of the eye from front to back. Anteriorly we have the *cornea*, C, behind which is the anterior chamber of the eye, bounded behind by the *crystalline lens*, L, and the ciliary processes, *cc*, to which the lens is attached. This chamber is filled with a watery fluid called the *aqueous humour*, A; and in front of the crystalline lens

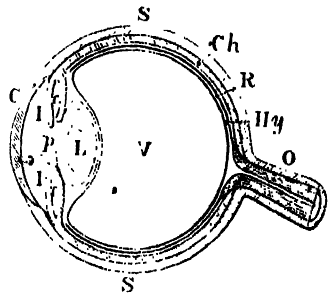

Fig. 164.

lies the *iris*, I I, a circular curtain with a central aperture, *p*, called the *pupil*. The iris is seen in the eye as the coloured ring surrounding the pupil; it is a muscular structure made up of circular and radial fibres, so arranged that the size of the pupil can be increased or diminished as required. The *crystalline lens*, L, is the lens of the eye; it consists of a soft transparent substance enclosed in a thin transparent membrane, and is held in position by the ciliary processes which are attached round its circumference. In structure it is somewhat complex; the posterior face is more convex than the anterior, and it is built up of a large number of concentric shells, increasing in density as they approach the centre, the outer shell having the same density as the surrounding medium. By this arrangement the optical action of the lens is more powerful than if it were com-

posed of a homogeneous medium of the same density as the nucleus, and also the loss of light by reflection at the surfaces of the lens is diminished. Behind the crystalline lens, in the posterior chamber, is the *vitreous humour*, V; this is a watery fluid very similar to the aqueous humour.

The walls of the eyeball are made up of three coats. The outer, S, called the *sclerotic*, is a tough white coat, giving consistency to the ball. The cornea, in the front, is fitted into this coat like a watch-glass into the case of the watch. The middle coat, *Ch*, the *choroid*, is a thin pigmented layer which divides in front into two layers; the anterior layer goes to form the iris, and the posterior constitutes the collar of ciliary processes carrying the crystalline lens. Adjacent to the ciliary processes is a muscular collar attached to the sclerotic coat, and inserted at the circumference of the crystalline lens. This collar is the *ciliary muscle*, and serves, by its action, to vary the convexity of the surfaces of the lens. The inner coat, R, is the *retina*, a delicate membrane which is practically a fine network expansion of the optic nerve, O. It covers the whole of the inner posterior surface of the eye as far as the ciliary collar, and is lined by a very delicate membrane, H*y*, called the *hyaloid membrane*. At the centre of the retina is the *yellow spot*, a small slightly raised yellowish spot, having a minute depression, called the *fovea centralis*, at its summit. This yellow spot, which is about $\frac{1}{20}$th of an inch in diameter, is the region of a most distinct vision, and the fovea centralis is the most sensitive spot on the retina. About $\frac{1}{10}$th of an inch on the inner side of the yellow spot is the *blind spot*, the point at which the optic nerve enters the eye. This spot is not sensitive to light.

The structure of the retina is very complicated. The surface next the vitreous humour consists of thin connective tissue called the hyaloid membrane. Below that extend the ramifications both of the optic nerve and the artery which enters the eye with the nerve. The nerve filaments end in ganglion cells, and fresh processes proceed from these through the rest of the retinal thickness and, penetrating the external layer of connective tissue, end in a layer called the *Bacillary Layer* or *Jacob's Membrane*. This layer

or membrane consists of elongated bodies, some shaped like rods and some like cones. Their extremities are embedded in a layer of pigment cells. The rods and cones form the sensitive part of the retina. An element of the retina consisting of rods, piles, and cones is about ·004 mm. in diameter, and the eye cannot distinguish between two objects unless the retinal images are separated by a greater distance than this. Experimental evidence indicates that the rods are most sensitive to faint lights, while the colour sensations are produced by the cones.

Considered optically, then, the eye consists of a double convex lens, the crystalline lens, protected in front by a circular diaphragm, the iris, and having a sensitive screen, the retina, on which the images of external objects are cast. The impressions conveyed to the brain by these images give rise to the sensation of sight.

The following are the mean values of the optical constants of human eyes:—

	When viewing a distant object.	When viewing an object 15 cm. away.
(*a*) Index of refraction of the aqueous and vitreous humours and of the cornea	1·337	1·337
(*b*) Index of refraction of the crystalline lens	1·437	1·437
(*c*) Thickness of cornea	0·4 mm.	0·4 mm.
(*d*) Radius of the outer surface of cornea	−7·8 mm.	−7·8 mm.
(*e*) Radius of anterior surface of lens .	−10·0 mm.	−6·0 mm.
(*f*) Radius of posterior surface of lens.	+6·0 mm.	+5·5 mm.
(*g*) Distance of anterior surface of lens from anterior surface of cornea .	3·6 mm.	3·2 mm.
(*h*) Thickness of lens	3·6 mm.	4·0 mm.

146. Vision. The condition of distinct vision of any object is that a clearly defined image of it is formed on the retina.

Fig. 165 represents such a case. When light enters the eye refraction occurs at the surfaces of the cornea and crystalline lens. The centres of these surfaces lie on a straight line called the optic axis, which meets the retina between the yellow and blind spots. Since no change of refractive index occurs from the cornea to the aqueous humour, the aqueous humour may be regarded as extending to the anterior

surface of the cornea. If the object is large the image is distorted, due partly to obliquity of the extreme rays and partly to spherical aberration. The distortion is, however,

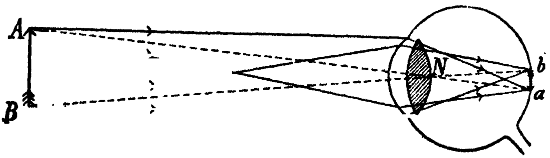

Fig. 165.

greatly corrected by the heterogeneity of the crystalline lens and by the spherical shape of the eyeball.

In all cases in which real objects are distinctly seen real inverted images are produced on the retina; that we see objects erect is due to the interpretation which the brain puts upon the stimuli it receives.

Under ordinary conditions it is evident that, with an eye as above described, only objects at a certain definite distance from the eye can be seen distinctly; for, the distance between the image and the lens being fixed, the distance between the object and the lens must also be fixed. We know, however, from experience that objects can be seen distinctly by the normal eye at all distances greater than a certain minimum limit known as *the distance of nearest distinct vision.* This is due to the power of *accommodation* possessed by the eye; the ciliary muscle, we have seen, is able to alter the curvature of the surfaces of the lens, making the front surface much more convex, and bringing the lens as a whole nearer to the cornea; thus the focal length is *accommodated* to the distance of the object on which the eye is focussed. For a normal or *emmetropic* eye the limits of distinct vision are from a point distant about 10 inches from the eye to infinity. When the eye is at rest, it is supposed to be adjusted for parallel light—that is, for distinct vision of very distant objects.

The eyes of various individuals vary much in their accommodative power. Young children can see distinctly objects placed two or three inches in front of their eyes,

ordinary adults can see objects as near as ten inches; but as the age of a person advances the power of accommoda-tion of the eye decreases, probably because of a loss of elasticity in the outer layers of the crystalline lens. This defect of vision is called *presbyopia*, and it causes the nearest point of distinct vision to gradually recede from the eye. Thus in order to read a book an old man is often compelled to hold it at arm's length.

147. Magnifying power. The absolute size of an object is, of course, a constant quantity, but the *apparent size*, which is proportional to the magnitude of the retinal image, depends upon the distance of the object from the eye. A measure of this apparent size is given by the angle which the object sub-tends at the eye. This angle is called the *visual angle*. Thus if AB, Fig. 166, represent the object and E the eye, the apparent size is measured by the visual angle AEB, and this angle evidently decreases as the distance of AB from E increases. When the angle is small the visual angle is approximately measured by the ratio, $\frac{AB}{EN}$, or may be taken as proportional to the tangent of the angle AEB—that is,

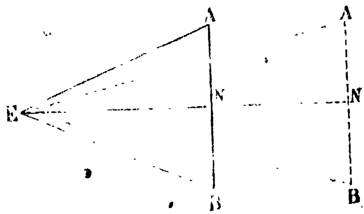

Fig. 166.

$$\text{tangent (visual angle)} = \frac{\text{Length of object}}{\text{Distance of object from eye.}}$$

When we look at a building a mile away, the visual angle under which we see it is very small, consequently its image is very minute, and we can perceive nothing but its general form. At 100 yards it subtends a much larger angle, and its image may perhaps occupy almost the whole of the retina, and we are able to perceive doors, windows, and smaller features. At 20 yards only a portion can occupy the retina at one time, and that portion subtends a much greater angle than before. It may perhaps con-

tain a printed bill, and the larger type may be easily read. At 1 yard the retina may be wholly occupied by the image of the bill, and all but the smallest type may be read. At 10 inches one of the smallest letters subtends such an angle that its form is plainly perceptible. At 4 inches the retinal image of that letter is larger still, *but its form is no more distinct*—it is less so, because the rays proceeding from the letter now diverge so widely from it that they cannot be focussed on the retina. Therefore nothing is gained in the way of distinct vision by any closer approach than 10 inches.

We have adopted this line of discussion in order to give the student a clear idea of the mode of action of the **Magnifying Glass.** A magnifying glass is simply a convex lens of short focal length employed to obtain magnified *virtual* images of small objects. The lens is placed at a distance less (usually *very little* less) than its focal length from the object to be viewed, A B (Fig. 167), and, as explained in Art. 85, a virtual magnified image is formed at A′ B′, which, when the position of the lens is properly adjusted, can be clearly seen by an eye at E. But it will be noticed that the enlargement of the image is counteracted by its increased distance, and the visual angle A′ O B′ under which it is seen, and therefore the size of the retinal image, is only the same as that of the object itself. Where, then, does the magnification come in? If the object were brought as close (perhaps 1 inch) to the unaided eye as it is to the lens, its retinal image, though equally large, would be indistinct because of the inability of the eye to focus such highly divergent rays. What the lens does is to reduce this great divergence X A O, Z B O, to the much smaller divergence X A′ O, Z B′ O, and so allow the rays to be focussed on the retina, while preserving the great visual angle.

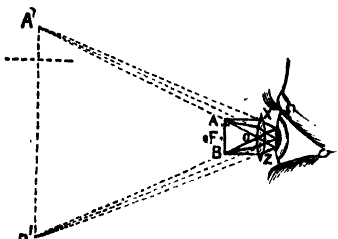

Fig. 167.

148. Magnifying power of lens. Since the size of the retinal image is inversely proportional to the distance of the object, and this distance is determined by the focal length of the lens, than which it is very little less, the magnifying power may be taken as the quotient of the near point of distinct unaided vision (usually taken as 10 inches) divided by the focal length of the lens. Therefore the magnifying power of a lens of 2 inch focus is $\frac{10}{2} = 5$, and of a lens of $\frac{1}{2}$ inch focus is $\frac{10}{\frac{1}{2}} = 20$. This, which is what is usually understood as magnifying power, refers to linear dimensions. Of course the superficial magnification is the square of the linear.

The above is only approximate, for the object is not placed quite at the principal focus. Let the distances of object and image from lens be denoted by u, D, respectively, where D is the least distance of distinct vision, and let f = focal length of the lens. Then

$$\frac{1}{D} - \frac{1}{u} = \frac{1}{f}.$$

$$\therefore u = \frac{fD}{f - D}.$$

But the magnification, in general equal to $\frac{v}{u}$, is in our case equal to $\frac{D}{u}$,

$$i.e., \frac{\text{Image}}{\text{Object}} = \frac{D(f - D)}{fD} = \frac{f - D}{f} = 1 - \frac{D}{f}.$$

Or, since we are dealing with a convex lens, we may write magnifying power $= 1 + \frac{D}{f_1}$, where f_1 is the *numerical* value of the focal length, and D is the least distance of distinct vision for the eye considered. Thus if f_1 is equal to 2 in., the magnifying power is 6, and not 5, as obtained by the approximate method above.

When an optical instrument is said to *magnify* an object, it is meant that the visual angle of the object as seen through the instrument is greater than its visual angle when seen directly by the naked eye, and the *magnifying*

power of the instrument is measured by the ratio of the visual angle as seen through the instrument to the visual angle when seen directly. This definition is evidently not sufficiently precise, for the visual angle of an object, seen directly, depends upon its distance from the eye; it is therefore necessary to specify this distance. In the case of a telescope, where the object viewed is distant, the magnifying power is defined as the ratio of visual angle of the image seen in the telescope to the visual angle of the object seen directly *at its actual distance from the eye.* In the case of the microscope, however, the object viewed is near at hand, and it is assumed that when seen directly it is placed at the distance of nearest distinct vision, where its visual angle is greatest. Hence the magnifying power of a microscope is defined as the ratio of the visual angle of the image seen in the microscope to the visual angle of the object seen directly *at the least distance of distinct vision.*

149. Defects of vision; Spectacles. The most common defects are known as (1) **Myopia** or **Short-sightedness,** (2) **Hypermetropia** or **Long-sightedness,** (3) **Presbyopia,** (4) **Astigmatism.**

A normal or **emmetropic** eye brings parallel light to a focus on the retina. By means of its power of accommodation the eye can also focus light from nearer points. Let

Fig. 168.

N be the nearest point of distinct vision (Fig. 168), then images of all points on the line from N to $+\infty$ can be focussed on the retina by the unaided eye.

1. In **myopic** eyes either the axis of the eye is too long or the crystalline lens is too convergent. Light from a

distant object is brought to a focus in front of the retina (Fig. 169), and thus the object is either not seen at all or seen very indistinctly. As the object approaches the eye the image travels backwards from the focus of the lens, and when the object reaches a point P, a certain distance away, the image falls exactly on the retina. This point P is the point of farthest distinct vision, which, if the eye were normal, would be at infinity. If the accommodating mechanism be perfect, the eye will, up to a certain limit, be able to adjust

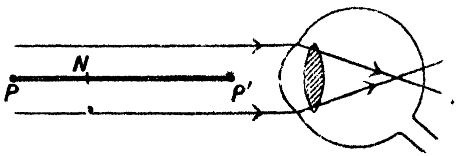

Fig. 169.

itself so as to give distinct vision for objects nearer the eye than this point, and even nearer than N, the near point for normal eyes. Let P′ be nearest point of distinct vision for a myopic eye. For the unaided eye only those points can be distinctly seen which lie between P and P′. Rays from a distant point are brought to a focus in front of the retina, even when the muscles of the eye make it as little convex as possible. This defect can be remedied by the use of spectacles. To determine the nature of the lenses required, we must notice that the necessary condition for remedying the defect is that rays diverging from a point on the *normal* range of vision, *i.e.* ∞ N, should, after refraction through the lens, apparently diverge from a point within the range of the short-sighted eye, *i.e.* P P′. Suppose this latter range to be from 3 to 8 inches from the eye. Now we cannot make this coincide with the normal range at *both* ends; we must therefore decide either for coincidence of the nearest or of the farthest points of distinct vision. It is usual to choose the latter, because it corresponds to the quiescent state of the eye; hence, in this case we require a lens such that rays coming from infinity—that is, parallel rays—will, after refraction through it,

diverge from a point F, nearly 8 inches from the eye (Fig. 170). The required lens is therefore a *concave* lens

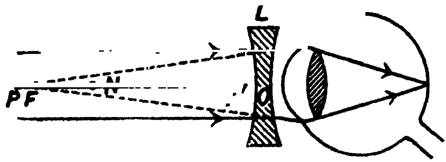

Fig. 170.

of 8 inches focal length; and, if x denote the nearest point of distinct vision with this lens, we have—

$$\frac{1}{3} - \frac{1}{x} = \frac{1}{8}.*$$

$$\therefore\ x = 4\cdot 8 \text{ in.}$$

That is, the range of vision is now from 4·8 inches to infinity, instead of from 3 inches to 8 inches in front of the eye.

A practical consequence of the use of a concave lens is that the retinal images are diminished, and thus objects appear smaller than they do to normal eyes.

2. In the case of **hypermetropia** or long-sightedness, the axis of the eye is too short or the lens not sufficiently convergent. When unaccommodated the only light which can

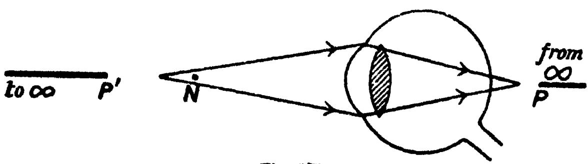

Fig. 171.

be focussed on the retina is that which is converging to a point P (Fig. 171) *behind* the eye.

By means of accommodation, rays can be focussed which

* Determined by the condition that rays diverging from a point x inches in front of the lens must, after refraction, appear to diverge from a point 3 inches in front of the eye.

are converging to points on the line from P to $-\infty$, and rays which are diverging from points between $+\infty$ and P′, the nearest point of distinct vision. P′ is at a greater distance from the eye than N, the nearest point of distinct vision for the normal eye.

Rays diverging from points nearer than P′ are focussed behind the retina. The power of the eye lens (to converge the rays) is too little, and so a convex lens must be requisitioned to help it. Suppose, for example, P′ is 30 inches in front of the eye and P 10 inches behind; then, to render the farthest points of distinct vision for the unaided normal eye and the aided hypermetropic eye coincident, we must employ a lens such that rays coming from infinity will, after refraction through it, converge to the point P, 10 inches behind the lens—that is, a *convex* lens of 10 inches focal length must be used, and the distance of the nearest point of distinct vision (x) is given by the relation—

$$\frac{1}{30} - \frac{1}{x} = -\frac{1}{10};$$

$$\therefore \quad x = 7\tfrac{1}{2} \text{ in.}$$

That is, the range of vision is now from $7\frac{1}{2}$ inches to infinity, instead of from 30 inches in front of the eye up to and through $+\infty$, and from $-\infty$ back to 10 inches behind the eye.

The action of the lens is shown in Fig. 172. The effect of L is to push P to $+\infty$ and to bring P′ nearer to N.

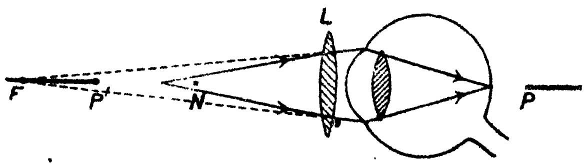

Fig. 172.

Another effect of the convex lens is to make objects appear larger than they appear to the normal eye.

3. In the case of **presbyopic** eyes (usually found in old people, Art. 144) distant objects can be distinctly seen, but

light from near objects cannot be focussed on the retina—that is, the nearest point of distinct vision has receded. To remedy this defect by means of spectacles, the focal length of the lenses must be adapted to the purpose for which the spectacles are required. For reading purposes the lenses must be chosen so as to cause rays diverging from the point of normal nearest distinct vision to appear to diverge, after refraction through them, from the nearest point of distinct vision of the defective eye. For example, suppose the nearest point at which a person can see distinctly is 30 inches; then, if f denote the focal length of the lenses required, we have—

$$\frac{1}{30} - \frac{1}{10} = \frac{1}{f};$$

$$\therefore \ f = -15 \text{ in.}$$

That is, convex lenses of 15 inches focal length are required.

When the accommodating mechanism is not perfect, there may be practically only one point of distinct vision, and the defect can be remedied only for particular cases. For example, suppose a person is able to see distinctly only at a point distant about 4 inches from the eye, then, if he requires spectacles to enable him to see distant objects distinctly, the focal length of the concave lenses to be used must be about 4 inches. If, however, he requires spectacles for reading purposes, and he wishes to hold his book in the same position as a normal-sighted person holds his, then the focal length of the lenses is determined from the fact that light coming from objects at the normal distance of distinct vision, 10 inches, should, after refraction, appear to come from a point 4 inches from the eye. Therefore—

$$\frac{1}{4} - \frac{1}{10} = \frac{1}{f},$$

or—

$$f = 6\tfrac{2}{3} \text{ in.}$$

That is, concave lenses of $6\frac{2}{3}$ inches focal length are required.

4. In the case of **astigmatic** eyes—due mainly to non-sphericity of the cornea—a vertical section being usually

more curved than a horizontal section—lines inclined in one direction can be seen much more plainly than lines in a direction perpendicular to this. There are very few eyes that do not suffer from this defect, horizontal lines being usually brought to a focus in front of the focus of vertical lines. A test may be made by drawing four or five parallel lines close together on a sheet of paper, which is then placed facing the patient about 4 or 5 yards away and slowly rotated. The patient with one eye open watches the lines, and in general it will be found that for quite a large range of rotation they appear very indistinct. To remedy this defect a cylindrical lens (Art. 139) is required, its position being arranged so that the refraction which it produces is in the same way as the weaker refraction of the cornea. If the eye is myopic or hypermetropic as well as astigmatic, a lens cylindrical on one side and spherical (concave or convex) on the other side will be required.

In all cases of defect of vision the magnitude of the defect may not be the same for both eyes, so that lenses of different focal length may be required to accurately correct the vision.

150. Miscellaneous experiments and observations with the eye.

(1) **To show that the eye is over-corrected for spherical aberration.** Bring a printed page so close to the eye that the print is indistinct. Now interpose a sheet of paper with a pinhole in it between page and eye and just in front of the latter. The print seen through the hole is quite distinct. This shows that rays going through the centre of the lens are converged more than those going through the peripheral portions. The opposite occurs with ordinary lenses. (Art. 90.)

(2) **To show that the eye is not achromatic for the extreme rays though very nearly so for intermediate rays.** Looking at a window frame with a bright background and holding a finger close in front of the eye, gradually move it across the field of view. As the advancing finger approaches a bar of the window-frame the near edge of that bar will bear a blue fringe, and the far edge a red fringe. In trying to understand this remember that the image on the retina is inverted.

Another method of showing the existence of chromatic aberration in the eye is to bring a well-illuminated printed page close up to the eye and stare at it. Blue and yellow fringes will then be seen to all the letters.

(3) Images of real objects are inverted. Experience, however, tells us that the objects are the right way up. If therefore an erect shadow could be thrown on the retina, it should appear inverted. To verify this make a pinhole in a piece of paper and holding it about an inch or so in front of the eye view through it a brightly illuminated surface such as a white lamp globe. Take now a pin and holding it head upwards introduce it between the eye and the pinhole. As shown in Fig. 173 the pin appears inverted.

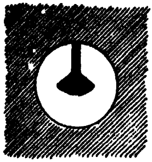
Fig. 173.

(4) It is evident that since the blood vessels of the retina are in front of the sensitive layer, light entering the eye should cast shadows of them upon this layer. That we are not always aware of these shadows is due to the fact that ordinarily they are formed by diffused light and hence are indistinct. In the following simple experiment parallel or nearly parallel light is used; thus the shadows are distinct and therefore easily seen:—View a very bright white surface through a pinhole in a piece of paper held just in front of the eye. Move the pinhole about; the bloodvessels will be seen as black shadows on the bright surface viewed. Now give a quick circular motion to the pinhole and observe that the bloodvessels seem to extend from the periphery to the centre, getting smaller and more ramified as they approach it (Fig. 174), the centre, however, being free from them. Now hold the paper still; the appearance vanishes, due to the rods and cones becoming fatigued.

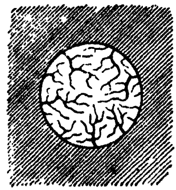
Fig. 174.

This is a subjective method of observation. The interior of the eye of another person may be, however, examined by means of the *ophthalmoscope*, invented by Helmholtz, which consists essentially of a concave mirror about an inch and a half in diameter, pierced through the centre by a small hole. Light from a lamp is reflected by this mirror into the patient's eye, and the observer looking through the small hole is able to minutely examine the various parts of the retina.

(5) Although the yellow spot is most sensitive to ordinary lights, it is not as sensitive to very faint lights as the surrounding portion of the retina. This is due to the fact that the bacillary layer at this point is composed entirely of cones (see Art. 145). For this reason, faint stars, looked at a little obliquely, appear brighter than if viewed directly.

(6) **To prove the existence of the blind spot.** Close the right eye and keeping *the left eye fixed on* B (Fig. 175) move the book to and

Fig. 175.

from the eye. When the book is about 12 inches from the face A will be invisible, but will come into view again for greater or lesser distances. Repeat with only the right eye open and fixed on A.

(7) The retina continues to feel the effects of the light after the exciting agent has been removed. This phenomenon is called the **persistence of impressions**, the impressions lasting about one-tenth of a second. Thus the glowing end of a match stick yields a bright circle when the match is swung around, and not a bright point changing its position. Again the colours of a rapidly rotating colour disc (Art. 120) blend into one; and an alternating electric current gives a steady light, the fluctuations of the light being too rapid to be noticed, except it be used to illuminate a rapidly moving object.

(8) Look at a bright object, such as a bright lamp globe, and then close the eyes, and, in addition, cover them. An image of the globe will appear; this is the *positive after-image*. After a time it disappears, and may be followed by a very dark image of the globe on the fairly dark background; this is the *negative after-image*. The positive image is due to continued nerve irritation, similar to the persistence of impressions, whilst the negative image is due to the fatigue of those nerves upon which the bright image fell. This prevents them being excited to the same extent, as the unacted-on nerves are, by the dull light penetrating the eyelids; hence the dark image. Repeat the experiment with a strongly illuminated coloured form, such as a church window. The positive after-image in this case is similar in colour to the window; it then fades away, and the negative after-image appears in the complementary colours.

Repeat by looking steadily at a bright red spot, and then shifting the eyes to a dull grey surface. A greenish-yellow spot will be seen on it. Other contrast results may be obtained by placing a small white circle of paper upon larger pieces of coloured papers. In each case the white seems to be illuminated by the colour complementary to that of the background. Thus on yellow it appears blue, and on green it appears of a ruddy hue. Again, if a hole be made in a piece of bluish green glass, and the glass be then held between the sun and a white screen, the shadow of the hole will look dark pink.

(9) Close the eyes and press with the fingers into one of the hollows on the upper side of the eyes next the nose. A circle of light will appear at the opposite side of the eyeball. This shows that the nerves may be stimulated by mere pressure from the outside.

(10) If we have two bodies of the same size, one bright and the other dark, the former will look the bigger of the two. This is caused by the rods and cones which are being excited causing their neighbours to be excited. The most striking example of **irradiation** occurs when the moon is in her first quarter, and is called "the old moon in the young one's arms." The illuminated crescent (Fig. 176) then appears to be a part of a much larger circle than that of the faintly illuminated remainder.

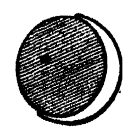

Fig. 176.

(11) A peculiar wavelike motion is often observed in a row of close-set railings when one is walking near by and looking through them at a farther row. At certain places bars in the two rows are behind each other, and thus the maximum amount of light is able to penetrate them. At other places bars in the farther row are behind spaces in the front row, and the illumination is smaller, varying to a minimum when the bars are behind the centres of the spaces. The same phenomenon is also very noticeable when a meat safe of perforated zinc is under observation, and when a piece of wire gauze is lying upon another. The appearance is very much like that of "watered silk."

(12) The **distance of objects** is judged partly by the amount of accommodation it is necessary to impress upon our eyes in order to see them distinctly, and partly by the amount of convergence between the optic axes of the two eyes. When the distance, however, exceeds a certain amount the accommodation is constant, and the optic axes are sensibly parallel. Hence other methods of judgment must be used, the usual being that of comparing the size of a known object at the far distance with the apparent size it would have when close to the observer. In judging distances, therefore, practice counts a lot. On a very clear day distant objects appear nearer than they really are, and hence we judge their magnitude smaller. In a fog vision is indistinct, and, as we associate indistinction with distance, we unconsciously estimate the object to be some distance away, and therefore larger than it really is.

Again, the sun and moon usually look larger when low down than when high up in the sky. This false impression is not in accordance with measurements of the angular diameter made by a micrometer. When near the horizon the eye is apt to estimate the size and distance of the sun and moon by comparing them with the neighbouring terrestrial objects (trees, hills, etc.). When the sun and moon are at a considerable altitude no such comparison is possible, and a different estimate of their size is instinctively formed.

(13) **To find the least distance of distinct vision.** Make two pinholes, $\frac{1}{16}$ in. apart, in a piece of paper. Hold the paper close up to the eye, the holes being in a horizontal line, and look through them at a vertical pin held just in front. The pin appears double and indistinct. Gradually remove the pin; the images become more distinct and approach each other, coinciding at a certain distance—the least distance of distinct vision—and afterwards remaining in coincidence. Explanation:—Two narrow pencils of light from the pin pass through the holes and after refraction in the eye converge to a common focus. If the pin is at or beyond the least distance of distinct vision the eye accommodates itself so that this common focus is on the retina; if, however, the pin is within this least distance the pencils reach the retina before meeting; thus two images are seen, both indistinct. Now repeat the experiment and block out, say, the left-hand hole. The right-hand image disappears, and *vice versa*. In proving this, remember that the brain inverts the images.

CALCULATIONS.

151. THE following relation, proved in the preceding chapter, should be noted :—

The magnifying power of a simple lens of numerical focal length f_1 is given by—

$$m = 1 + \frac{D}{f_1}. \qquad \text{(Art. 148.)}$$

In connection with vision and spectacles the following summary should be remembered.

1. *The normal or emmetropic eye.* In the quiescent stage it is focussed on infinity, and by accommodation all points from infinity to about 10 inches in front of the eye can be seen.

2. *The myopic or short-sighted eye.* Parallel light is brought to a focus in front of the retina. The eyeball is too long or the lens too convex, and a concave spectacle lens is required.

3. *The hypermetropic or long-sighted eye.* Parallel light is brought to a focus behind the retina. The eyeball is too short or the lens not sufficiently convex, therefore a convex spectacle lens is required.

4. *The presbyopic eye.* This possesses scarcely any accommodation. Being set for distant objects the focus for near objects is behind the retina, hence convex lenses are required.

5. *The astigmatic eye.* Cornea not spherical, hence the foci of lines in different directions are at different distances behind the crystalline lens. Cylindrical or sphero-cylindrical lenses must be employed, generally in conjunction with spherical ones.

In nearly all cases the problem of finding the right lens can be solved by a judicious use of the formula—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

EXAMPLES VIII.

1. A person's range of distinct vision is from 4 inches to 8 inches from the eye ; find the focal length of the lenses he should wear, and the range of his vision with those lenses.

Here the lenses required are such as will cause rays coming from infinity to diverge from a point 8 inches from the eye—that is, *concave* lenses of 8 inches focal length are required.

With these lenses let x denote the nearest distance of distinct vision; then—

$$\frac{1}{4} - \frac{1}{x} = \frac{1}{8},$$

or—

$$x = 8.$$

Therefore the range of vision with the lenses is from 8 inches to infinity.

2. Explain, by help of a diagram, the effect of a convex lens held close to the eye and employed as a simple microscope.

Prove an approximate formula for the magnifying power of the lens, its principal focal length and the distance of distinct vision by the naked eye being given.

3. A person whose nearest distance of distinct vision is 18 inches uses a reading lens of 6 inches focal length; what magnification does he obtain?

4. A person can see objects distinctly only at a distance of about 4 inches from the eye; calculate the focal length of lenses he should use for reading, walking, and for viewing distant objects. Assume that 10 inches is the normal distance of nearest distinct vision, and that in walking the average distance at which he requires to see clearly is 15 feet.

5. An aged person sees distinctly from infinity up to about 20 inches from the eye. What spectacles should be worn to remedy this defect?

6. A lantern slide is $3\frac{1}{4}$ inches square and an enlarged image is to be formed on a screen 20 feet distant from the lens by the aid of a lens of 6 inches focal length. What kind of lens should be used, at what distance from the slide must it be placed, and what will be the size of the image?

7. Explain why it is possible to take a photograph with a pinhole pierced in an opaque screen in place of a lens, and why it is not possible to do so when the hole is large.

8. A pinhole camera is made in the form of a cube (edge 1 foot) with a hole in the centre of one side. It is placed opposite a building 60 feet high at a distance of 100 yards. Find the size of the image.

9. A total reflection prism is employed to deviate a ray through 60°. What is the shape of its section?

10. A person is under water. Do objects in the water look at the same distance from him as they would in air? Why are near objects indistinct?

EXAMINATION QUESTIONS.

QUESTIONS SET AT VARIOUS UNIVERSITY EXAMINATIONS.

1. Explain the action of a lens when used as an eyeglass. A man who can see most distinctly at a distance of 5 inches from his eye wishes to read a notice at a distance of 15 feet off. What sort of spectacles must he use, and what must be their focal length?

2. Explain with a sketch the action of an optical lantern. Why should the radiant be as small as possible?

3. Draw and explain a figure illustrating the principle of a simple magnifying lens, and calculate the magnifying power of a lens of half-inch focal length held close to an eye whose best vision distance is 8 inches.

4. A short-sighted person has distinct vision at 5 inches. What kind of lens should he use and of what focal length, to enable him to read a book 20 inches from his eyes?

5. The maximum distance of distinct vision for a certain person is 20 centimetres. To enable him to see distant objects distinctly he will require a lens. Calculate either (*a*) the power, in *dioptries*, of that lens, or (*b*) its focal length, in centimetres. Explain also, with the aid of a diagram, why this lens will enable him to have distinct vision.

6. Give a brief general account of the eye as an optical instrument. Suppose a short-sighted eye can see an object clearly only when it is placed at a distance not exceeding 8 inches, what kind of lens should be used, and of what power, in order that, if placed close to the eye, it would enable objects that are 48 inches away to be clearly seen?

7. A convex lens of 2 inches focal length is held 1 inch from the eye by a person with distance of distinct vision of 9 inches so as to look at a small object. Where must the object be placed? Illustrate your answer by a figure.

8. Explain how a person who is short-sighted is enabled by using a lens to see things at a distance. Of what sort and of what focal length should the lens be if the person can only see things clearly up to 2 feet away?

9. Describe a sextant and explain how it is used. Explain carefully how it is that you see the two images largely superposed and not merely beside each other.

10. Show by a drawing how you would employ a right-angled isosceles glass prism to bend a beam of light at right angles. Will any light be lost at the hypotenuse? State fully the reasons for your answer.

CHAPTER XII.

MORE COMPLEX OPTICAL INSTRUMENTS.

152. In this chapter we shall consider the more complex optical instruments—telescopes, microscopes, spectroscopes, etc., and consider the various devices by which spherical and chromatic aberrations are brought to a minimum.

153. Pocket microscopes. We have already (Art. **147**) discussed the use of a single convex lens on a magnifying glass. A lens of high power used in this way is called a **simple microscope**, and is most efficient if made plano-convex and used with its plane side towards the eye. The magnifying power is inversely proportional to the focal length. In practice it is found that, as the focal length is decreased, distortion and chromatic defects creep in, and a single lens only acts well if its focal length be not less than one inch, so that for greater magnifying powers recourse must be had to combination of lenses.

The simplest forms of pocket magnifiers are—

1. The **Coddington lens.** This was invented by Wollaston and is simply a sphere of glass (Fig. 177 A) in which a

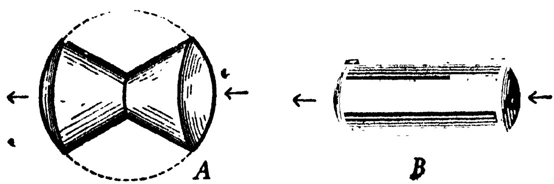

Fig. 177.

deep groove has been cut all the way round leaving only a small central aperture though which the rays may pass.

2. **The Stanhope lens.** This consists of a cylinder of glass (Fig. 177 B) whose ends are ground to spherical surfaces of unequal radii. The dimensions are so chosen that when a small object is placed on the end of the lesser curvature, an eye placed close to the other end sees a magnified and well-defined image.

3. **Wollaston's doublet.** This was the first combination of two lenses used for this work, and in appearance is very similar to an inverted Huyghens eye-piece (Art. **160**). The deviation of the rays is borne equally by the two lenses and thus defects of aberration and achromatism are minimised.

154. The telescope. Telescopes are employed for the purpose of obtaining distinct vision of distant objects, especially stars and other celestial bodies. They are of two kinds—*refracting* and *reflecting*, but the same general principle underlies them all.* A real image of the object is formed by a convex lens (*object glass*) or concave mirror (*speculum*), and is examined by a magnifying glass (*eye-piece*). There are several different *forms* of telescopes, the details of construction in each case being adapted to the purpose for which that particular form is intended.

155. The Refracting Astronomical Telescope. This instrument was invented by Kepler, the astronomer, in 1611,

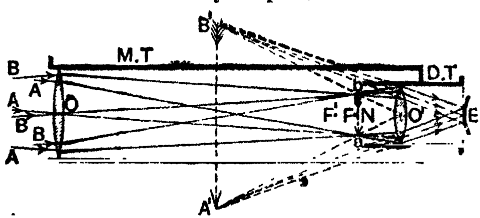

Fig. 178.

but was first used by Huyghens in 1655. In its simplest form it consists of a convex lens, O, fixed at one end of a brass tube (Fig. 178), and another and smaller convex lens,

* The opera or field glass is not included in this general statement. It is also possible to construct a telescope entirely of prisms.

O′, fitted in a tube which slides inside the former. The lens O, which first receives the rays from the distant object, is called the *object glass*, and may be a simple convex lens; but in the best instruments it is a compound achromatic lens (Art. 108), having its focus at F. The lens O′, at the other end of the tube, may also be a single lens, but generally consists of a system of two lenses, called an *eye-piece*, the focal length of the system, O′ F′, being considerably smaller than that of the object glass. Considering the simple form shown in Fig. 178, the optical action of the instrument may be explained as follows. Let A, A, A, denote rays coming from a point A on a distant* object A B.† These rays, after refraction through the object glass, O, are brought to a focus at *a*. Similarly the rays B, B, B, coming from a point B on the object, are brought to a focus at *b*, and a real inverted image of the distant object is obtained at *a b*. By moving the eye-piece tube the position of the lens O′ can be adjusted so that the image *a b* falls just within its focal length—that is, the distance of *a b* from O′ should be slightly less than the focal length of O′. With this adjustment an eye looking through O′ sees a virtual magnified image of *a b* at A′ B′ (Art. 85, 1, 2). If the instrument is focussed so that A′ B′ is seen at an infinite distance from the eye, then *a b* is at the focus of O′. Also, if the object A B is distant, then *a b* is practically at the focus of O. Hence, under these conditions, the image *a b* is at a point which is the common focus of O and O′, and the length of the telescope is equal to the sum of the focal lengths of the object glass and eye-piece. For more distinct vision, however, the eye-piece is often focussed so that A′ B′ is seen at the nearest distance of distinct vision, the distance of the image *a b* from O′ being then less than the focal length of the eye-piece, and the length of the telescope thus slightly less than the sum of the focal lengths of the object glass and the eye-piece. If a near

* If the object is very distant, then the rays A, A, A, are practically parallel.

† Not shown in the figure.

object is viewed, the distance of $a\,b$ from O is greater than the focal length of the object glass; hence in this case the length of the telescope is greater than the sum of the focal lengths of the object glass and eye-piece It will be noticed that the image $a\,b$ is inverted, and that, since A′ B′ is an erect image of $a\,b$, the image seen on looking through the instrument is inverted; this is immaterial in astronomical observations, but for terrestrial purposes it is necessary to have an erect image. In order, therefore, to adapt an astronomical telescope to ordinary use, it must be fitted with an *erecting eye-piece* similar to that described below (Art. **159**, 8).

156. Determination of the magnifying power of a telescope. In order to determine the magnifying power of a telescope, it is only necessary to obtain the ratio of the visual angle (Art. **147**) of the image to that of the object, the latter being seen at its actual distance from the eye. If this distance is great, compared with the length of the telescope, then, in Fig. 178, A O B is practically the angle which the object A B subtends at the eye. Similarly A′ O′ B′ is the angle which the image A′ B′ subtends, and the magnifying power is given by the ratio $\frac{A'O'B'}{A\,O\,B}$. But the angle A O B is equal to $a\,O\,b$, and A′ O′ B′ is identical with $a\,O'b$ (cf. Figs. 106, 167); therefore we have $\frac{A'O'B'}{A\,O\,B} = \frac{a\,O'b}{a\,O\,b}$; and, the angles involved being small, the magnifying power is therefore approximately equal to $\frac{a\,b}{N\,O'}\Big/\frac{a\,b}{N\,O} = \frac{N\,O}{N\,O'}$. Now if the object is very distant, the image $a\,b$ is formed close to the principal focus of the object glass; and, if the position of O′ is adjusted so that $a\,b$ is very near to its principal focus, then F and F′ coincide with N, which then becomes the *common* focus of O and O′, and *the magnifying power is given by the ratio of the focal length of the object glass to the focal length of the eye-piece.* That is, if m denote the measure of the magnifying power, F_1 the numerical value

of the focal length of object glass, and f_1 the numerical value of the focal length of the eye-piece, then—

$$m = \frac{F_1}{f_1}.*$$

This, it must be remembered, is true only when the object is very distant, and when the eye-piece is placed so that the image $a\,b$ is at its focus, and the virtual image A′B′ is therefore at infinity. A slight increase in magnifying power is obtained by focussing so that A′B′ is seen at the nearest distance of distinct vision.

With this adjustment the magnifying power is $\frac{F}{f}\left(1-\frac{f}{D}\right)$ where D is the shortest distance of distinct vision. For, the image A′ B′ being now at a distance D from O′, the distance of $a\,b$ from O′ is given by—

$$\frac{1}{D} - \frac{1}{u} = \frac{1}{f}$$

where u denotes the required distance. That is—

$$\frac{1}{u} = \frac{1}{D} - \frac{1}{f} = \frac{f-D}{f\,D}.$$

$$\therefore u = \frac{f\,D}{f-D},$$

and the magnifying power, being now determined by the ratio $\frac{F}{u}$, is equal to $\frac{F\,(f-D)}{f\,D}$; or, substituting numerical values and neglecting sign,

$$m = \frac{F_1(D+f_1)}{f_1\,D} = \frac{F_1}{f_1}\left(1 + \frac{f_1}{D}\right).$$

Exp. 44. To find the magnifying power of a telescope.

(1) Determine F and f by ordinary methods and substitute in the above equations.

(2) Focus the telescope on a distant object, such as a slate roof or a brick wall. Arrange that the image is formed in the plane of the

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* The sign convention is here neglected. Strictly m is *negative* because the image is *inverted*; this is, seen from the ratio $\frac{N\,O}{N\,O'}$, for $N\,O = F$, and $N\,O' = -f$; that is, $m = -\frac{F}{f}$.

object and thus all parallax is avoided. With one eye at the telescope and the other unassisted, then note how many slates or bricks as seen by the unaided eye occupy the same length as one slate or brick seen through the telescope. This number is the magnifying power.

(3) Focus the telescope for infinity and then point it towards a bright cloud or a strongly illuminated surface. Observe the bright circle on the eye-piece. Measure its diameter $a\,b$ (Fig. 179) by means of a fine scale, and also the diameter A B of the object glass. Then clearly,

$$\frac{\mathrm{A\,B}}{a\,b} = \frac{\mathrm{F}_1}{f_1} = m.$$

Nearly all telescopes contain stops (Arts. **46**, **90**) whose function is to cut off the marginal rays proceeding from the object glass. In Fig. 179 the presence of S virtually diminishes the diameter of the object glass from A B to A′ B′, with a proportional decrease from $a\,b$

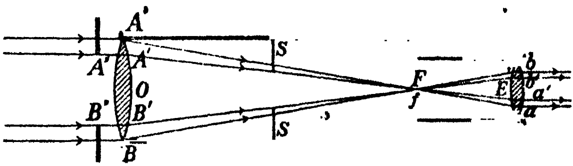

Fig. 179.

to $a'\,b'$ of the diameter of the circle of light on the eye-piece. To eliminate this error place a wide adjustable slit in front of the object glass and narrow it down until its shadow begins to encroach on the circle $a'\,b'$. Then carefully adjust it so as just not to encroach. The ratio $\frac{\mathrm{A'\,B'}}{a'\,b'}$ is the magnifying power.

(4) Focus the telescope for infinity and point it towards the light. Remove the object glass, and obtain a real image of the circular opening in which the object glass fits upon a transparent glass millimetre scale placed somewhere near E (Fig. 178). Denote the diameter of the circle by $c\,d$. Measure $c\,d$ and A B. Then—

$$\frac{\mathrm{A\,B}}{c\,d} = \frac{\mathrm{F}_1}{f_1} = m.$$

The proof is as follows: Let v be the distance of the image from the eye-lens. Then, since the object is at a distance $\mathrm{F}_1 + f_1$ from the lens, we have—

$$\frac{1}{v} + \frac{1}{\mathrm{F}_1 + f_1} = \frac{1}{f_1} \therefore v = \frac{f_1(\mathrm{F}_1 + f_1)}{\mathrm{F}_1}.$$

But—

$$\frac{\mathrm{A\,B}}{c\,d} = \frac{\mathrm{F}_1 + f_1}{v} \therefore \frac{\mathrm{A\,B}}{c\,d} = \frac{\mathrm{F}_1}{f_1}.$$

157. The reflecting telescope. This assumes many forms. The earliest described was invented by Gregory in 1663. In the most common, or *Newtonian*, form (Fig. 180), invented in 1668, the end of the tube turned towards the object is open, and at the other end is a concave mirror or *speculum*, S, of glass silvered on the front surface.

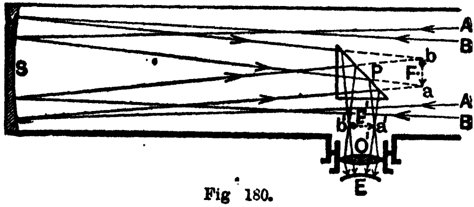

Fig. 180.

This, if it were allowed to, would form at its principal focus a real image $a\ b$ of the object to which it is directed. But before reaching the focus, the rays fall on a total reflection prism (Art. 140), which diverts the image to $a'\ b'$, where it is examined through an eye-piece E in the side of the tube.

A great advantage of a reflecting telescope over a refracting telescope is that no chromatic effects are introduced at reflection, also if required for observation on celestial bodies, aberration can be completely eliminated by using a paraboloidal mirror (Art. 138).

158. Why astronomical telescopes are made large. When an image is highly magnified, it is, of course, proportionately reduced in brilliancy. Many terrestrial, and most celestial, objects are themselves comparatively faint. It is therefore desirable that the telescope should be able to grasp as much light as possible from the object. This is the reason for the employment of object glasses or specula of large diameter or *aperture*, for the light-grasping power is proportional to the square of the diameter. By so doing many stars which are invisible to the naked eye are revealed. A long focus, and therefore a long tube, is used to give great magnifying power, and also to decrease aberration by keeping the curvature of the reflecting or refracting surfaces low.

As a rule, astronomical telescopes are provided with several eye-pieces of different focal lengths. This allows the observer a range in magnifying power which is most convenient, as some objects will stand a greater magnification than others.

Since light is a species of wave motion, the focus of a beam or the image of an object is simply the place where all the secondary waves reinforce each other.* It therefore follows that the image of a point source will always possess a finite size. By increasing the aperture of the mirror or object glass, it may be proved that, quite irrespective of its focal length, the dimensions of the images of point-sources are decreased in the inverse ratio, hence the great advantage of mirrors and object-glasses of large aperture for stellar observations. An increase of magnifying power decreases the brightness of images of extended objects, for the field of view is now occupied by a smaller portion of the object. It, however, increases the relative brightness of images of point-sources such as stars, for in this case, although the actual image is not made brighter, the brightness of the surrounding field is decreased.

Achromatic refracting telescopes give better results than reflecting telescopes of the same size, but owing to the immense care required in the manufacture of good lenses over two feet in diameter, most very large telescopes are reflectors. Lord Rosse's large reflector erected at Parsonstown, Ireland, in 1845, has a mirror of 6 feet in diameter and focal length of 56 feet. The largest refracting telescopes are very costly and hence are to be found chiefly in America. The Lick and Yerkes instruments have apertures of 36 and 40 inches respectively, and focal lengths of 58 and 62 feet respectively.

159. Equivalent lens. The image formed by a single lens is subject to several defects, due to spherical and chromatic aberrations. To remedy these defects it has been found necessary in optical instruments to employ achromatic lenses, or combinations of two or more ordinary lenses arranged along a common axis.

To understand the actions of such systems it is necessary to understand what is meant by an *equivalent lens.* A lens is said to be *equivalent* to a combination of lenses when it produces the same *deviation* in a ray incident parallel to its principal axis as that produced by the combination, the equivalent lens being placed in the position of the first lens

* See Catchpool's "Textbook of Sound," Art. 121.

on which the light falls. The deviation produced by a lens is thus determined. Let P Q (Fig. 181) represent a ray incident at Q on the lens L in a direction parallel to the principal axis; then, if F represent the focus of the lens, this ray is reflected along Q F, and the deviation produced is measured by the angle P′ Q F—that is, by the angle

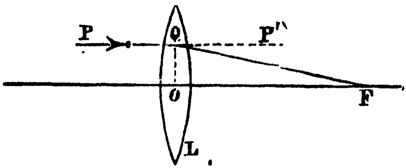

Fig. 181.

Q F O. If this angle is small, it is approximately equal to its tangent O Q/O F. That is, *if a ray, parallel to the axis of a lens of focal length f, be incident on it at a distance x from the axis, then the deviation produced is approximately given by the ratio x/f.* This is also approximately true when the inclination of the rays to the axis is small.

Consider now a system of two lenses placed at a distance a apart on a common axis, and let it be required to determine the focal length of the combination. In order that all the quantities involved may be positive, we shall take two con-

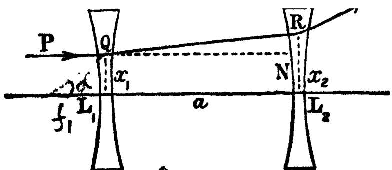

Fig. 182.

cave lenses, L_1 and L_2 (Fig. 182), of focal length f_1 and f_2. Then, the deviation of the ray P Q produced by L_1 is equal to x_1/f_1 and the deviation of Q R by L_2 is approximately equal to x_2/f_2, that is, the total deviation is given by

$$\frac{x_1}{f_1} + \frac{x_2}{f_2}.$$

field-lens, they are refracted through b', which, in order that the rays may emerge parallel, should be in the focus

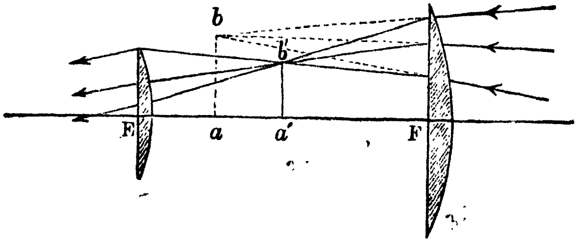

Fig. 185.

of the eye-lens. To determine the position of $a\,b$, in order that $a'\,b'$ may be at the focus of E, we have—

$$\frac{1}{\mathrm{F}\,a'} - \frac{1}{\mathrm{F}\,a} = \frac{1}{3f}$$

where f is the focal length of the eye-lens.

Also, if $a'\,b'$ is at the focus of E, then $\mathrm{E}\,a' = -f$; and, since $\mathrm{E\,F} = -2f$, $\therefore a'\,\mathrm{F} = -f$; that is, $\mathrm{F}a' = f$.

Therefore—

$$\frac{1}{f} - \frac{1}{\mathrm{F}\,a} = \frac{1}{3f},$$

or—

$$\frac{1}{\mathrm{F}\,a} = \frac{1}{f} - \frac{1}{3f} = \frac{2}{3f}.$$

$$\therefore \quad \mathrm{F}\,a = \frac{3f}{2}$$

That is, the rays coming from the object glass should, if uninterrupted, meet at a point behind the field-lens, or in front of the eye-lens, at a distance from either lens equal to half the focal length of the lens considered. If the object viewed is very distant, then this may be expressed by saying that the focus of the object glass should lie between the lenses of the eye-piece at a distance from either lens numerically equal to half the focal length of that lens.

It will be seen from what has been said above that the rays from the object glass converge to a point behind the

field-lens, and the image $a\,b$ has therefore no *real* existence, the rays being refracted by F to form the image $a'\,b'$. For this reason Huyghens' eye-piece has been called a *negative* eye-piece.

It should be noticed that the conditions of achromatism of this eye-piece have been deduced by applying the general conditions of achromatism to the formula giving the focal length of a single lens *equivalent* to the system of lenses forming the eye-piece. Now this formula for the equivalent lens has been obtained in accordance with the definition of equivalence given in Art. 159, and only implies equal deviation by the system and its equivalent. Hence the rays for which the eye-piece is achromatic will suffer equal deviations, and will therefore, on emergence, be parallel, *but not necessarily coincident.* The eye, however, treats all parallel rays as coincident, so that the arrangement is practically achromatic, although the achromatism is not perfect; moreover, since both lenses are of the same material, and

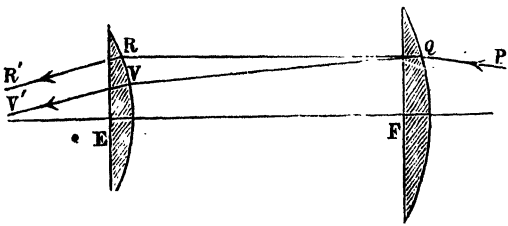

Fig. 186.

therefore of the same dispersive power, the system is equally achromatic for all colours. Fig. 186 illustrates the action of the eye-piece as an achromatic system. The ray P Q, incident on the field-lens F at Q, suffers refraction and dispersion, and the violet light travels along Q V, while the red light travels along Q R, the intermediate rays lying between these extreme rays. Now Q R is less deviated than Q V, and therefore, in order that they may be parallel after emergence from E, the deviation of Q R at R must be greater than that of Q V at V. This, as explained in

Art. 79, will evidently be the case; and, if the lenses are properly chosen and placed, the emergent rays R R′ and V V′, together with all intermediate rays, will emerge parallel, and appear to an eye receiving them to come from the same point.

6. This eye-piece, though theoretically perfect, cannot be used in telescopes where measurements by the aid of cross-wires are made. The measurements are effected by means of a frame carrying cross-wires of fine platinum wire, silk thread, or spider-threads in the focal plane of the object glass. The real image of the object being formed in this plane is coincident with these threads, and when viewed through a suitable eye-piece both image and cross-wires are magnified to the same extent, and any slight distortion produced by refraction through the lenses of the eye-piece is the same for both. Hence points at which the real image and the cross-wires actually coincide are also the points seen to be coincident on looking through the eye-piece. The actual distance between any two points on the real image is readily measured by means of this arrangement. The frame carrying the cross-wires can be moved in its own plane by a micrometer screw with a graduated head;* and, by bringing a particular cross-wire into coincidence first with one point and then with another, the distance between the two points can be read off in terms of the graduations on the micrometer head. This distance is readily converted into angular distance; for, if F be the focal length of the object glass and d the given distance, then $\frac{d}{F}$ gives approximately the required angular distance. Now it is evidently impossible to adopt the arrangement described above with Huyghens' eye-piece, for the real image of the object by the object glass is not formed, and the cross-wires could not be placed at $a' b'$ (Fig. 185), for points on $a' b'$ have not necessarily the same relative position as the corresponding points of $a b$ would have.

7. For purposes of measurement, therefore, astronomical

* See "Higher Text Book of Heat," Art. 18 (2).

telescopes are usually fitted with what is known as **Ramsden's eye-piece**, which is sometimes called a *positive* eye-piece. It is usually made up of two plano-convex lenses of equal focal length, placed with their convex surfaces facing each other, and separated by a distance equal to two-thirds the focal length of either. The conditions of achromatism require that the distance between them should be equal to the focal length of either; but if this arrangement were adopted the field-lens would be at the focus of the eye-lens, and would therefore interfere with distinct vision, especially if the glass was not quite clear, or if any dust happened to lie on its surface. The distance between the lenses is for this reason reduced to two-thirds the focal length of either, and, although the system thus arranged is not perfectly achromatic, it is very nearly so.

Let E and F (Fig. 187) represent the eye-lens and field-lens of Ramsden's eye-piece. Rays coming from the object

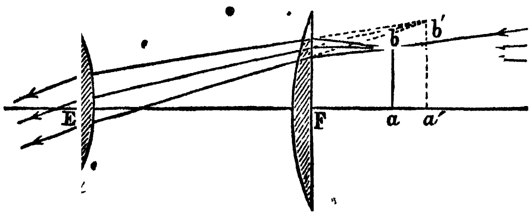

Fig. 187.

glass converge to a focus at b in front of the field-lens F, and passing through b fall on F, where they suffer refraction, and after passing through it diverge from b', which, in order that the rays may emerge parallel from E, should be in the focus of that lens. That is, E a' should be equal to $-f$ where f is the focal length of either of the lenses. But—

$$EF = -\frac{2}{3}f. \quad \therefore Fa' = -\frac{1}{3}f.$$

Hence, to determine the position of $a\,b$, we have—

$$\frac{1}{Fa'} - \frac{1}{Fa} = \frac{1}{f};$$

that is—

$$-\frac{3}{f}-\frac{1}{\mathrm{F}a}=\frac{1}{f}.$$

$$\therefore \frac{1}{\mathrm{F}a}=-\frac{3}{f}-\frac{1}{f}=-\frac{4}{f}$$

$$\therefore \mathrm{F}a=-\frac{f}{4}.$$

That is, the real image formed by the object glass should fall *in front* of the field-lens at a distance from it numerically equal to one-fourth its focal length. This is also the proper position for the frame carrying the cross-wires. This eye-piece does not satisfy the conditions of minimum aberration, but the curvature of the faces of the lenses is arranged to remedy these defects as far as possible, and the indistinctness due to this cause is inappreciable.

8. The only other eye-piece that need be mentioned is the *erecting eye-piece*, referred to in Art. **155**, which is used to adapt an astronomical telescope for observation of terrestrial objects. It consists of four lenses: the two nearest the object glass, A and B (Fig. 188), are of equal focal

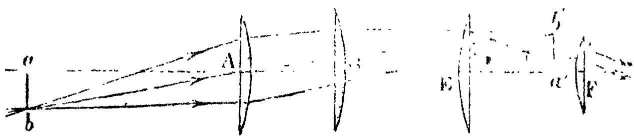

Fig. 188.

length, and placed at any distance from each other; the two nearest the eye, E and F, form a Huyghens' eye-piece. Rays coming from the object glass meet at b in front of the lens A, and at a distance from it equal to its focal length. Passing through b, the rays are refracted through A, and, emerging parallel, fall upon B, which brings them to a focus at b'. In this way an inverted image of $a\,b$—that is, an erect image of the external object—is formed at $a'b'$, and the system E F is adjusted to this image in the way indicated in the figure, and explained above in reference to Fig. 185. The four lenses are usually fixed in a tube in their proper

relative positions, and the adjustments relative to the image $a\,b$ are effected by sliding this tube in and out.

9. In estimating the magnifying power of a telescope fitted with an eye-piece, the focal length of the lens equivalent to the eye-piece system must be taken.

On looking at Fig. 178, it will be seen that the rays, after emergence from the eye-piece, cross in the plane indicated by E E; the section of the beam in this plane is approximately circular, and, as it marks the proper position of the eye, it is called the *eye-ring*. Telescopes are usually so constructed that the aperture in the cap of the eye-piece indicates the position of the eye-ring; hence in looking through a telescope the eye should be placed close up to this aperture. From this figure it will be seen that the eye-ring, or *bright spot* as it is sometimes called, is the image of the object glass formed by the eye-piece, and it follows from Exp. 44, 4, that the ratio of the diameter or radius of the object glass * to that of the bright spot gives the magnifying power of the instrument.

161. Reading or observing telescope. A small astronomical telescope is very useful for reading distant thermometers, scales, etc. A long tube, A (Fig. 189), carries an

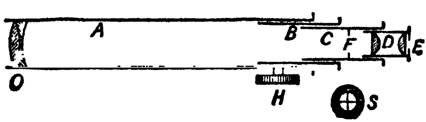

Fig. 189.

achromatic object-glass, O, at one end. A tube, B, slides into A, C into B, and a Ramsden's eye-piece, D into C. At F is fixed a ring, S, carrying the cross-wires. The tubes B, C, D can be moved by turning the milled head, H—the so-called *focussing screw*. Thus the distance between the objective, O, and the cross-wires can be adjusted

* The diameter of the object glass is sometimes called the *aperture* of the telescope.

without affecting the distance between the eye-piece and cross-wires.

To set a telescope up for use, first adjust the tube, D, by sliding it in or out C, until the cross-wires appear clear and distinct to an eye placed at E; then direct the tube to the object, and turn the milled head, H, until the image of the object and cross-wires appear distinct *simultaneously*. Also move the eye about in front of the eye-piece: there must be no movement of the image relatively to the cross-wires, *i.e.* no parallax.

162. Galileo's telescope. This form of telescope was invented by Galileo in 1609. It consists, like the astronomical telescope, of an object glass and an eye-piece. The object

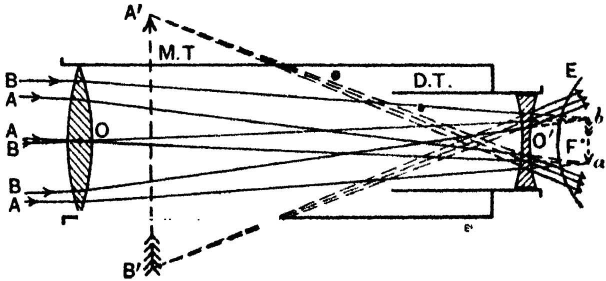

Fig. 190.

glass is exactly similar to that of the astronomical telescope, but the eye-piece is different; it is simply a double-concave achromatic lens placed between the object glass and its principal focus, and at a distance from the latter equal to or slightly greater than its own focal length. The optical action of this arrangement will be understood on reference to Fig. 190.

O represents the object glass, having its focus at F, and O′ the eye-piece, placed so that the distance O′ F is equal to or slightly greater than its focal length. We shall suppose that O′ F is equal to the focal length of O′, and that F is therefore the common focus of O and O′. The rays A, A, A,

and B, B, B, coming from points A and B of a distant object, would, if uninterrupted, form an image $a\,b$ at a point very near to F, but falling on O′, these rays are refracted so as to diverge from the points A′, B′. A virtual erect image of A B is thus seen at A′ B′ by an eye E. The position of this image depends upon the position of O′ relative to $a\,b$; if the distance from O′ to $a\,b$ is equal to the focal length of O′, then A′ B′ is at infinity; but, if this distance is greater than the focal length, then A′ B′ is nearer, and may by adjusting the position of O′ be brought to the nearest point of distinct vision, as in the figure. For the purpose of easily effecting this adjustment the eye-piece is mounted in a sliding tube D.T. (Fig. 190) called a draw tube, which slides in the main tube M.T. of the telescope.

The magnifying power is given, as in the case of the astronomical telescope, by the ratio $\frac{a\,O'\,b}{a\,O\,b}$; that is, if the object viewed is very distant and the image A′ B′ is at infinity—

$$= \frac{a\,b}{O'\,F} \Big/ \frac{a\,b}{O\,F} = \frac{O\,F}{O'\,F} = \frac{F}{f}$$

where F and f are respectively the focal lengths of the object glass and eye-piece.

The chief advantages of this form of telescope over the astronomical are, that the construction of the instrument is simplified, its length is reduced, and an erect image is obtained directly without the aid of a special eye-piece. On the other hand, the disadvantages are numerous; the errors of aberration are imperfectly remedied, cross-wires cannot be used for the reason explained in connection with Huyghens' eye-piece, and the magnifying power and field of view are very limited. This last disadvantage arises from the fact that the eye-ring is virtual, and therefore lies inside the instrument, so that only a portion of the rays diverging from it can be received by an eye placed at the eye-piece.

For these reasons the Galilean telescope is best adapted for observation of terrestrial objects and where only a

small magnifying power is required. Opera glasses, field glasses, and some marine glasses are the principal forms of the instrument. These forms are usually *binocular*—that is, they consist of two telescopes with parallel axes, mounted so that both eyes may be conveniently used in looking at any object.

163. The compound microscope. The compound microscope is in principle exactly similar to the astronomical telescope. The difference between the two instruments results from the adaptation of the object glass, or *objective* as it is called in the case of the microscope, to the purposes of the instrument. The objective of a microscope, O (Fig. 191), is essentially a convex lens of very short focal length. The small object to be viewed, A B, is placed close to O at a distance slightly greater than the focal length of that lens, and a real image of this object is formed at $a\,b$ in front of the eye-piece O′. An eye looking through O′ sees, at A′ B′, a magnified virtual image of this already magnified real image, and by adjusting the position of O′ this virtual image can be seen at the nearest distance of distinct vision.

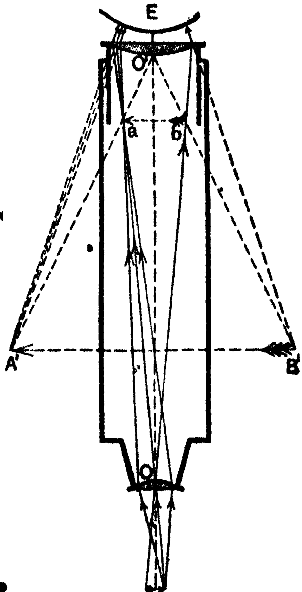

Fig. 191.

In actual instruments the objective is generally a complicated system of several lenses, constructed and arranged in order to diminish as far as possible the errors of aberration. Owing to the nearness of the object to the objective, the obliquity of the incident pencils is very great, and hence special precautions have to be taken to prevent excessive

spherical aberration. The eye-piece is usually an ordinary Huyghens' eye-piece, but when measurements have to be made a Ramsden's eye-piece must be substituted.

Owing to the great magnification produced, the object requires to be very brilliantly illuminated, or the image at $A'B'$ would be too faint to be of any use; hence most microscopes are provided with a reflecting mirror, mounted so that it can be easily placed so as to concentrate light on the object viewed. The magnifying power is evidently determined, as in the case of the simple lens (Art. 147), by the ratio $\frac{A'B'}{AB}$, for both object and image are supposed to be seen at the nearest distance of distinct vision. But $\frac{A'B'}{AB} = \frac{A'B'}{ab} \cdot \frac{ab}{AB}$. Now $\frac{A'B'}{ab}$ is the magnification produced by the eye-piece, and is approximately equal to $\left(1 - \frac{D}{f}\right)$ where D is the nearest distance of distinct vision and f is the focal length of the eye-piece. Also $\frac{ab}{AB} = \frac{Oa}{OA}$. Therefore the magnifying power is approximately given by—

$$m = \left(1 - \frac{D}{f}\right)\frac{Oa}{OA}.$$

But D, f, and Oa are constants, for the same adjustment of the same eye-piece.

$$\therefore\ m \propto \frac{1}{OA}.$$

That is, the magnifying power varies inversely as the focal length of the objective. For example, the magnification * produced by an objective of $\frac{1}{3}$ inch focal length is *approximately* three times that produced by an objective of 1 inch focal length.

Exp. 45. To determine the magnifying power of a microscope. Adopt the method of Exp. 44, 2, using, however, two scales—one, a fine one for observation through the microscope; the other, a coarser,

* It must be understood that throughout this chapter "magnification" means *linear* magnification.

one, for observation with the naked eye. The magnifying power is the true ratio of apparently equal lengths.

Instead of cross-wires microscopes are often provided with a minutely divided scale in the eye-piece called a *micrometer scale*, for the purpose of measuring bodies of small dimensions. Before the absolute lengths can be got, the micrometer scale divisions must first be expressed in millimetres; and this is done by focussing the microscope upon a finely divided scale called a *stage micrometer*, graduated in millimetres and tenths, and noting how many divisions of the eye-piece scale are apparently equal to a millimetre. The object to be measured is then placed on the stage of the microscope, and its dimensions obtained in terms of the eye-piece scale divisions, and then by simple arithmetic calculated in millimetres.

There is no limit to vision. Any particle, however minute, can be seen as long as it can be suitably illuminated. If the dimensions of the particle are much less than half a wave-length of light it is only seen as a whole, *i.e.* its separate parts cannot be discriminated. The visibility of such particles is effected by focussing an intense beam of light upon them, and then viewing them through a suitably placed microscope, when they appear as bright points. The case is analogous to that of dust-motes, which though as a rule invisible to the naked eye, are easily seen when a strong beam of sunlight passes through the air in which they are situated.

164. The spectroscope. This is an instrument constructed for the production and careful examination of pure spectra (Art. **103**). It consists essentially of three parts—the *collimator*, the *prism*, and a *telescope*. The *collimator*, C (Fig. 192), is a tube, like a telescope tube with a *slit*, S, at one end, and a lens, L, at the other. The slit is an important part of the instrument; it consists of metal jaws with exactly parallel edges, and its width can be adjusted by means of an attached screw arrangement. The length of the collimator tube is such that when properly focussed the slit is at the principal focus of the lens.

The *prism*, P, is a short prism of glass of triangular cross-section, similar to those referred to in Art. **73**.

The *telescope*, T, is a small astronomical telescope, exactly similar to that described above in Art. **161**. The arrange-

ment of these three parts is shown in the figure; the telescope and collimator are attached to a central pillar and table, on which the prism is placed. The source of light whose spectrum is required is placed at S, so that the light from it falls directly on the slit of the collimator. The rays diverging from the slit are refracted through the lens L, and, emerging parallel, fall upon the prism P, where they again suffer refraction. Here each ray of the incident beam has the same angle of incidence, and rays of the same refrangibility will therefore be deviated to the same extent;

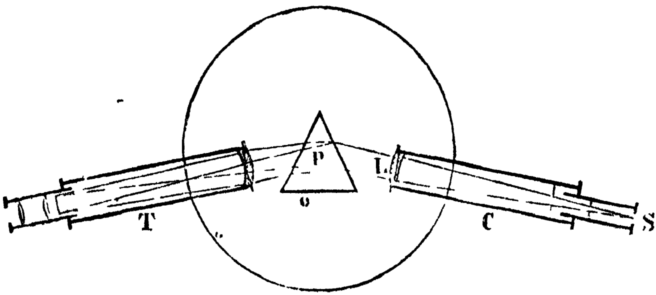

Fig. 192.

hence, on emergence from the prism, the rays of each constituent of the dispersed beam will be parallel among themselves, though not to those of the other constituents. The position of the telescope is adjusted to receive this emergent beam; and, if the eye-piece is focussed for parallel light, a person looking into the telescope will see a magnified image of the pure spectrum which is formed in the focal plane of the object glass.

For purposes of measurement the instrument is often fitted with another tube, T′ (Fig. 193), carrying a scale at one end and a collimating lens at the other. This tube is placed with the lens facing the prism, and its position is so adjusted that an image of the scale, illuminated by some convenient source of light, is reflected from one face of the prism into the telescope. When the scale tube is properly focussed, the image of the scale will be seen in the telescope

in coincidence with the spectrum, and the position of any line or band of the latter can be referred to its position on the scale.

By means of this scale the lines of any spectrum can be *mapped*—that is, their position, as given by the divisions of the scale, can be noted down. A map constructed in this way has, however, no absolute value, and would be different for different instruments, or for a different adjustment of the same instrument. We can, however, by a graphical construction, reduce it to an absolute scale of wave-lengths

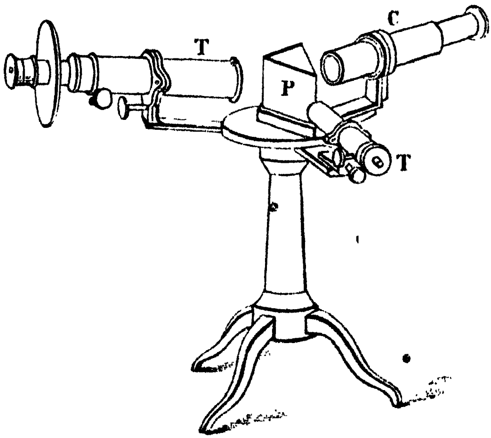

Fig. 193.

By observing the position of a number of lines *of known* wave-length we can construct a curve, having as its abscissæ the scale divisions and as ordinates the wave-lengths corresponding to given positions on the scale; then, by noting the position of any line on the scale, and measuring the ordinate of the curve corresponding to that position, we can determine approximately the wave-length of the line considered.

Very often it is necessary to compare two spectra. This is best done by arranging to obtain both spectra together in the same field. For this purpose a small right-angled total-reflection prism is fitted over the other half of the slit

of the collimator, and a source of light, giving one of the required spectra, is placed on one side, so that the rays from it are totally reflected into the upper half of the slit. The other half is illuminated directly by the source giving the other spectrum, and thus both spectra are seen together, the one in the upper half of the field and the other in the lower half. See also Art. 167.

Fig. 193 shows a simple form of spectroscope.

In the best forms of the instrument there are two or more prisms instead of one. This is necessary in order to obtain greater dispersion; with one prism only a comparatively short spectrum can be obtained, and any peculiarities are not readily noticed. With a train of prisms so that the light passes successively through them the dispersion is increased by each prism, and a very long spectrum is obtained.

165. The spectrometer. This is a modification of the spectroscope adapted for accurate measurement.

Fig. 194 illustrates Wilson's* form for students. The collimator, C, is fixed to the base. The telescope, T, is mounted on a rotating

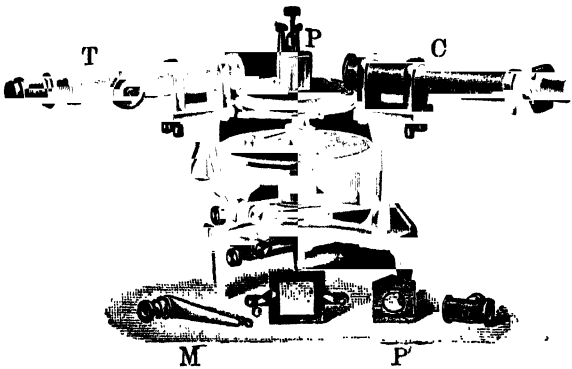

Fig. 194.

piece which carries a circular scale graduated in degrees. The prism, P, is mounted on an adjustable table which can be levelled

* W. Wilson, 1, Belmont Street, Chalk Farm, N.W.

by three screws, and this in turn may be raised or lowered and afterwards clamped to the spindle of a circular plate which carries two verniers (180° apart) which slide alongside the circle attached to the telescope. F, F are fine adjustments to be used after the prism and telescope are clamped in position. The verniers read to minutes of arc and for accurate work are read by a small magnifier, M.

Adjustments of the spectrometer. Before the spectrometer can be used it must be accurately adjusted. Any time spent over this process will be saved over and over again by the rapidity and accuracy with which the readings can be taken. The order in which the adjustments are performed is as follows :—

(1) The eye-piece is focussed on the cross-wires.

(2) The telescope is focussed upon a distant object; the cross-wires and eye-piece are in a small tube by themselves so that this adjustment does not disturb the previous one. The adjustment is correct when on moving the eye transversely across the eye-piece the image of the distant object remains at rest relative to the cross-wires.

(3) The collimator and telescope are now brought into a straight line. The slit of the former is now illuminated and the adjustable tube on the collimator moved until the image of the slit as viewed through the telescope is distinctly focussed, the adjustment being tested as above.

If a distant object is not available, the following method due to Schuster may be used: Fix a prism with its refracting edge vertical upon the turn-table, and illuminate the slit of the collimator with sodium light. Find the position of the slit at minimum deviation, and fix the telescope at about three-quarters of the diameter of the field of view beyond it. Then on turning the prism round in one direction the slit moves towards the direction of the incident light, stops still, and then comes back. It, therefore, may be brought twice to the centre of the field. To distinguish between the two positions of the prism when this occurs, call the position when the prism face upon which the light falls is more normal to the incident rays the "normal" position, and the other position the "slanting" position. Place the prism in the slanting position, bring the slit to the centre of the field, and focus telescope on it till image is quite sharp. Now rotate prism to the normal position. In general the slit is out of focus. Adjust collimator till image is good. Now turn back again to slanting position and focus the telescope, and then back again to normal position and focus the collimator. After this has been done two or three times the collimator slit will be in focus, without alteration of collimator or telescope, in both positions of the prism; and when this is the case the rays leaving the collimator and entering the telescope are parallel.

The proof is simple. Since the slit remains in focus, it follows that the virtual image formed by the prism is at the same distance

from the telescope in the two positions of the prism, that is to say, the distance between the prism and the virtual image of the slit is not altered by altering the angle of incidence. But it can be proved—by calculation or practical geometry—that the distance of the image from the prism varies with the angle of incidence of the rays, except in the one case when the image is at infinity, and consequently the incident rays are parallel The collimator is therefore sending out parallel rays of light, and the telescope is adjusted to focus such rays.

166. Experiments with the spectrometer.*

(1) The angles of a prism are easily determined by the spectrometer. Two methods are available; the agreement of the two sets of results is a test of the accuracy of the instrument and observer.

(*a*) Mount the prism on the table, T, with the angle, A, which is to be measured pointing towards the collimator, and level T until the refracting edge of the prism is exactly vertical. Parallel light

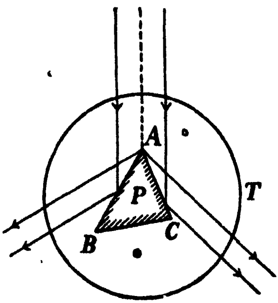

Fig. 195A.

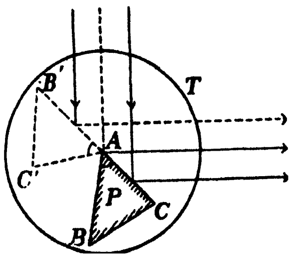

Fig. 195B.

leaving the collimator is split into two parallel beams (Fig. 195A) by reflection at the prism faces A B, A C. Clamp the prism table in position. Sight the telescope upon each beam in turn, bring the images of the slit to the cross-wire and read the verniers. It is obvious that the angle through which the telescope has been rotated is equal to twice the angle of the prism (cf. Art. 98).

(*b*) Mount the prism so that its refracting edge is just over the centre of the table Rotate the telescope to a position about 90° from the collimator and clamp it. Now rotate the prism until the light reflected from the face A C, Fig. 195B, is sent down the telescope. Adjust until the image of the slit is on the cross-wire,

* See Bower and Satterly's "Practical Physics," §§ 150-62, for a more detailed list of experiments.

and read the verniers attached to the prism table. Next rotate the prism table until light reflected at the face A B is sent down the telescope and again read the verniers. The angle through which the prism has been turned is obviously equal to the supplement of the angle B A C. The other angles may be measured in like manner.

(2) The refracting angle of a prism being known, the determination of the refractive index of the material composing the prism is easily made. The slit is illuminated by monochromatic light, say by a sodium flame. First set the telescope to get a direct reading of the collimator slit, place the prism on the table in a suitable position, and locate the refracted image of the slit by the naked eye. Then bring the telescope around to view it and adjust to the position of minimum deviation as described in Art. 97. Take the reading of the telescope. The angle through which the telescope has been rotated from the first position is equal to D in the formula

$$\mu = \frac{\sin \frac{D + A}{2}}{\sin \frac{A}{2}}$$

from which μ can be readily calculated. If the refracting index of a liquid is required, the liquid may be placed in a hollowed glass prism (P′ Fig. 194), or in a prism whose sides are composed of three parallel-sided plates cemented together at the edges. For a more commercial method see Art. 168.

(3) By a similar experiment to (2) the dispersive power of the substance composing the prism may be determined. If a very accurate value of the dispersive power is not required the slit may be illuminated by an ordinary luminous batswing flame. Set the prism in a position of minimum deviation for the mean or brightest rays. The orange-yellow sodium light will do well for this. Put a little common salt into the flame and take the reading of the sodium line. Now remove the salt and take the readings of extreme limits of the red and violet ends of the spectrum. The refractive indices μ_r, μ, μ_v may now be calculated, and by means of the formula

$$\omega = \frac{\mu_v - \mu_r}{\mu - 1}$$

the dispersive power can be found. To get a more accurate result two definite kinds of light must be used, say the red and the violet lines given by the spectrum of hydrogen, or potassium.

(4) As in the preceding article, spectra may be mapped; the only difference now being that the prism is fixed in the position of minimum deviation for the D-light, and the slit being illuminated by the various lights in turn, the angle of deviation is read for

each. The curve can then be plotted between the wave-length and the angular deviation. For some metals the salt can be heated to incandescence in a Bunsen flame, others require the electric arc; in the case of liquids the spectra are obtained from the light yielded by electric sparks between two platinum points, one of which is in the liquid, and the other just above; while in the case of gases, vacuum tubes are used, the narrow part of the tube being placed in front of and parallel to the slit (Fig. 196).

Fig. 196.

If the ordinary eye-piece be removed and a photographic objective substituted, photographs of different spectra may be obtained. These photographs confine themselves chiefly to the violet and ultra violent ends of the spectrum, the red light, as mentioned in Art. **112**, being photographically weak.

167. The Doppler effect* in spectroscopy. If a source of light is moving towards an observer the wave-length of the light sent to him is decreased, and if moving away from him the wave-length is increased. Consequently the lines in the spectrum of this light are displaced, towards the violet end of the spectrum in the former case and towards the red end in the latter case, the displacement being approximately proportional to the relative velocity of approach or regression. If then, by means of a comparison spectrometer similar to Fig. 193, the spectrum of a moving body is compared with the spectrum of a stationary flame giving some of the same lines as the body, the displacement of the lines can be easily measured, and from that the velocity of the body ascertained. By such means Dr. Huggins has shown that the bright star Sirius is receding from the earth with a speed of thirty miles a second; and Professor Keeler has shown that the rings of Saturn consist of a multitude of meteorites revolving around the parent body. The Doppler effect taken in conjunction with the Kinetic Theory of Gases † also explains why the lines in the spectrum of a gas widen when the temperature of the gas is raised.

* See Catchpool's "Textbook of Sound," Art. 25.
† See Wagstaff's "Properties of Matter," Art. 171.

168. Refractometers. The refractive index of a given liquid at a given temperature and for the same light is a constant; hence the refractive index may be employed as a means of identification of a liquid. For a more rapid measurement of the index than is possible with the spectrometer several instruments have been devised of which probably Pulfrich's Refractometer (Fig. 197) is the best. A short glass tube some 15 mm. in diameter and containing about 1 or 2 grammes of the liquid is cemented on a right angled prism placed so that one of the faces enclosing the right angle is horizontal. A beam of monochromatic light, made to pass at "grazing incidence" along the surface between the liquid and the glass prism, is refracted downwards into the prism at an angle, r, with the normal which cannot be greater than the critical angle from glass to liquid, and finally emerges into the air at an angle e with the normal of the vertical face of the prism. This latter angle is observed by means of a telescope T moving round a graduated scale shown diagrammatically in the figure. If ${}_a\mu_l$, ${}_a\mu_g$ represent the refractive indices of the liquid and the glass of the prism respectively, then we have for the first refraction—

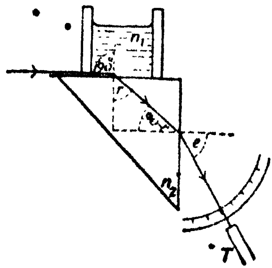

Fig. 197.

$$\frac{{}_a\mu_g}{{}_a\mu_l} = \frac{\sin 90^\circ}{\sin r} = \frac{1}{\sin r},$$

and for the second refraction—

$${}_a\mu_g = \frac{\sin e}{\sin (90^\circ - r)} = \frac{\sin e}{\cos r}.$$

$$\therefore\ {}_a\mu_l = {}_a\mu_g \sin r = {}_a\mu_g \sqrt{1 - \frac{\sin^2 e}{{}_a\mu^2_g}}$$

$$= \sqrt{{}_a\mu^2_g - \sin^2 e}.$$

169. The direct vision spectroscope. This is a convenient form of spectroscope for the qualitative examination of flames and incandescent bodies. As explained in Art. 106 a crown and a flint glass prism may be combined to give dispersion without deviating the mean ray. By using several prisms with their edges alternately in opposite directions a very large dispersion may be obtained. The prisms are enclosed in a brass tube B, Fig. 198, provided at one end with an adjustable slit placed parallel to the refracting edges of the prism. Light leaving the slit S is rendered parallel by means of the lens L. It then falls on the prism combination, which may consist of three crown glass prisms united to two prisms of flint glass. The refracting angles and indices of refraction are so chosen that

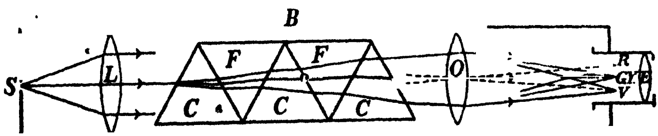

Fig. 198.

the brightest ray in the spectrum passes through without deviation, while the red and violet rays are deviated in opposite directions. Fig. 198 only shows the path of the rays arising from the central incident ray. The spectrum formed may be viewed directly or magnified by a short telescope, as in the figure. For their size these spectroscopes can be made very powerful and are of great service in chemical, physiological and rainband observations (Art. 114).

170. Binocular vision. The stereoscope. If our eyes were fixed in their sockets our field of view, were we to keep the head fixed, would be very limited. The eyes, however, can be moved about 55° in every direction about their mean positions, and distances are usually judged by the amount of convergence we have to impress upon the optic axes. It is extremely difficult to judge a distance accurately with one eye, as the reader will find if he tries to quickly place the

point of his pen upon any small object on the table. Now to every point on one retina there is a corresponding point on the other, so that although two images of an object are formed by our eyes the brain is only cognisant of one.

When we look at a relatively near object of three dimensions, *i.e.*, one having length, breadth and thickness, the images formed on the retinas of the two eyes are not exactly alike, as the positions of the eyes are slightly different. This is rendered very apparent if we stand near and look at a clump of trees and quickly open and shut each eye in turn. The brain, however, blends these images into one, and the effort required for this gives us an idea of the solidity of the object. In the case of an ordinary picture the two images are almost exactly alike, hence the flatness which is nearly always very apparent; indeed, artists sometimes try to surmount this difficulty by exaggerating the perspective effects.

The **stereoscope** is an instrument invented by Wheatstone by which ordinary photographic pictures may be made

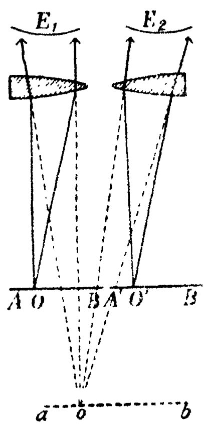

Fig. 199.

to yield to the eye the appearance of depth. Two photographs A B, A′ B′ (Fig. 199), of the same object are taken

in two slightly different positions—the positions that a person's two eyes would be in if he were actually observing it from about the same position as that in which the camera is placed.

These are then correctly mounted on one card and then viewed through separate, very acute-angled prisms. These prisms are set with their angles inward; so that the rays from a point O in the right-hand picture, A B, are deviated outwards and enter the right eye E_1, as if they were coming from a virtual image much to the left of O. Similarly the rays from the corresponding point O′ in the picture A′ B′ enter the left eye E_2 as if they were coming from a virtual image to the right of O′. By varying the distance of the pictures A B and A′ B′ from the prisms, it is possible to make the virtual images of O and O′ coincide at a point *o*, say. At the same time, other virtual images will coincide, and thus, instead of two different pictures A B, A′ B′ being seen, only one—*a b*, a virtual image of these two—is perceived, and the impression produced on the mind of an observer is the same as if he were looking at the object itself, the front parts of the object appearing to stand out, and the back parts to sink back.

The surfaces of the prisms are usually curved convex, as indicated in the diagram, so that the images are magnified as well as superposed, thus making the detail much clearer. The best results are usually obtained when the two pictures are so mounted that the distance between corresponding points is nearly equal to the distance between a person's two eyes.

CALCULATIONS.

171. THE following relations, proved in the preceding chapter, should be noted :—

(1) The magnifying power of a telescope adjusted for normal vision of a distant object is $\frac{F}{f}$, where F and f are respectively the focal lengths of the object glass and eye-piece. (Art. **156.**)

(2) If F denote the focal length of a single lens equivalent to a system of two lenses of focal lengths f_1 and f_2, and placed at a distance a apart, then—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 . f_2}. \qquad \text{(Art. 159.)}$$

(3) Conditions of achromatism for two lenses in contact, of focal lengths f_1 f_2 and of materials of dispersive powers ω_1 ω_2 respectively—

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0. \qquad \text{(Art. 160, 1.)}$$

(4) Condition of achromatism for two lenses of the same material of focal lengths f_1, f_2, separated by a distance a –

$$a = -\frac{f_1 + f_2}{2} \qquad \text{(Art. 160, 2.)}$$

(5) Condition of minimum aberration for the same lenses –

$$a = f_2 - f_1. \qquad \text{(Art. 160, 3.)}$$

(6) Conditions (4) and (5) are satisfied if $f_1 = 3f_2$. (Art. **160**, 4.)

(7) The Huyghen's (or negative) eye-piece consists of two plano-convex lenses ($f_1 = 3f_2$) separated by a distance equal to $2f_2$. (Art. **160**, 5.)

(8) The Ramsden's (or positive) eye-piece consists of two plano-convex lenses ($f_1 = f_2$) separated by a distance equal to $\frac{2}{3}f_1$. It allows of the use of cross-wires and eye-piece scales. (Art. **160**, 7.)

(9) The magnifying power of microscope adjusted for normal vision is—

$$\left(1 - \frac{D}{f_1}\right)\frac{l}{F_1},$$

where F_1 f_1 are the numerical values of the focal lengths of objective and eye-piece, D is the least distance of distinct vision, and l is a constant dependent on the length of the instrument. (Art. 163.)

172. Miscellaneous propositions. The following propositions are frequently convenient:—

(1) *Refraction at a single spherical surface, the distances being measured from the centre of the refracting surface.* As in Art. 66, we obtain with reference to Fig. 79—

$$\mu = \frac{\text{A O}}{\text{N A}} \cdot \frac{\text{N A}'}{\text{A}'\text{O}}.$$

Now distances being measured from O, let O A be denoted by p, and O A′ by q. Then—

$$\text{A O} = -p, \ \text{N A}' = q - r,$$

$$\text{N A} = p - r, \ \text{A}'\text{O} = -q.$$

$$\therefore \ \mu = \frac{\text{A O}}{\text{N A}} \cdot \frac{\text{N A}'}{\text{A}'\text{O}} = \frac{-p\,(q-r)}{-q\,(p-r)} = \frac{p\,(q-r)}{q\,(p-r)}.$$

$$\therefore \ \mu = \frac{p\,(q-r)}{q\,(p-r)}.$$

$$\therefore \ p\,q - p\,r = \mu\,p\,q - \mu\,q\,r.$$

$$\therefore \ \mu\,q\,r - p\,r = \mu\,p\,q - p\,q.$$

Therefore, dividing by $p\,q\,r$, we get—

$$\frac{\mu}{p} - \frac{1}{q} = \frac{\mu - 1}{r}.$$

This relation is similar to the formula—

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r},$$

obtained in Art. 66. The two formulæ are identical in form, the v and u of the latter being replaced by p and q—a phonetic method of remembering the connection. It must, however, be remembered that p corresponds to u, and q to v—that is, in both formulæ the first in alphabetical order gives the distance of the object, while the other gives the distance of the image.

(2) *Refraction through a spherical lens.* Refraction through a spherical lens is a case of some practical importance. Let O represent the centre of a spherical lens N B C N′, and let A B be a ray incident on the lens at B. The ray here suffers refraction, and, travelling along B C in the lens, appears to diverge from *a*. On emergence at C, it is again refracted, and, travelling along C D, appears to diverge from A. The point A′ is thus the conjugate focus of A. Cp. Fig. 98.

* To determine the position of a, we must apply the formula deduced above. Let O A be denoted by p, O N by r, and O a by q'; then—

$$\frac{\mu}{p} - \frac{1}{q'} = \frac{\mu - 1}{r} \qquad (1)$$

where μ denotes the index of refraction from the external medium into the lens. Again, to find A′, we must consider refraction at the second surface of the lens. Here, denoting O a, as before, by q' and O A′ by q, we have—

$$\frac{\frac{1}{\mu}}{q'} - \frac{1}{q} = \frac{\frac{1}{\mu} - 1}{-r}.\text{*}$$

Or, multiplying by μ, we get—

$$\frac{1}{q'} - \frac{\mu}{q} = \frac{\mu - 1}{r}. \qquad (2)$$

Adding equations (1) and (2), we get—

$$\frac{\mu}{p} - \frac{\mu}{q} = \frac{2(\mu - 1)}{r},$$

$$\therefore \frac{1}{p} - \frac{1}{q} = \frac{2(\mu - 1)}{\mu r},$$

or—

$$\frac{1}{q} - \frac{1}{p} = -\frac{2(\mu - 1)}{\mu r}.$$

This establishes a relation for a spherical lens similar † to the usual formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, for ordinary lenses. Hence the focal length of a spherical lens of radius r (r being considered positive) is given by—

$$f = -\frac{\mu r}{2(\mu - 1)}.$$

A spherical lens has the advantage that any line through O may be considered as a principal axis, and therefore the formula obtained above is true for refraction along any diameter. The focal plane of the lens is thus a spherical surface concentric with the surface of the lens, and of radius numerically equal to the focal length of the lens. The Coddington lens (Fig. 177A) is a common form of spherical lens.

* For, $\frac{1}{\mu}$ is the index of refraction from the lens into the external medium (Art. 53), and O N′ = − O N = − r.

† It must, however, be remembered that in this case distances are measured from the centre of the lens.

EXAMPLES IX.

1. A biconvex lens silvered on one of its faces is often used as a concave mirror on the magnet-suspensions of electrical instruments. If the focal length of the lens is l and the refractive index of the glass is μ, find a rule for calculating the focal length of the mirror formed.

If r and s are the radii of the front and back surfaces of the lens we have by Art. 82—

$$\frac{1}{l} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right). \tag{1}$$

Now considering each surface in turn we have :—

(*a*) Parallel light reaching the front surface of the lens (Fig. 200)

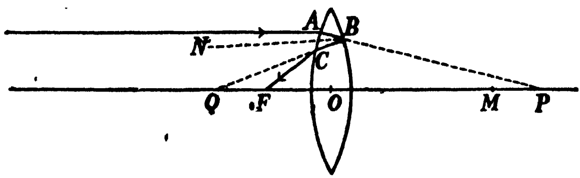

Fig. 200.

is refracted and converged to a point P distant v_1 from O the centre of the lens, where—

$$\frac{\mu}{v_1} - \frac{1}{\infty} = \frac{\mu - 1}{r}. \tag{2}$$

(*b*) The light is reflected at the second face and now converges to a point, Q, distant v_2 in front of the lens where v_2 is given by—

$$\frac{1}{v_2} + \frac{1}{v_1} = \frac{2}{s}. \tag{3}$$

(*c*) The beam is again refracted at the front surface and now converges to a point F in front of the lens. The distance from F to the lens is equal to f the focal length of the effective mirror. f is given by—

$$\frac{\mu}{v_2} - \frac{1}{f} = \frac{\mu - 1}{r}. \tag{4}$$

From equations 1, 2, 3, and 4 we must now eliminate v_1 and v_2.

Multiplying (3) by μ we get—

$$\frac{\mu}{r_2} + \frac{\mu}{r_1} = \frac{2\mu}{s}.$$

Subtracting this from (4)—

$$-\frac{1}{f} - \frac{\mu}{r_1} = \frac{\mu - 1}{r} - \frac{2\mu}{s}.$$

Substituting for $\frac{\mu}{r_1}$ the value given by equation (2), we have—

$$-\frac{1}{f} = 2\frac{\mu - 1}{r} - \frac{2\mu}{s}$$

$$= 2\left\{\frac{\mu - 1}{r} - \frac{\mu - 1}{s} - \frac{1}{s}\right\}$$

$$= 2\left\{\frac{1}{l} - \frac{1}{s}\right\} \text{ by (1).}$$

$$\therefore f = \frac{sl}{2(l - s)}.$$

2. Find the focal length of a single lens equivalent to a combination of a convex lens of 8 inches focal length and a concave lens of 12 inches focal length placed 4 inches apart on a common axis.

Here, applying the formula—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 f_2},$$

$$f_1 = -8, \quad f_2 = 12, \quad a = 4.$$

$$\therefore \frac{1}{F} = -\frac{1}{8} + \frac{1}{12} + \frac{4}{-8 \times 12}$$

$$\therefore F = -12.$$

That is, a *convex* lens of 12 inches focal length is equivalent to the given combination.

3. An astronomical telescope is used to view an object placed at 20 yards distance, and the eye-piece is adjusted for nearest distinct vision of the image. Find the magnification, given that the focal length of the object glass is 2 feet, and that of the eye-piece 4 inches. What will be the magnifying power of this instrument when adjusted for normal vision of a very distant object?

Here, in Fig. 178, if N denote the position of the real image formed by the object glass, we have—

$$\frac{1}{ON} - \frac{1}{60} = -\frac{1}{2}, \textit{ i.e.,}\ ON = -\frac{60}{29} \text{ feet.}$$

Also, if the nearest distance of distinct vision be taken as 10 inches, then—

$$\frac{1}{10} - \frac{1}{\mathrm{O'\,N}} = -\frac{1}{4}.$$

$$\therefore \frac{1}{\mathrm{O'\,N}} = \frac{1}{10} + \frac{1}{4} = \frac{7}{20},$$

or—

$$\mathrm{O'\,N} = \frac{20}{7} \text{ inches}$$

$$= \frac{5}{21} \text{ feet.}$$

But, as shown in Art. **156**, the magnifying power is given by the ratio $\frac{\mathrm{O\,N}}{\mathrm{O'\,N}}$.

$$\therefore m = \frac{\mathrm{O\,N}}{\mathrm{O'\,N}} = -\frac{60}{29} \times \frac{21}{5} = -\frac{252}{29}$$

$$= -8\frac{20}{29}.$$

That is, the image of the object is inverted, and appears about $8\frac{3}{4}$ times greater than the object itself. When adjusted for normal vision of a very distant object, the magnifying power is approximately given by—

$$m = \frac{\mathrm{F}}{f} = \frac{24}{4} = 6 \text{ (numerically).}$$

The student should notice from this example that the magnifying power of a telescope varies with the distance of the object and with the adjustment of the eye-piece.

4. The dispersive power of crown glass is about ·03 and that of flint glass about ·05. Show how to construct a converging achromatic lens of 60 cm. focal length.

From the formula—

$$\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2} \qquad \text{(Art. 160, 1)}$$

we have—

$$\frac{f_1}{f_2} = -\frac{\cdot 03}{\cdot 05} = -\frac{3}{5} \qquad (1)$$

where f_1 and f_2 are respectively the focal lengths of the crown-glass and flint-glass lenses.

Also, from—

$$\frac{1}{\mathrm{F}} = \frac{1}{f_1} + \frac{1}{f_2}$$

we get—

$$-\frac{1}{60} = \frac{1}{f_1} + \frac{1}{f_2}. \qquad (2)$$

From (1) we get $f_2 = -\frac{5}{3} f_1$; therefore, substituting, we get—

$$-\frac{1}{60} = \frac{1}{f_1} - \frac{3}{5f_1} = \frac{2}{5f_1}.$$

That is—

$$5f_1 = -120.$$

$$\therefore f_1 = -24 \text{ cm.}$$

Also

$$f_2 = -\frac{5}{3}f_1 = \frac{5}{3} \times 24 = 40 \text{ cm.}$$

Therefore the compound lens must be made up of a *convex* lens of *crown* glass of 24 cm. focal length, and a *concave* lens of *flint* glass of 40 cm. focal length.

5. The images formed by the objective of a microscope are 8 inches from the objective. Find the magnifying power of the instrument, given that the focal length of the objective is $\frac{1}{2}$ inch, and that of the eye-piece 2 inches.

6. The focal length of the object glass of a telescope is 3 feet, and that of the eye-piece is 3 inches; draw a curve showing how the magnifying power varies with the distance of the object.

7. Find the focal length of a lens equivalent to a combination of two lenses, each of focal length f, and placed at a distance $2f$ apart.

8. Describe a simple form of microscope with two lenses, and trace pencils from different points of an object through it. If the rays emerge parallel to one another, what change must be made in the position of the lenses in order that the object may be clearly seen?

9. Describe the astronomical telescope; trace the course of a pencil of rays through it from any point of a distant object; and find the magnifying power.

10. What is meant by the chromatic aberration of a lens, and how is it corrected in the object glass of a telescope? The mean refractive indices of two specimens of glass are 1·52 and 1·66 respectively; the differences in the indices for the same two lines of the spectrum are ·018 for the first, and ·022 for the second; find the focal length of a lens of the second glass, which, when combined with a convex lens of 50 cm. focal length of the first, will make an object glass achromatic for these two lines.

11. Describe the astronomical telescope fitted with Ramsden's eye-piece, and draw a figure showing the path of a pencil of rays from a distant object through it. What advantage has Ramsden's over Huyghens' eye-piece, and why is the latter usually employed in microscopes?

12. An achromatic lens of 1 metre focal length is to be constructed of lenses of crown and flint glass, whose refractive indices are 1·5 and 1·65, and whose dispersive powers are as 5 to 8. The crown-glass lens is to be equi-convex and one side of the flint-glass lens is to fit it. Find the radii of curvature.

13. A concavo-convex lens, of focal length 30 cm. and radii of curvature 10 and 30 cm., is silvered on the concave surface. Show that the lens acts as a plane mirror.

14. What is the magnifying power of a telescope whose object glass is of 12 feet focal length, and its eye-piece of $\frac{1}{2}$ inch focal length?

15. In a Newtonian reflector whose speculum is of 10 feet focal length, what must be the focal length of the eye-piece to give a power of 250?

16. Compare the light-grasping power of two mirrors whose diameters are 13 inches and 36 inches, and of the human eye when the pupil is $\frac{1}{4}$ inch in diameter.

17. A telescope is held with its object-glass end under the surface of the water of a pond; the water wets the outer surface of the glass, but does not come inside the telescope. The telescope is focussed so that objects at the bottom of the pond are clearly seen. Is the telescope now longer or shorter than when used for viewing objects at the same distance in air? Does it make any difference what kind of convex lens is used for object glass?

18. Compare the dispersive powers of carbon bisulphide and water from results obtained from the following data, the prisms being placed in the position of minimum deviation for the yellow light. Deviations of the red, yellow, and violet light with the carbon bisulphide prism (refracting angle, 40° 24′), 21° 45′, 28°, and 30° 47′, with the water prism (refracting angle, 39° 33′), 13° 52′, 13° 57′, 14° 20′.

19. Find the focal length of a sphere of glass of radius 10 cms. ($\mu = 3/2$).

EXAMINATION QUESTIONS.

QUESTIONS SET AT VARIOUS UNIVERSITY EXAMINATIONS.

1. What is the centre of a lens? Under what circumstances is the centre of a lens midway between its surfaces? Two equal lenses are placed side by side in the same plane, with their centres 3 inches apart. Two objects of the same size and shape, but of different colours, are placed behind the lenses at a distance of twice its focal length from each lens, and with their centres 6 inches apart, the line joining the centres of the objects being parallel to that joining the centres of the lenses. How will the images be situated, and what will be seen by an observer situated at a considerable distance in front of the lenses?

2. Describe Galileo's telescope (or an opera glass). State the connection between the relative positions of the two lenses and their focal lengths, and estimate the magnifying power.

3. What do you understand by the chromatic dispersion of a lens? What is the best means of correcting it?

4. If you were observing a small luminous object by a telescope not corrected for chromatic aberration, what appearances would present themselves on sliding the eye-piece in and out?

5. Explain the construction of an achromatic object glass.

6. A compound achromatic lens of focal length 40 cm. is to be constructed of two thin crown-glass and flint-glass lenses in contact, the surfaces that are in contact having a common radius 25 cm. The optical characters of the glasses employed being as follows, namely:

	Dispersive power.	Refractive index for middle of spectrum.
Crown glass . .	0·21 . .	1·5
Flint glass	0·45 . .	1·6

—calculate the radius of the second face of each lens, and establish the formulæ employed in the calculation. The compound lens is supposed to act as a convex lens.

7. What is meant by the aperture of a lens? What has the aperture of a lens to do with (*a*) the definition, and (*b*) the brightness, of the images which it is used to form?

8. How does the brightness of a star as seen in a telescope depend upon the magnifying power and upon the aperture? Loss of light by reflection and refraction need not be considered.

9. You are given an astronomical telescope focussed for infinity and pointing at a daylit sky. You are also given a rule, a finely divided scale, and a pocket lens. How would you find the magnifying power of the telescope, and how would you satisfy yourself that the result was not vitiated by stops in the telescope?

Give the theory of the method when the eye-piece is a single convex lens.

10. Draw a careful diagram showing the passage of the rays through a telescope, with lenses of 1 inch and 9 inches focal length respectively, when used to view a distant source of light.

If the tubes containing the lenses can be drawn out until the lenses are 19 inches apart, show that an object 18 inches away can be focussed in the telescope.

11. Find the focal length of a spherical bubble of air in water, the radius of the bubble being 3 mm

CHAPTER XIII.

POLARISATION.

173. IT has been stated in previous chapters that light consists in a wave-motion in the ether, and that this wave-motion is transverse in character. A wave motion may in general be either (1) *longitudinal, i.e.* one in which the vibrations are in the direction in which the waves are travelling, as is the case with sound waves in air (Fig. 201); or (2) *transverse*, in which the constituent vibrations are perpendicular to the direction of propagation, as

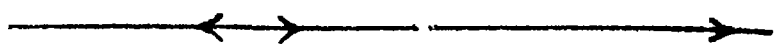

Fig. 201.

in waves travelling along a stretched string (Fig. 202). In the latter case the vibrations are not necessarily confined to one direction, but might take place in any direction in a plane perpendicular to the line of advance of the waves. In a longitudinal train of waves, however, it is clear that the vibrations are necessarily restricted to one

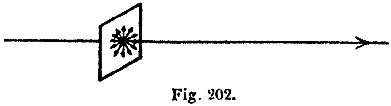

Fig. 202.

direction. It is, therefore, possible that one train of transverse waves may differ from another, otherwise similar, in that the directions of vibration in the two trains are different; such a distinction between two trains of longitudinal waves would be impossible.

Transverse waves in which the vibrations are confined to one direction would not be symmetrical, and might be

expected to exhibit properties having some relation to direction. This characteristic of "sidedness," the dependence of certain properties on direction, is called *polarity*, and such a train of waves is said to be *polarised.* If, as in this case, the vibrations are executed in one direction only, the wave-motion is *plane-polarised.*

Some of the phenomena of polarised waves may be reproduced by the aid of the following apparatus (Fig. 203):—A length of rubber cord or rubber tubing filled with sand (to reduce the velocity of waves along it), two pieces of wood or other material, each having a parallel-sided slit a few inches long in it, the slit being of such width as just to allow the cord to pass freely through. Fix one end of the cord to a support, pass it through the two slits, which

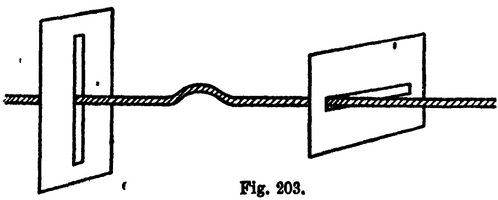

Fig. 203.

must at first be pushed up to the fixed end, and hold the other end in the hand. By moving this end to and fro through a small distance and continually changing the direction of the movement, while keeping it always in a plane perpendicular to the cord, a series of transverse waves will be sent along the cord, and the vibrations will be in various directions indifferently, on the whole as often in one direction as another. This is an unpolarised wave-motion.

Now move one of the slits along the cord, and arrange that it is held rigidly in a stand, and that the cord passes freely through it. Send a train of waves as before, when it will be seen that the waves which get through the slit have their vibration confined to a direction parallel to the slit: they are plane-polarised. [illegible] the second slit midway between the end and the first one, and support it so that its direction is perpendicular to that of the first. Repeat the experiment, and it will be seen that scarcely any wave-motion gets through the second slit. In this position the slits may be said to be *crossed*. Rotate the second slit gradually until it is parallel with the first, from time to time sending waves as before: it will be found that the amplitude of the waves transmitted through the second slit continually increases, until, when the slits are parallel, the plane-polarised waves coming through the first slit pass on unaltered through the second.

174. Polarisation of light. Double refraction. A train of plane-polarised rope waves thus differs from an unpolarised train in that it will not pass through a slit held in one certain direction, while it passes unaltered when the slit is turned through a right angle. It was observed in 1669 by Bartholinus that under certain conditions it was possible to obtain a beam of light that manifested similar properties of "sidedness" or polarisation. This occurred when light passed through a crystal of Iceland spar (a form of calcium carbonate).

This substance occurs in rhombohedral crystals (Fig. 204), the six faces of the rhomb being parallelograms, having angles 102° and 78°. Two, and two only, of the opposite corners are contained by three obtuse angles; a line (O A, O′ A′), drawn through either of these corners so as to make equal angles with the three sides meeting at the corner, is called the *optic axis* of the crystal, and the crystal, so far as its optical properties are concerned, may be taken to be symmetrical about such an axis. It is clear that this axis is a *direction*, and not any fixed line in the crystal. If now a crystal be cut so as to have two plane and parallel faces, parallel to this axis (or even if we take a crystal as it ordinarily cleaves, and look through a plane face), and we view normally through it an illuminated pin-hole, two images of this are seen. One of them is normally above the object, and is found to occur according to the ordinary laws of refraction; the other is to one side of the normal, and is not formed in accordance with these laws. This phenomenon is called *Double Refraction*. When the crystal is rotated above the object the "ordinary" image remains still, but the "extraordinary" image, as it is called, rotates with it round the ordinary one.

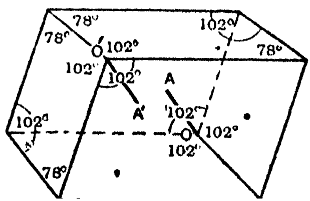

Fig. 204.

If now we place a similar crystal above the first one, we can study by its help the two beams of light diverging

from these two images. We find that when the two crystals are placed with their optic axes parallel the two images are simply separated more widely, for we have merely got a thicker crystal. As the upper crystal is slowly rotated, each of the images gives rise to two (an ordinary and an extraordinary from each of the original images). When the axes of the crystals are at 45°, these four images are of equal brightness; as their inclination to one another approaches 90°, one of each pair gradually becomes weaker and disappears when the axes are at right angles. There are then left two images, an "extraordinary" one of the first "ordinary" image, and an "ordinary" one of the first "extraordinary" image.

This shows that the two beams of light separated by the first crystal have properties depending upon direction, since they give rise to different types of image according as they pass through a second crystal placed similarly to the first, or in a direction at right angles, whereas the original beam of light gives two images however the first crystal is turned. They are, in fact, plane-polarised, the one in a plane at right angles to that of the other. The crystal is so built up that it transmits (in a direction at right angles to its optic axis) wave-trains whose vibrations are in one direction with a different velocity from those whose vibrations are in a direction at right angles. When the unpolarised light falls on it, the vibrations may be regarded as being resolved in these two directions, and the two sets of components travel on differently, leading to different laws of refraction, and to the formation of two distinct images. Similar or more complicated double refraction occurs in the majority of cases when light travels through crystalline media.

Various pieces of apparatus which will only allow light whose vibrations are in one plane to pass through will shortly be described. If a beam of ordinary light passes through such an arrangement only the components of the vibrations in one definite plane are transmitted, and thus a beam of plane-polarised light is obtained. This is, of course, of weaker intensity than the original beam, containing only half its energy.

The apparatus thus used is called a *Polariser*. (But exactly the same arrangement may be used to examine any beam of light to discover whether it is polarised, for if the apparatus be slowly rotated no change will be observed in the intensity if the light is unpolarised; while if it is plane-polarised it will be completely blocked out for one position of the apparatus, and completely transmitted for the position at right angles. If the light is partially plane-polarised it will be changed in intensity as the apparatus is rotated. Used in this way, the Polariser is called an *Analyser*.) It should be mentioned that, except in the case of plane-polarised light, the information given by the use of an Analyser alone is capable of more than one interpretation.

If ordinary light passing through such a Polariser A (Fig. 205) (*e g*, a tourmaline crystal, see § 175), then

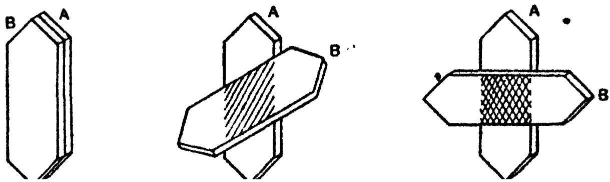

Fig. 205.

falls on a second similar arrangement B placed exactly similarly, practically all the light transmitted by A passes through B and, of course, remains polarised in the same direction as before. This corresponds to the case of the parallel slits (§ 173). If, however, we rotate B relatively to A, then B gradually quenches the light which has passed through A, and when B has been turned through 90°, no light that has passed through A can get through B. The polariser and analyser are then said to be crossed (cf. the case of the crossed slits).

If we employ an analyser to examine the light from the two images formed by Double Refraction, in Iceland spar or in any other case, we find that for one position of the analyser one image disappears, and when the analyser is

turned through a right angle the other image disappears. Thus the light from each is plane-polarised and in perpendicular directions.

Huyghens (1629-1695) showed that double refraction could be explained in terms of the undulatory theory of light, but he was not able to suggest a reason why the two beams should present these characteristics of "sidedness," since he supposed the waves constituting light to be longitudinal. It was not until much later that it was realised that these phenomena were to be brought into line with the wave theory, and this theory itself to gain in precision, by interpreting them to indicate that *light waves are transverse* and not longitudinal.

175. Methods of producing polarised light. (*a*) *By utilising the phenomena of Double Refraction.*

(i) A crystal of Iceland spar, as has been seen, splits up a beam of light into two beams plane-polarised in directions at right angles. The two beams are, however, in any ordinary sized crystal, very close to one another, and in order to separate them sufficiently a very large crystal would be necessary. Such crystals are rare, and therefore this method, though very efficient otherwise, is not practically serviceable.

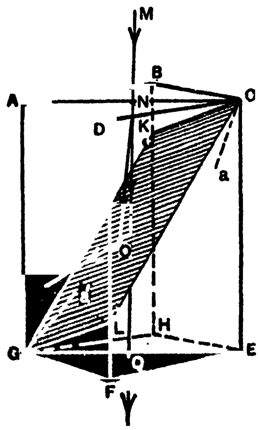

Fig. 206

(ii) A plate of *tourmaline* is frequently used. Tourmaline is a doubly refracting crystal of a slightly greenish tinge. It has the property of very rapidly absorbing one of the two beams of polarised light into which it separates a beam of ordinary light, so that a comparatively thin plate of the material only allows one of the beams to pass through it and so produces from ordinary light a beam of plane polarised light.

(iii) The Nicol Prism. This prism, invented in 1828, is a device for getting rid of one of the polarised beams produced in a crystal of Iceland spar. In Fig. 206, C and G are the opposite corners of a rhomb of Iceland spar, which are contained by three obtuse angles, and C a, G a' are in the direction of the optic axis. The plane A C E G containing this axis is called the *principal section* of the crystal. To form a Nicol prism a section C K G L of the crystal is made at right angles to the principal section. The two halves are then cemented together with Canada Balsam, which forms a thin parallel transparent film between them.

If now a beam M N of unpolarised light strikes the face A B C D it will, on entering the crystal, since it is travelling in a direction inclined to the optic axis, be divided into an ordinary ray N O and an extraordinary ray N Q polarised in planes perpendicular to one another. Now μ for Canada Balsam is less than that of Iceland spar for the ordinary ray, and greater than that of Iceland spar for the extraordinary, hence if the inclination of N O to C K G L exceeds a certain value, total reflection of the ordinary ray N O will occur, while the extraordinary ray will pass on through the crystal and emerge at Q, thus yielding a pure beam of plane-polarised light. The dimensions of the crystal are so chosen as to make the angle of incidence of N O greater than the critical angle. The crystal is mounted in a tube blackened on the inside, so that the reflected beam is absorbed. The Nicol Prism is one of the most valuable sources of polarised light.

(*b*) *By reflection.* In 1810 Malus noticed that light reflected from a glass window showed signs of polarisation when examined through a doubly refracting crystal, whereas the direct light showed no such signs. Thus it became clear that a change in the character of the light had occurred in the process of reflection: the reflected light was partially plane-polarised.

If ordinary light reflected from a polished glass surface be examined with an analyser, the degree of polarisation is found to vary with the angle of incidence, and to be complete when the angle is about $57\frac{1}{2}°$. This is known

as the *polarising angle* for glass. It varies for different substances, being related to the refractive index of the reflecting substance by the simple relationship $\tan i = \mu$ where i = polarising angle. This is *Brewster's Law.* The polarising angle, therefore, evidently varies slightly with the colour of the light. It follows from this law that at the polarising angle the reflected and refracted rays are at right angles for (Fig. 207).

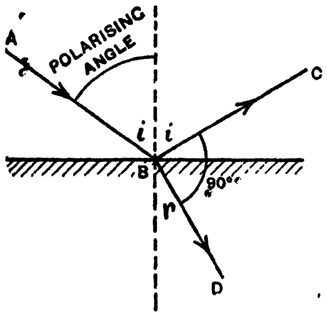

Fig 207.

$$\sin i = \mu \sin r \qquad (1) \text{ (Law of refraction).}$$
$$\tan i = \mu \qquad (2) \text{ (Brewster's Law).}$$

hence $\cos i = \sin r$

and therefore i and r are complementary, and $\angle CBD = 90°$.

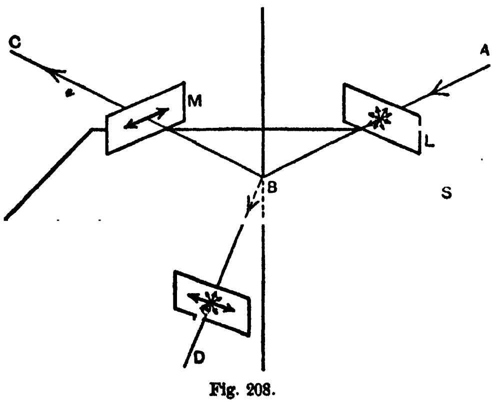

Fig. 208.

Let AB (Fig. 208) be a ray of unpolarised light incident upon the surface S *at the polarising angle.* In the wave front L the vibrations are successively occurring

in all directions. At B all vibrations which are not parallel to the surface are transmitted, together with a certain proportion of the parallel vibrations: thus the refracted ray is only partly plane-polarised. The reflected ray BC consists entirely of vibrations parallel to the surface and it is therefore completely plane-polarised. Those vibrations in the incident wave front L which are inclined to the plane of incidence may be regarded as being resolved into vibrations respectively parallel and perpendicular to this plane, when the components suffer reflection or refraction as above.

Light polarised by reflection is defined as *polarised in the plane of incidence*, but the vibrations constituting it are perpendicular to this plane, *i.e.* the plane perpendicular to the direction of vibration is called the *plane of polarisation*.

If we receive the reflected beam BC on a second surface of the same sort with its normal in a plane perpendicular to the plane of incidence on the first surface, *i.e.* so that the planes of reflection are perpendicular, vibrations which were perpendicular to the first plane of incidence will be in the plane of incidence for the second surface. In consequence, if we adjust the inclination of this latter surface so that the beam strikes it at the polarising angle, no light is reflected. The two surfaces now act like polariser and analyser crossed.

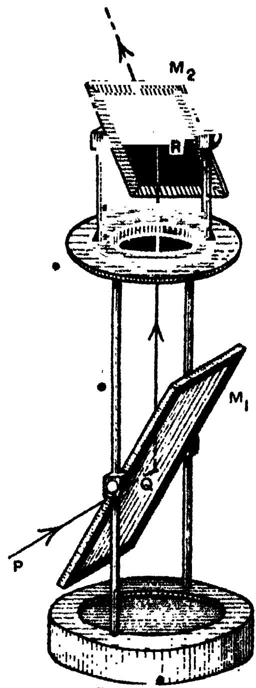

Fig. 209.

These facts may be verified with the following apparatus —M_1 (Fig. 209) is a plane glass plate backed with black paint and capable of being turned about a horizontal axis,

M_2 is a similar plate which can be turned about a horizontal and also a vertical axis. In each case the amount of rotation can be measured on a scale of degrees. (The scales for measuring the rotations round the horizontal axes are not shown in the diagram.) M_1 and M_2 are so arranged as to be equally inclined to the horizontal, and a beam of parallel light P Q is directed on to M_1 at the polarising angle so that the plane of incidence is normal to the surface and so that the reflected ray Q R is vertical and falls on M_2, which it must then necessarily strike at the same angle. As M_2 is rotated about a vertical axis it is found that the intensity of the beam reflected by it varies from a maximum when the normals to M_1 and M_2 are parallel, to zero when the planes of incidence are at right angles. This apparatus is called a Polariscope. It is clear that by its use one can experimentally determine the polarising angle without the aid of any other form of polariser or analyser.

It has already been pointed out that the refracted ray B D (Fig. 208) is by no means completely polarised, although it contains a greater proportion of light whose vibrations are in the plane of incidence. If now we allow this light to fall on another plate parallel to the first, still more of the light whose vibrations are perpendicular to the plane of incidence will be filtered out by reflection. It is found that if we use about two dozen such parallel plates, about 90 per cent. of the light transmitted is constituted of vibrations in the plane of incidence.

Such a "pile of plates," as it is called, thus yields an easily made piece of apparatus for producing nearly plane polarised light. Microscope slide-cover glasses (carefully cleaned) are recommended; from 12 (giving light rather more than 80 per cent. polarised) to 24 (giving light about 90 per cent. polarised) should be placed in contact and fixed by means of corks, in a small tube, at an angle of about $32\frac{1}{2}°$ with the axis. Two such will serve as polariser and analyser.

Since reflected light is always more or less plane-polarised, it has been suggested that the inconvenient glare from sunlight reflected

from the surface of water might be largely reduced by the use of polarising spectacles (made, *e.g.*, of thin plates of tourmaline). In this way it is said, visibility into the water is improved, and also to some extent the visibility towards the horizon is increased.

(*c*) *Polarisation produced by the scattering of light.* It has been explained in § 125 that the brightness and the blue colour of the sky are due to the light scattered by very small particles in and of the air. We may regard these particles as being set in vibration relative to the ether by the light waves falling on them, and, in consequence, acting like separate sources of light.

When light from the sky is examined by an analyser it is found to be partially plane-polarised. The extent to which it is polarised depends upon the direction in which we are looking, it being a minimum when we receive light from a direction facing or opposite to the sun, and a maximum when we receive it from directions at right angles to the sun. In this latter case it is not completely plane-polarised, so that what is observed as the analyser is rotated is a distinct darkening of the field in one direction, but not perfect extinction.

The same phenomenon can be observed if the light scattered by the suspended precipitate in the thiosulphate solution (§ 125) is examined. When viewed from the side it will be found to show a very considerable degree of polarisation.

The occurrence of polarisation in scattered light is to be expected as a consequence of the fact that the vibrations in the light waves are transverse, for thus a scattering particle can only be set in vibration in a plane at right angles to the direction of the incident light, and when the scattered light is viewed in a direction perpendicular to this incident light, the vibrations received are confined to one direction, and so the light received is plane-polarised.

A very interesting experiment in illustration of this consists in passing a beam of plane-polarised light through a tube containing the suspended precipitate of sulphur and then examining the scattered light in various directions in the plane at right angles to the incident light. It is found that when viewed in a direction parallel to that of the vibrations in the incident light, the track of

the beam through the solution is practically invisible (*i.e.* there is no scattered light), while in the direction at right angles its brightness is a maximum. In the first case we are looking in a direction end-on to that of the vibration excited in the particles, and no light is propagated in such a direction.

The haze which envelops distant objects is principally occasioned by scattered light, and a further application of the principles of polarisation has been proposed in order to improve the visibility in such cases. It is suggested that this might be accomplished by allowing the light to pass through polarisers which would cut off the bluish atmospheric haze, thus improving the detail and colour of the distant objects.

176. Circularly and elliptically polarised light. If ordinary unpolarised light is examined with an analyser, no change of brightness of the field of view occurs as the analyser is rotated, since on the average the amount of vibration in all directions is constant, for, though the number of consecutive vibrations in any given direction is no doubt great, these occupy a very short time, and thus the direction of vibration only remains constant for a very small fraction of a second.

If, however, the vibrations were circular in character, such as could be resolved into two equal simple Harmonic vibrations of the same period, at right angles to one another, and having a phase difference of one-quarter of a period,* the same thing would be observed on rotating the analyser. Such light is said to be *circularly polarised.* It can be distinguished from unpolarised light by passing it through a thin plate of quartz (a doubly refracting crystal), cut parallel to the optic axis and of such thickness as to slow down the propagation of one of the components over that of the other until they are in phase, and so combine to form plane-polarised light,† which can be detected by the use of the analyser in the ordinary way. Since a difference of phase of one-quarter of a period must be introduced between the components, the thickness of the crystal must be such as to contain one-quarter of a wave-length more in the extraordinary than in the ordinary beam, and such a plate is, therefore, called a

* See Catchpool's *Sound*, Ch. I., § 12 (2). † *Ibid.*, § 12 (1).

quarter-wave plate. Conversely, if plane-polarised light is incident on a quarter-wave plate at 45° with the principal plane, it will emerge as circularly polarised light.

If the vibrations in a beam of light are elliptical, then they can be resolved into two unequal simple Harmonic vibrations of the same period, at right angles, and having a phase difference of one-quarter of a period. Thus, when viewed through an analyser, the light transmitted will be brighter for one direction of the analyser than for that at right angles. To distinguish such *elliptically polarised light* from a mixture of polarised and unpolarised light, it must be passed through a quarter-wave plate, when it will, for one position of the plate, be reduced to plane-polarised light, as in the preceding instance.

177. Colour effects due to polarisation. As has been stated, the characteristic properties of doubly-refracting crystals depend upon the fact that such crystals in general transmit light whose vibrations are in one given direction with a different velocity from light with vibrations in the direction at right angles. But these velocities and their differences depend upon the wave-length, *i.e.* upon the colour of the light. It follows that if plane-polarised light is transmitted through such a crystal cut parallel to the optic axis, the character of the emergent light depends upon the colour of the light, and the thickness of the crystal.

If white light falls on such a crystal and the transmitted light be examined with an analyser, very brilliant colour effects are therefore seen. The study of these effects is beyond the scope of this book, but one interesting application may be mentioned.

Many *isotropic* bodies (*i.e.*, substances whose properties are the same in all directions) act like doubly refracting crystals when they are strained, *e.g.*, by bending, twisting, or compression. If examined between crossed polariser and analyser such a substance under these circumstances will exhibit the characteristic colour-fringes, the position of which will mark out the lines of strain. This fact is put to practical use in examining glass for imperfect annealing, and in examining the conditions of strain in small model structures built up of a suitable transparent material.

178. Rotation of the plane of polarisation. If a beam of unpolarised light is viewed through crossed Nicols or other polarising devices, the field of view is, of course, completely dark. If now a crystal of quartz, cut *perpendicular to the axis*, is interposed between the polariser and the analyser, it is found that the light is no longer completely cut off by the analyser, but that when this latter is turned through a definite angle (depending upon the thickness of the quartz) the light is again extinguished.

This indicates that the plane-polarised light yielded by the first Nicol is still plane-polarised after traversing the quartz, but the direction of vibration (and, therefore, the plane of polarisation) has been rotated through a certain angle, which is measured by that through which it is necessary to turn the analyser in order to extinguish the light. If, as we look towards the oncoming light, the rotation is clockwise, it is described as right-handed, and the substance producing it is said to be dextro-rotatory; if in the opposite direction the rotation is left-handed, and the substance laevo-rotatory.

The amount of rotation varies (1) directly as the thickness of the crystal, (2) approximately inversely as the square of the wave-length of the light. It follows from (2) that, if white light is used, chromatic effects will be seen as the analyser is rotated.

The property of rotating the plane of polarisation, often known as "optical activity," is possessed by other substances than quartz, particularly by certain organic compounds, *e.g.*, sugar, quinine, turpentine, tartaric acid, camphor. The rotation produced is sometimes right and sometimes left-handed. Some such substances show markedly different rotatory powers in the solid state and in solution respectively.

The amount of rotation produced by an optically active substance such as sugar for light of a given wave-length, and at a given temperature, depends upon (1) the strength of the solution, (2) the length of liquid through which the beam of light passes. The rotation produced by a length of 1 decimetre of a pure substance, divided by the density of the substance, is called its *specific rotation*.

The rotation produced by 1 decimetre of a solution of an active substance in an inactive solvent divided by the weight of the substance in 1 c.c. of the solution, is called the *specific rotation* of the dissolved substance. The specific rotation, as has been said, varies with the colour of the light used, and it is, therefore, usual to employ sodium light.

By measuring the amount of rotation produced under standard conditions by a given length of the liquid it is possible to deduce the strength of the solution. This process is particularly applied to the estimation of sugar, when it is known as *saccharimetry*. Observations upon rotation of the plane of polarisation also afford other information of value to the chemist.

An arrangement of crossed Nicols (Fig 210) constitutes a simple *polarimeter* or *saccharimeter*. It is, however,

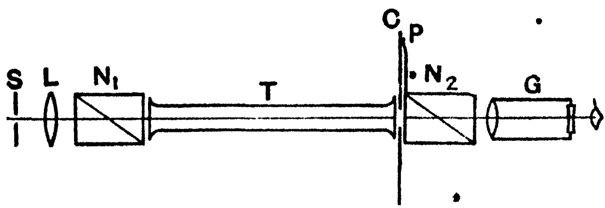

Fig. 210.

impossible to tell within several degrees when the field of view is at its minimum brightness, and hence this simple form of apparatus is wanting in sensitiveness. Modifications have been introduced whereby the field is divided into two halves, and adjustment of the analysing Nicol has to be made until these appear equally bright, when the plane of polarisation will be in a definite position relative to the analyser. Such a comparison between two halves of the field can be carried out much more accurately than the estimation of maximum darkness.

A very important relationship between light and magnetism was discovered by Faraday, who in 1845 showed that when a beam of polarised light passed through a

transparent medium in a direction parallel to the lines of a magnetic field, its plane of polarisation suffered rotation by an amount depending upon the distance traversed and the strength of the field.

179. Polarisation of other ethereal waves. Further evidence of the continuity of the whole range of ethereal waves, from the longest "Electric" waves to the shortest X-rays, is provided by the study of polarisation. It is found that, on the one side, X-rays can be shown under certain circumstances to be polarised; and, on the other side, that both the radiant energy, known as heat radiation, and the ordinary electric waves, radiated by an electric spark or by any other means, can also be made to exhibit the same phenomenon.

ANSWERS.

EXAMPLES I. (page 29).

7. 100 cm.; 20 cm.

8. Diameter of umbra = 1 cm.
Diameter of penumbra = $4\frac{1}{3}$ cm.

9. ·00894 sq. cm. (nearly); A′ B′ = $13\frac{1}{3}$ cm.

10. $2\frac{12}{25}$ ft.

11. 750 ft.

13. 3 : 4.

14. $I_D : I_B : I_C :: 3\sqrt{3} : 8 : 8$.

15. $a^2 : b^2$.

16. $(115)^2 : (201)^2$.

17. 80 cm. from less intense light.

18. (*a*) Screen between the lamps, $2\frac{4}{5}$ ft from 16-candle-power lamp. (*b*) Screen outside lamps, 24 ft. beyond 16-candle-power lamp.

19. $4\frac{33}{64}$.

20. 12·96.

21. 11·2.

22. 408·25 in.

23. 5 : 7 : 6.

EXAMPLES II. (page 58).

4. 12 in., 24 in., 36 in.; 12 in., 12 in.

5. 60°.

6. 3 ft. (The man's eye is supposed to be at the top of his head.)

7. 100°.

8. 30°.

10. 2 ft.

11. 8° 36′.

Examples III. (page 86).

8. 13·5 in.

9. $\dfrac{I}{O} = -\dfrac{f}{u-f} = -\dfrac{f}{3f-f} = -\dfrac{1}{2}$.

10. 12 in.

11. ·025 in.; 9 in. behind.

12. $r = 5\frac{1}{3}$ ft.; mirror $13\frac{1}{3}$ ft. from wall.

13. $3\frac{3}{7}$ in. behind; $\frac{3}{7}$ diameter of a penny.

14. 2 ft. real image, 1 ft. virtual.

15. Real image, size $\frac{1}{3}$ object = ·66 cm.

17. $\dfrac{1}{2}$, virtual, $\dfrac{f}{2}$ behind.

18. 7·5 cm., 30 cm. from mirror.

20. $2\sqrt{3}$ ft. from plane mirror.

27. 10·08 in. real, inverted, reduced.

28. 12·15 in.

29. $16\frac{4}{5}$ in.

30. $9\frac{1}{3}$ in. behind.

31. 7·9 in., ·61 in., 2·2 in., behind in each case.

33. 21·18 in., $1\frac{1}{17}$ in. high; real, inverted.

Examination Questions (page 93).

3. Between candle and gas flame, 2 ft. from former and 6 ft. from latter; or, on the line passing through the lights, 4 ft. from the candle and 12 ft. from the gas flame.

4. Use a Bunsen's photometer and compare the amounts transmitted, when the glass is in different positions, with a standard source. By subtraction the amounts reflected can be found.

6. 1·18 metres, 1·27 metres.

22. Real, and one-third as large as the object; 1 ft. from the mirror; inverted.

23. 6 in.

25. One-third of the diameter from the pole.

26. 1 ft. from the mirror; inverted; three times as large; at the centre of curvature.

27. Object 3 in. in front of mirror; image $1\frac{1}{2}$ in. behind the mirror

EXAMPLES IV. (page 130).

5. $1\frac{1}{3}$ in. nearer.
6. ·654 nearly.
7. $\frac{9}{7}$.
8. 1·68.
10. $\frac{2\sqrt{3}}{3}$ or 1·15.
11. 4·38 in.
12. 225,000,000 metres per sec.
13. 1·2 in.
15. Front thickness $3\frac{3}{4}$ cm., side thickness 3 cm.
16. 32·2 mm. from surface.
18. 1·50.
19. 1·495.
20. 1·336. Water.
22. Equal.
24. 1·73.
27. Relative refractive index $= \frac{9}{8}$. Angle of refraction $= 34\frac{1}{2}°$.

EXAMPLES V. (page 168).

9. ·6°.
10. 1·41.
12. 1·5.
13. $- 83\frac{1}{3}$ cm.
15. $(3 \pm \sqrt{3})$ ft. from wall.
16. Real; 5 ft. from wall; 5 in. long.
17. If image is real, $f = -8$ cm.
 „ „ virtual, $f = -13\frac{1}{3}$ cm.
18. Virtual, $\frac{10}{7}$ object; $4\frac{2}{7}$.
19. $- 9$ in.
20. 15 in.; combination acts as a concave lens.
21. $- 12$ cm.
22. 24 cm.
23. $\frac{2}{3}$ in.
25. If it returns through the lens, $2\frac{2}{11}$ in. in front of lens. If it does not return through the lens, 10 in. in front of plane mirror.
26. 15 or 10 cm. from mirror.
27. 9 in. from the eye. Half linear dimension of object.
29. 4 in.
32. (*a*) $12\frac{6}{7}$ in. behind, real, inverted, $\frac{6}{7}$ in. high; (*b*) 20 in. behind, real, inverted, 3 in. high; (*c*) $22\frac{1}{2}$ in. behind, real inverted, $3\frac{3}{4}$ in. high; (*d*) 40 in. in front, virtual, erect, 15 in. high.
33. (*a*) $7\frac{1}{2}$ in., $\frac{1}{2}$ in. high; (*b*) $6\frac{6}{25}$ in., $\frac{16}{25}$ in. high; (*c*) 6 in., 1 in. high; (*d*) $4\frac{4}{17}$ in., $1\frac{9}{17}$ in. high, virtual and erect in each case.
34. 12 in. from lens on same side as object; twice as large.
37. 8 ft.

Examination Questions (page 174).

3. 25 : 1.

5. – 5 in.

6. 5 ft. from the bright point.

9. The images formed are 6 in. and $3\frac{1}{2}$ ft. below the surface of the water.

11. 3/4.

15. Its brightness will be diminished.

17. 18 in., to obtain a *real* image with a convex lens; also 6 in., to obtain a *virtual* image with a convex lens.

19. Convex.

20. 9/8.

23. First image 15 in. from first lens; diameter $1\frac{1}{2}$ in. Second image $19\frac{1}{11}$ in. from second lens; diameter $\frac{4}{11}$ in.

26. 4 in.

28. See Ex. V. 5.

31. 1·48.

33. 2 ft. from lens on the same side as the object; 6 in. high.

34. (1) Ends 36 and $16\frac{7}{8}$ in. from lens. Length $19\frac{1}{8}$ in.
(*b*) 20 in. from lens. Length 16 in.

35. The distance between the lenses must be equal to the algebraical sum of the focal lengths.

36. 5 cms. behind the sphere.

37. Angle just smaller than 90°.

38. Observer looks up through a cone whose semi-vertical angle is equal to the critical angle, *viz.* $48\frac{1}{2}$°.

When in the submarine the circle of light extends infinitely and the observer can look out at all angles.

40. 1·732. If angle of prism is greater than the critical angle it is not possible.

Examples VI. (page 193).

1. – 20 cm.

2. – 10·02 cm.

3. 20·84 cm.

4. 99·1 cm.

5. 100 cm.

6. 10·7 cm., 1·53.

7. – 112·1 cm., 121·9 cm., 1·54.

8. – 35·3 cm.; 11·9 cm., 35·2 cm.; 1·51.

9. 1·331.

10. 1·628.

EXAMINATION QUESTIONS (page 195).

3. Diameter of image = ·63 in. Brightness $\propto \left(\dfrac{\text{diameter}}{\text{focal length}}\right)^2$.

6. $-$ 11 in.

7. On the same side as object, and 12 in. from centre.

9. Convex; 75 cm.

10. $2\frac{1}{4}$ in. or $4\frac{1}{2}$ in., according as image is real or virtual.

12. 6 in.

13. Before proceeding to draw the curve draw up a table as shown. In it, d_1, d_2, f_1 are the distances of object and image from the lens, and the focal length respectively. No signs are prefixed in the table, but the position of the object and image whether in front or behind the lens is given.

	d_1	d_2	
In front	∞	f_1	Behind
In front	$< \infty > 2f_1$	$> f_1 < 2f_1$	Behind
In front	$2f_1$	$2f_1$	Behind
In front	$< 2f_1 > f_1$	$> 2f_1 < \infty$	Behind
In front	f_1	∞	Behind
In front	$< f_1 > \frac{f_1}{2}$	$< \infty > f_1$	In front
In front	$\frac{f_1}{2}$	f_1	In front
	0	0	
Behind	$\frac{f_1}{2}$	$\frac{f_1}{3}$	Behind
Behind	f_1	$\frac{f_1}{2}$	Behind
Behind	∞	f_1	Behind

The curve is similar to that obtained in Ex. III. 7.

17. Either lens by itself will throw an inverted and magnified image on the screen if $u > f_1$ but $< 2f_1$.

Both lenses must be used to get an erect and magnified image.

18. Use the magnification method; or, failing that, the displacement method.

EXAMPLES VII. (page 234).

9. Angular radius = 51° 10′.

Examination Questions (page 235).

1. 1·5°.

10. The colours on the screen are—going from one side to the other—white, blue, black, yellow, white.

11. Bunsen's photometer may be used. 2·56 per cent.

12. $\frac{1}{1024}$. No!

13. (*b*) No.

Examination Questions (page 253).

(1) Least interval between successive eclipses is 40 hrs. minus 13·9 secs; greatest interval 40 hrs. plus 13·9 secs.

(5) 1,000 revolutions per second.

Examples VIII. (page 281).

3. 4.
4. $6\frac{2}{3}$ in., $4\frac{4}{11}$ in., 4 in., concave lenses.
5. Convex; 20 in. focal length.
6. Convex; $6\frac{2}{13}$ in., $126\frac{3}{4}$ in. square.
8. 2·4 in.
9. An equilateral triangle.
10. Nearer. The rays converge to a point behind the retina.

Examination Questions (page 283).

1. Concave; $5\frac{1}{7}$ in. focal length.
3. 17.
4. Concave; $6\frac{2}{3}$ in. focal length.
5. Concave; focal length 20 cm., dioptric power $-$ 5.
6. Concave; $9\frac{3}{8}$ in. focal length, dioptric power $-$ $4\frac{1}{3}$.
7. $1\frac{3}{8}$ in. in front of lens.
8. Concave; 2 ft. focal length.

Examples IX. (page 325).

5. 138

7. $F = \dfrac{f}{4}$.

10. $66\frac{2}{3}$ cm.

12. Focal lengths: convex, $-37\frac{1}{2}$ cm.; concave, $+60$ cm.
Radii of curvature: convex lens 37·5 cm., back surface of concave lens $+975$ cm.

13. See Art. 95, II. 2. $\dfrac{1}{c} = \dfrac{1}{R} - \dfrac{1}{f} = -\dfrac{1}{30} - \left(-\dfrac{1}{30}\right) = 0$

$\therefore c = \infty$;

or Ex. IX. 1. $l = s \quad \therefore f = \infty$.

14. 228.

15. $\frac{13}{28}$ in.

16. 2701 : 20730 : 1.

17. Front surface of lens convex, telescope longer when in water front surface concave, telescope shorter.

18. CS_2, $\omega = 0{\cdot}105$. H_2O, $\omega = 0{\cdot}033$. Ratio $= 3{\cdot}16$.

19. 15 cm. measured onwards from centre.

Examination Questions (page 331).

1. The images will be superposed on each other. The colour will change as the observer moves transversely to the line joining him to the lenses.

6. For convex lens of crown glass. $f = -21\frac{1}{3}$, $r = -18\frac{23}{33}$ cm.; for concave lens of crown glass $f = 45\frac{5}{7}$ cm., $s = 282\frac{6}{17}$ cm.

7. See Art. 158.

8. See Art. 158.

11. 4·5 mm. measured to the front from the centre (diverging lens).

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